Poly-instanton Inflation

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Abstract. We propose a new inflationary scenario in type IIB Calabi-Yau compactifications, where the inflaton is a Kähler modulus parameterising the volume of an internal four-cycle. The inflaton potential is generated via poly-instanton corrections to the superpotential which give rise to a naturally flat direction due to their double exponential suppression. Given that the volume mode is kept stable during inflation, all the inflaton-dependent higher dimensional operators are suppressed. Moreover, string loop effects can be shown to be negligible throughout all the inflationary dynamics for natural values of the underlying parameters. The model is characterised by a reheating temperature of the order $T_{\rm rh} \simeq 10^6$ GeV which requires $N_e \simeq 54$ e-foldings of inflation. All the inflationary observables are compatible with current observations since the spectral index is $n_s \simeq 0.96$, while the tensor-to-scalar ratio is $r \simeq 10^{-5}$. The volume of the Calabi-Yau is of order 10^3 in string units, corresponding to an inflationary scale around 10^{15} GeV.

Contents

1 Introduction

It is now 30 years since cosmic inflation was proposed to solve outstanding problems of the standard cosmological model by postulating a period of exponential expansion in the early Universe [1–3].

Since the proposal of the original ideas, a plethora of inflationary models was put forward, among which we find the class of slow-roll inflationary models. At their heart is a slowly evolving field in a nearly flat potential that generates a quasi-de Sitter phase in the early history of the Universe. Due to its simplicity, the slow roll mechanism is among the preferred ways to generate an inflationary epoch. While as a class of models, slow roll provides a successful and elegant realisation of inflation, pinning down the details of the mechanism is a challenging question. In particular, identifying the inflaton field, and its connections with a fundamental theory of particle physics, are still questions open to debate. The best that can be done is to propose and analyse models that are compatible with current observational and experimental bounds, and compute their signatures so that they can be tested by forthcoming observations.

Moreover, the very shallow inflaton potential yields a scalar mass lighter than the Hubble scale H, $m_\phi \ll H$. However, as in the Higgs case, it is notoriously hard to keep scalars light by preventing them from getting large contributions when integrating out heavy ultra-violet physics. This is the famous 'η-problem' [4] whose solution is crucial in order to trust any inflationary scenario. Due to this ultra-violet sensitivity of inflation, it is possible to find a robust solution to the η-problem only by embedding models in an ultra-violet complete theory.

For these reasons, a promising avenue to embed inflation into a model of particle physics is string theory (for reviews see [5, 6]). A key feature of string theory is the need for extra spacetime dimensions whose geometry is parameterised by scalar moduli fields. It is the vacuum structure of the moduli potential that determines the masses and couplings of the low-energy effective field theory. Over the past decade, significant progress has been made

towards the understanding of the moduli potential [7, 8], allowing for a promising contact between string theory and particle phenomenology. The progress in moduli stabilisation also opened up the possibility of realising inflation in the moduli sector. String inflationary models based on single-field slow-roll can be broadly classified, based on the origin of the inflaton field [9], into open string models [10–12] and closed string models [13–20].

For closed string inflation there is a direct connection between the physics that stabilises the extra dimensions and inflation. Of particular interest for string phenomenology and inflationary applications is the LARGE Volume Scenario (LVS) of type IIB string theory [8] that through a combination of perturbative and non-perturbative effects allows for solutions with exponentially large volumes. In the context of the LVS, inflation can be driven by Kähler moduli rolling towards their minima.

As recently reviewed in [6], these models provide an interesting solution to the η-problem which does not rely on an axionic shift symmetry [21]. The two reasons why these models can evade the η-problem can be summarised in the following way:

1. The characteristic no-scale structure of the Kähler potential of type IIB supergravity is broken at leading order by α' effects which develop a potential *only* for the over-all volume mode. Thus all the other $(h^{1,1} - 1)$ directions in the Kähler moduli space orthogonal to the volume are flat at leading order, and so constitute natural inflaton candidates. The 'extended no-scale structure' guarantees that string loop effects generate only a subleading potential lifting some of the remaining flat directions [22]. Hence any possible direction orthogonal to the volume is a good inflaton candidate.

