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Properties of codimension-2 braneworlds in six-dimensional Lovelock theory

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Abstract. We consider maximally symmetric 3-branes embedded in a six-dimensional bulk spacetime with Lovelock dynamics. We study the properties of the solutions with respect to their induced curvature, their vacuum energy and their effective compactness in the extra dimensions. Some simple solutions are shown to give rise to self-accelerating braneworlds, whereas several others solutions have self-tuning properties. For the case of geometric self-acceleration we argue that the cross-over scale in between four-dimensional and higher-dimensional gravity and the scale of late-time geometric acceleration, fixed by the present horizon size, are related via the conical deficit angle of the six-dimensional bulk solution, which is a free parameter.

1. Introduction

Cosmological and astrophysical data indicate that more than two thirds of the content of the Universe is of the form of dark energy. The best fit to the data indicates that this component has the form of a small cosmological constant. This results to a significant theoretical problem since its natural value, from the point of view of Quantum Field Theory, is of the order of the ultraviolet cutoff we would impose for our quantum field theory (anything from SUSY breaking scale to the Planck scale). In addition, one has to understand why this cosmological constant is of the order of the dark matter energy density now, taking into account the completely different cosmological evolution of these components of the Universe.

The above, of course, are correct statements given that we have a homogeneous Universe described by the Einstein's field equations. Taking into account the above difficulties, modifying gravity in the infrared is a legitimate theoretical hypothesis that should be taken seriously. In the context of braneworld models, there have been some interesting ideas. Firstly, it was shown that five-dimensional models with an induced gravity term on the brane (DGP models) can have a *self-accelerating* phase [2], where the current acceleration of the Universe is not due to a compontent of the Universe energy density, but rather a geometrical effect. An even more ambitious proposal, that of the *self-tuning* [3, 4], was to find models where the vacuum energy of the brane can be large without affecting the curvature of the brane and without fine-tuning of it with other brane or bulk parameters. Both proposals change the way that we see the interplay between vacuum energy and curvature in gravity. There were not, however, without problems. Self-accelerating braneworlds were typically infested by ghosts [5] and self-tuning ones had hidden fine-tunings or curvature singularities [6].

In our recent work [1], we examined a completely novel possibility of obtaining acceleration due to geometry as well as certain self-tuning properties. The modified gravity theory that we will study is Lovelock theory [7] in six dimensions, which is the natural extension of General Relativity in higher dimensions. The Lovelock theory in six dimensions has, in addition to the Einstein-Hilbert term, the Gauss-Bonnet combination (for a recent review on Lovelock theory see [8]). Although the latter is a topological invariant in four dimensions, it becomes dynamical for higher dimensions and modifies the gravitational theory. This theory is special because, on the one hand it provides geometric novel solutions which are absent in Einstein theory, and on the other hand, it gives an induced gravity term on the brane. In the context of this theory, we found examples of both self-accelerating and self-tuning cases. These examples open new possibilities for consistent self-acceleration and effective self-tuning which need to be considered in more detail in the future.

2. Lovelock braneworlds

Let us consider the six-dimensional dynamics of Lovelock gravity with a bare cosmological constant Λ and a Gauss-Bonnet term. The action of the system reads

$$S = \int d^6x\,\sqrt{-g}\left[\frac{1}{16\pi G_6}(R + \hat{\alpha}\mathcal{L}_{GB}) - 2\Lambda\right] \ , \tag{1}$$

where

$$\mathcal{L}_{GB} = R_{MNK\Lambda}R^{MNK\Lambda} - 4R_{MN}R^{MN} + R^2 \ , \tag{2}$$

is the Gauss-Bonnet Langrangian density, G_6 the six-dimensional Newton's constant and $\hat{\alpha}$ the Gauss-Bonnet coupling.

The procedure we use to generate brane world solutions, is to doubly Wick rotate black hole solutions (first studied by Boulware and Deser [9], see also [10]). In this procedure, the positions of the horizons r_h will be the endpoints of the internal space where there codimension-2 branes are in principle located. The brane tension is in fact related to the temperature of the black hole horizon, and a warped two brane setup will correspond to black hole solutions with double horizons. In fact, via a generalised version of Birkhoff's theorem [11], it can be shown that the most general axisymmetric solutions of (1) with maximally symmetric four-dimensional subspaces are

$$ds^2 = V(r)d\theta^2 + \frac{dr^2}{V(r)} + r^2 h^{(\kappa)}_{\mu\nu}dx^\mu dx^\nu \ , \tag{3}$$

where the potential is given by

$$V(r) = \kappa + \frac{r^2}{2\alpha}\left[1 + \epsilon\sqrt{1 + 4\alpha\left(a^2 - \frac{\epsilon\mu}{r^5}\right)}\right] \ , \tag{4}$$

with $\alpha = 6\hat{\alpha}$, $16\pi G_6 \Lambda = 20a^2$ the positive cosmological constant[1]. The four-dimensional metric brane metric $h^{(\kappa)}_{\mu\nu}$ is parametrised by $\kappa = 0, -1, 1$, for four-dimensional Minkowski, AdS_4 and dS_4 respectively with curvature $R[h] = 12\kappa$. Finally, $\epsilon = \pm 1$, giving rise to two distinct branches of solutions. The branch $\epsilon = +1$ (*Gauss-Bonnet branch*) does not have an Einstein theory limit as $\alpha \to 0$ (more recently the vacuum in this branch was shown to be unstable [12]). This limit is regular for the other branch with $\epsilon = -1$ (*Einstein branch*). The case where $1 + 4\alpha a^2 = 0$ is special, because the theory can be written in a Born-Infeld (BI) form [13]. It does not have an Einstein theory limit, however, this is the only case that we have a unique vacuum.