2. The tree-level Kähler potential K itself depends *only* on the overall volume. Therefore, if the inflaton is a combination of the Kähler moduli orthogonal to the volume mode (or, equivalently, if inflation takes place with the volume kept stable), no higher order inflaton-dependent operator gets generated by expanding the prefactor e^K of the F-terms scalar potential. Given that g_s corrections to K generically induce a dependence on all the Kähler moduli, inflaton-dependent higher order operators get indeed generated by expanding e^K once we consider the g_s-corrected Kähler potential. However, again due to the 'extended no-scale structure', these operators are suppressed with respect to the leading inflationary dynamics.

Two possible effects to develop a potential for the inflaton field at subleading order have already been proposed. In the first case, Kähler moduli inflation [14], the inflaton is a blow-up mode and its potential is generated by ordinary non-perturbative corrections to the superpotential, while in the second case, fibre inflation [15], the inflaton is a K3 divisor which develops a potential via perturbative string loop effects [1]. In fibre inflation a long enough period of inflation can be achieved rather naturally. On the other hand, in Kähler moduli inflation, a sufficiently long period of inflation can be driven by the non-perturbative potential only by fine-tuning the coefficients of the loop corrections, as pointed out in [15]. Notice that these g_s corrections would spoil inflation in the region rather close to the minimum where the non-perturbative potential would give rise to slow-roll. On the other hand, in regions further away from the minimum, due to the 'extended no-scale structure', these perturbative loop corrections would actually drive inflation in a way very similar to [15].

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[1]The authors of [19] tried to obtain inflation driven by the volume mode in order to solve the tension between inflation and TeV-scale SUSY [23]. However, for the reasons explained above, this model suffers from the η-problem, and so its realisation needs severe fine-tuning.

In this paper we shall provide the first model where the whole inflationary dynamics is driven entirely by non-perturbative effects since perturbative loop corrections can be shown to be negligible for natural values of the underlying parameters. The new key-ingredient is the use of poly-instanton corrections to the superpotential which are instanton corrections to the action of another instanton derived in type I compactifications [24]. The type IIB origin of these poly-instanton corrections has not been worked out in detail yet. However, as argued in [25], the poly-instantons might get generated either for a fluxed instanton on a K3 four-cycle or for an instanton on a T^4 divisor with deformations fixed by the background fluxes. We shall therefore focus on Calabi-Yau three-folds which admit a K3 or T^4 fibration over a $\mathbb{P}^1$ base similarly to the toric constructions of [26].

Here the inflaton is the volume of the fibre which supports the poly-instanton effects. The different topological origin of the inflaton between our model and the one developed in [14] results in a different canonically normalised inflaton field. This leads to a potential which in our case gives rise to slow-roll in a region much closer to the minimum than in the model of [14] even if in both cases the inflationary potential is generated by non-perturbative effects. This is the reason why in our model string loop corrections are less dangerous and can be shown to be negligible throughout all the inflationary dynamics for rather natural values of the underlying parameters.

The predictions of the model are rather independent on the choice of the microscopic parameters, as these influence mainly the scale of inflation but have little effect on the shape of the potential. Our model is characterised by a reheating temperature of the order $T_{\rm rh} \simeq 10^6$ GeV which requires $N_e \simeq 54$ e-foldings of inflation. The requirement of generating the correct amount of density perturbations fixes the Calabi-Yau volume of the order 10^3 in string units. This, in turn, leads to a high inflationary scale, $M_{\rm inf} \simeq 10^{15}$ GeV, corresponding to a small tensor-to-scalar ratio, $r \simeq 10^{-5}$, and a sub-Planckian motion of the inflaton field $\Delta\phi \simeq 0.5 M_p$. We also point out that the spectral index, $n_s \simeq 0.96$, is within the observationally allowed window.

This paper is organised as follows: we start in section 2 by reviewing the LARGE Volume Scenario of type IIB string theory with particular emphasis on fibred constructions. In section 3 we first show how the poly-instanton corrections to the superpotential can generate a potential for the fibre modulus, and then we study its inflationary applications both analytically and numerically. In the same section we also present a detailed discussion of the η-problem stressing the similarities and the differences of our model with the previous ones developed in [14] and [15], where the inflaton was still a Kähler modulus. We finally conclude in section 4.

2   Basics of the LARGE Volume Scenario

In this section we review the basic properties of the LVS, to be used in what follows for cosmological purposes.

2.1   4D effective action

The low-energy 4D $\mathcal{N} = 1$ supergravity, obtained by compactifying type IIB string theory on a orientifold of a Calabi-Yau three-fold $\mathcal{M}$, is characterised by a plenitude of scalar fields that parameterise the geometry of $\mathcal{M}$. These include the axio-dilaton S, the complex structure moduli U_a, $a = 1, ..., h^{1,2}(\mathcal{M})$ and the Kähler moduli T_i, $i = 1, ..., h^{1,1}(\mathcal{M})$. Due to the

no-scale structure of the tree-level Kähler potential for the Kähler moduli [28]:

$$K_0 = -2 \ln \mathcal{V}, \tag{2.1}$$

and the fact that the semiclassical superpotential W_0, generated by turning on non-zero background fluxes [29, 30], does not depend on the T-moduli, only the S and U-moduli are fixed supersymmetrically at tree-level by imposing vanishing F-term conditions. In fact, for a constant superpotential W_0, the F-term scalar potential for the Kähler moduli:

$$V = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2), \tag{2.2}$$

identically vanishes due to the no-scale structure relation following from (2.1):

$$K_0^{i\bar{j}} \partial_i K_0 \partial_{\bar{j}} K_0 = 3. \tag{2.3}$$

The dependence of the Calabi-Yau volume $\mathcal{V}$ on the Kähler moduli can be obtained by recalling its definition in Einstein frame:

$$\mathcal{V} = \frac{1}{6} k_{ijk} t^i t^j t^k, \tag{2.4}$$

where k_{ijk} are triple intersection numbers and t^i two-cycle volumes. The fields appearing in the 4D effective theory are the complexified Kähler moduli $T_i \equiv \tau_i + i b_i$ where b_i are axionic components while τ_i are four-cycle volumes, given by:

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t^i} = \frac{1}{2} k_{ijk} t^j t^k. \tag{2.5}$$

Inverting (2.5), and then inserting this result back in (2.4), one can work out the dependence of the volume on the T-moduli. As we shall discuss later on, the no-scale structure of the system (2.3) has important positive consequences for building models of inflation.

In order to have a phenomenologically viable model it is essential to stabilise all the moduli. Since the leading order terms in the Kähler potential and superpotential do not generate a minimum for the Kähler moduli, one must go one step further and see how the inclusion of subleading terms in K and W changes the scenario. These higher order corrections can be classified into perturbative and non-perturbative.

Since W is protected by supersymmetric non-renormalisation theorems it is immune from perturbative corrections: therefore these can only occur in the Kähler potential. The no-scale structure can be broken both by α' and g_s effects. The leading order $\mathcal{O}(\alpha'^3)$ correction to the Kähler potential has been computed in [31] and takes the form:

$$K = -2 \ln \left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}} \right), \tag{2.6}$$

where $\xi = -\frac{\zeta(3)\chi(\mathcal{M})}{2(2\pi)^3}$ with $\chi(\mathcal{M}) = 2 \left(h^{1,1} - h^{2,1} \right)$. String loop effects also break the no-scale structure by adding new contributions to K [22, 33].

The inclusion of non-perturbative corrections to the superpotential provides an alternative way of breaking the tree-level no-scale structure [7]. This is achieved by considering contributions to W coming from gaugino condensation on D7-branes or Euclidean D3-brane instantons:

$$W = W_0 + \sum_i A_i e^{-a_i T_i}, \tag{2.7}$$

where A_i and a_i are independent from the Kähler moduli, although they depend on the microscopic details of the model. In what follows we will take them to be constants, with $a_i = 2\pi/n_i$ and n_i natural numbers.