Defining the Gaussian Normal radial coordinate $\rho = \sqrt{4(r - r_h)/V'_{r_h}}$ and expanding around e.g. one root r_h of V, we get that the internal space is locally conical

$$ds_2^2 \approx \left(\frac{1}{4}V'^2_{r_h}\right)\rho^2 d\theta^2 + d\rho^2 \ . \tag{5}$$

If the angular coordinate has periodicity $\theta \in [0, 2\pi c)$, then the deficit angle which is induced at the brane position is $\delta = 2\pi(1-\beta)$ with $\beta = \frac{1}{2}|V'_h|c$. Note that the conical deficit is related as usual to the temperature of the Wick rotated black hole horizon. From the Lovelock equations supplemented by a brane tension term, one can separate the distributional Dirac parts and write down induced Einstein equations for the brane. These brane junction conditions are [14]

$$2\pi(1-\beta)\left(-\gamma_{\mu\nu} + 4\hat{\alpha}G^{ind}_{\mu\nu}\right) = 8\pi G_6 T^{brane}_{\mu\nu} \ , \tag{6}$$

where $\gamma_{\mu\nu} = r_h^2 h^{(\kappa)}_{\mu\nu}$ is the induced metric on the brane with curvature $R[\gamma] = 12\kappa/r_h^2 \equiv 12\kappa H^2$, and $G^{ind}_{\mu\nu} = -3\kappa H^2 \gamma_{\mu\nu}$ is the induced Einstein tensor. The brane position $V(r_h) = 0$ depends on the bulk parameters via (4). The induced Newton's constant on the brane can be determined from (6) to be

$$G_4 = \frac{3G_6}{4\pi\alpha(1-\beta)} \ . \tag{7}$$

Note that in order to have positive induced Newton's constant, we should have have angle deficit ($\beta < 1$) for $\alpha > 0$ and angle excess ($\beta > 1$) for $\alpha < 0$.

Substituting the $G^{ind}_{\mu\nu}$ back in (6), we find a relation between the Hubble parameter H on the brane and the action parameters ($T^{brane\ \nu}_{\mu} = -T\delta^\nu_\mu$)

$$\kappa H^2 = -\frac{1}{2\alpha} + \frac{8\pi G_4}{3}T \ . \tag{8}$$

The above equations (7), (8) are very important since they firstly relate the hierarchy in between the scales of the theory and secondly the curvature on the brane H^2 to its sources, namely the brane tension and the Gauss-Bonnet coupling. We see in particular from (8) that the junction conditions tell us that the effective expansion H is in one part due to the Gauss-Bonnet induced cosmological term and in another part due to the vacuum energy of the brane. On the other hand we expect (7) to be indicating a cross-over scale in between a four-dimensional gravity phase and a six-dimensional one, $r_c^2 = G_6/G_4$. Unlike the codimension-1 DGP model note the appearance of two scales α and β dictating the size of r_c.

[1] Here we have omitted the bulk charge parameter for simplicity.

3. Self-properties of the solutions

Let us now discuss the physical consequences of the above solutions. In particular, we wish to see whether we can obtain codimension-2 braneworlds exhibiting *self-accelerating* or *self-tuning* behaviour. The key relation for picking these solutions is (8).

3.1. Self-acceleration

For self-acceleration we need dS_4 vacua ($\kappa = 1$), where one has a positive geometrical contribution to the curvature, i.e. $\alpha < 0$ coming from the Gauss-Bonnet term in the action (1). In addition, this contribution should be dominant in comparison to the brane tension contribution (in other words the T-term in (8) should be negligible). There are two cases where this can happen.

First, if we have no bulk cosmological constant $a^2 = 0$ and we are in the Einstein branch $\epsilon = -1$. Then the internal space is non-compact and the brane position is bounded from below as $r_h(\mu) > \sqrt{2|\alpha|}$. The limit $r_h(\mu_s) = \sqrt{2|\alpha|}$, with $\mu_s \equiv \sqrt{2}|\alpha|^{3/2}$, is singular since it corresponds to the branch cut singularity of the square root of (4). The solution is self-accelerating when μ is in the neighborhood of μ_s. The bulk solution is simply a Wick rotation of the six-dimensional Schwarzschild black hole.