2.2 Swiss-cheese compactifications

In [8] it was shown that the combination of $\mathcal{O}(\alpha'^3)$ corrections to K and non-perturbative effects in W can generate an AdS minimum for the Kähler moduli at exponentially large volumes for geometries in which the volume can be written as:

$$\mathcal{V} = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{h^{1,1}} \gamma_i \tau_i^{3/2} \right). \tag{2.8}$$

These are the so-called Swiss-cheese geometries in which one Kähler modulus controls the overall volume while the remaining $(h^{1,1} - 1)$ Kähler moduli are the volumes of diagonal del Pezzo divisors. The scalar potential for these geometries behaves as:

$$V = \sum_i \frac{8(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3\mathcal{V}\alpha\gamma_i} - \sum_i \frac{4a_i A_i W_0 \tau_i e^{-a_i \tau_i}}{\mathcal{V}^2} + \frac{3\xi W_0^2}{4g_s^{3/2}\mathcal{V}^3}. \tag{2.9}$$

The minus sign in front of the second term is crucial for the existence of the minimum and comes from the minimisation of the axionic potential. Furthermore one must require $\xi > 0$ to have a minimum for the potential where the blow-up cycles are fixed at leading order at:

$$\langle \tau_i \rangle \simeq \frac{1}{a_i g_s} \left(\frac{\xi}{2\alpha\gamma_i \sum_j \gamma_i / a_i^{3/2}} \right)^{2/3}, \tag{2.10}$$

while the volume is stabilised at:

$$\langle \mathcal{V} \rangle \simeq \frac{3\gamma_i \alpha W_0}{2a_i A_i} \left(\frac{1 - a_i \tau_i}{1/2 - 2a_i \tau_i} \right) \sqrt{\tau_i}\, e^{a_i \tau_i}. \tag{2.11}$$

Notice that the combination of α' corrections to K and non-perturbative effects in W allows for the generation of a minimum for all the blow-up moduli plus the volume mode. For Swiss-cheese type geometries this is sufficient to fix all the T-moduli [2].

2.3 Fibred compactifications

The existence of an AdS minimum at large volumes for manifolds more general than Swiss-cheese geometries was studied in [33]. Of particular interest to us are Calabi-Yau manifolds that admit a K3 or T^4 fibration over a $\mathbb{P}^1$ base. These manifolds are characterised by the fact that their volume is linear in the two-cycle giving the volume of the $\mathbb{P}^1$ base. Explicit examples of this kind of Calabi-Yau three-folds with additional del Pezzo divisors have been analysed in [26] using the powerful tools of toric geometry. Here we shall just focus on the simplest of such manifolds whose volume takes the form:

$$\mathcal{V} = \lambda_1 t_1 t_2^2 + \lambda_2 t_3^3, \tag{2.12}$$

where t_1 is the volume of the $\mathbb{P}^1$ base, $\tau_1 = \lambda_1 t_2^2$ is the size of the K3 or T^4 fibre, and $\tau_3 = 3\lambda_2 t_3^2$ controls the volume of a blow-up mode (the other four-cycle volume is given by $\tau_2 = 2\lambda_1 t_1 t_2$). Notice that the fibre is a K3 surface if its Euler characteristic is $\chi = 24$ whereas it is a T^4 divisor if $\chi = 0$ [34]. We shall not specify the value of χ in order to be

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[2]See [32] for an example in type IIA string theory.

as generic as possible since our moduli stabilisation mechanism works in both cases. We can then rewrite the volume in terms of the correct Kähler moduli as:

$$\mathcal{V} = t_1\tau_1 - \alpha\gamma\tau_3^{3/2} = \alpha(\sqrt{\tau_1}\tau_2 - \gamma\tau_3^{3/2})\,, \tag{2.13}$$

where $\alpha = 1/(2\sqrt{\lambda_1})$ and $\gamma = \frac{2}{3}\sqrt{\lambda_1/(3\lambda_2)}$.