Second, if the mass parameter vanishes $\mu = 0$ and we are in the neighborhood of the BI point $a^2_{BI} = 1/(4|\alpha|)$ (in both branches $\epsilon = \pm 1$). Then, the brane position is a function of the bulk cosmological constant $r_h = r_h(a^2)$ and at the BI limit $r_h(a^2_{BI}) = \sqrt{2|\alpha|}$. In these cases the internal spaces are compact and since the space has no singularity at $r = 0$, we can extend the radial coordinate to $r < 0$ and consider the region of $-r_h \leq r \leq r_h$. The internal space is symmetric around $r = 0$, thus we have Z_2 symmetry around the equator of the internal space.

In both the above cases in order for the geometrical acceleration to account for the current acceleration of the Universe, the Gauss-Bonnet coupling appearing in the 6 dimensional action should be enormous, roughly of the order $\alpha \sim 10^{120} M_{Pl}^{-2}$. This hierarchy means that the bulk gravity is essentially dictated by the higher order Lovelock term that in turn gives ordinary four-dimensional gravity on the brane according to (6). The hierarchy in between G_4 and G_6 maybe reduced by sufficiently fine-tuning β to be close to 1^-. In fact combining (7) and (8), for $T \approx 0$, we have that,

$$\delta \sim \frac{3r_c^2}{2r_0^2} \tag{9}$$

where $r_0 = H_0^{-1}$ is the horizon size and δ the angular excess, $\delta < 0$. Hence the crossover scale, r_c, is now a combination of a purely topological number δ and the horizon size r_0, unlike the situation in codimension-1 DGP where the two scales are the same. We should emphasize however, that strictly speaking the cross-over scale should be obtained from the brane propagator of the above bulk solution (3) and is not necessarilly the scale appearing in the junction condition. Clearly this requires further investigation.

3.2. Self-tuning

For self-tuning to operate, one should be able to absorb variations of the brane tension in integration constants like c and μ, with the crucial demand of keeping the curvature of that brane as well as other bulk or brane parameters constant. We will be obviously interested in dS_4 ($\kappa = 1$) or flat ($\kappa = 0$) vacua. It turns out that if there is more than one brane present in the compactification, there are unavoidable fine-tunings in the model. Thus, the only possible self-tuning vacua can be the ones with only one brane present, or when an extra mirror brane is present, as in the Z_2-symmetric model that we mentioned in the previous subsection. A supplementary requirement for the self-tuning to be satisfying, is that the scales of H^2 and T should be dissociated. For the latter to happen, one should have $|\alpha| H^2 \ll 1$ (in other words the H-term in (8) should be negligible). In this case we can see from (8) and (7) that changes to T can be absorbed in c. Moreover, α can have much more natural values, i.e. $|\alpha| \sim M_{Pl}^{-2}$, than the ones for the previous self-accelerating vacua. In all these cases, since c should follow the variation of T, we will obtain a vacuum-energy-dependent Newton's constant G_4 as seen from (7). There are several examples where this can happen.

First, all the non-compact flat vacua, for $\alpha > 0$ or $\alpha < 0$, and with or without bulk cosmological constant, satisfy the above requirements. In this cases, we have exact self-tuning solutions.

For the dS_4 vacua, there are many possibilities with non-compact or compact internal space. A non-compact example is when we have no bulk cosmological constant $a^2 = 0$, we are in the Einstein branch $\epsilon = -1$, for both $\alpha > 0$ and $\alpha < 0$, and for large enough mass $\mu |\alpha|^{-3/2} \gg 1$. On the other hand, the only compact (Z_2-symmetric) case is when the mass parameter vanishes $\mu = 0$, we are in the Einstein branch $\epsilon = -1$, for $\alpha < 0$ and for small enough bulk cosmological constant $|\alpha| a^2 \ll 1$.

4. Discussion and Conclusions

We have obtained several self-accelerating and self-tuning solutions in a Lovelock six-dimensional theory with codimension-2 branes. The new setting of these models may help overcome the obstacles of the previous self-accelerating and self-tuning models in the literature.

An important property of the above models is that the codimension-2 junction conditions have an induced Einstein tensor term. Therefore, one should expect that the theory (even for non-compact or very large volume models) behaves in a four-dimensional way upto some cross-over scale which we have argued to be $r_c^2 = G_6/G_4$. We should emphasise that a characteristic element of the present setting is that this cross-over scale is in principle different from the scale of self-acceleration r_0 because the deficit angle δ which comes also into play. In principle the scale r_c could be much lower than the horizon size r_0 thus reducing the hierarchy between G_6 and G_4. How low we can set the crossover scale r_c, compared to r_0 depends on the modified higher dimensional gravity and how much it will modify large scale cosmological observables.

What is, therefore, the next step in the analysis of the codimension-2 brane models presented here, is the linear perturbation of these solutions as well as the cosmology analysis. The former, will tell us of the stability and the precise gravitational spectrum. For the latter, although one expects that the introduction of matter on the brane introduces singularities which need to be regularised, the fact that there exists an induced Einstein equation on the brane, may allow for the cosmology to be studied without the need of an explicit regularisation. Furthermore, knowledge of the modified Friedmann equations on the brane will tell us a lot more on the cosmological scales that we have hinted upon here and on the validity of codimension-2 braneworlds.

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