Due to the similar structure of the two volume expressions, Eqs. (2.8) and (2.13), an analysis of the scalar potential reveals that an AdS minimum at exponentially large volumes also exists for fibred geometries. There is however one crucial difference: while in the simpler geometries initially studied, with volumes described by Eq. (2.8), all moduli were stabilised, for K3 or T^4-fibered Calabi-Yau manifolds only the blow-up moduli and the volume $\mathcal{V} \sim \sqrt{\tau_1}\tau_2$ directions are fixed. Therefore the leading order stabilisation dynamics leaves one flat direction in moduli space which can be used to provide a stringy realisation of slow roll inflation.

Before proceeding to study the inflationary dynamics, we finally point out that in this discussion we neglected many aspects related to the construction of a globally consistent compactification with the local presence of chiral matter. We shall however assume that the problems that one generically encounters when trying to combine moduli stabilisation with D-brane model building can be cured along the same lines described in [35].

3 Poly-instanton inflation

In this section we introduce poly-instanton corrections to the superpotential. These are the essential ingredient to lift the flat direction left over after the study of the leading stabilisation dynamics. We then show that the resulting potential for the fibre modulus can support slow-roll inflation. We provide both analytic arguments and numerical results for several representative points in moduli space.

3.1 Fibre stabilisation

The existence of poly-instanton corrections was first established within the context of type I compactification in [24]. These are instantons which do not give rise to a single contribution to the superpotential but, due to the presence of extra fermionic zero-modes, they correct the action of another instanton wrapping a different internal cycle. The zero-mode constraints for generating poly-instanton corrections in the type IIB T-dual version has not been worked out in detail yet and it is beyond the scope of this paper. We shall therefore take a phenomenological approach and make the following two assumptions:

1. The field theory living on a stack of D7-branes wrapping τ_3 can be broken into two gauge groups which separately undergo gaugino condensation, giving rise to a race-track superpotential of the form:

$$W = W_0 + A\,e^{-aT_3} - B\,e^{-bT_3}\,, \tag{3.1}$$

where A and B are threshold effects, while $a = 2\pi/n_a$ and $b = 2\pi/n_b$, with $n_a, n_b \in \mathbb{N}$.

2. On top of these effects, a Euclidean D3-instanton wrapping the fibre τ_1 yields non-perturbative corrections to the gauge kinetic functions of the two condensing gauge groups, resulting in a poly-instanton corrected superpotential which looks like:

$$W = W_0 + A\,e^{-a(T_3+C_1e^{-2\pi T_1})} - B\,e^{-b(T_3+C_2e^{-2\pi T_1})}\,, \tag{3.2}$$

where C_1 and C_2 are free constants.

Notice that we are following [25], where the same assumptions led to stabilise the fibre four-cycle obtaining a very anisotropic compactification characterised by the presence of two micron-sized extra dimensions and strings around the TeV scale. The authors of [25] pointed out that our set-up is very likely to generate poly-instanton corrections to the superpotential since the two extra fermionic zero-modes of the type I computation are Wilson line modulini and these get mapped to Wilson line and deformation modulini in the type IIB picture. Hence, given that a K3 fibre admits deformations while a T^4 surface has Wilson lines, it is not unreasonable to expect that these poly-instanton effects can indeed get generated in our set-up either for a magnetised instanton on K3 or for an ED3 on T^4 with deformations fixed by the same background fluxes turned on to stabilise the dilaton and the complex structure moduli [25].

The scalar potential computed by using Eq. (2.2) can be separated as:

$$V = V_{\mathcal{O}(\mathcal{V}^{-3})} + V_{\mathcal{O}(\mathcal{V}^{-3-p})} \,, \tag{3.3}$$

where the first piece scales with the volume as $\mathcal{V}^{-3}$, while the other as $\mathcal{V}^{-3-p}$, with p a positive parameter to be defined later. After minimising with respect to the two axions b_1 and b_3, these two pieces are:

$$\begin{aligned}
V_{\mathcal{O}(\mathcal{V}^{-3})} = &\frac{8\sqrt{\tau_3}\left(A^2 a^2 e^{-2a\tau_3} + B^2 b^2 e^{-2b\tau_3} - 2AB\,ab\,e^{-(a+b)\tau_3}\right)}{3\mathcal{V}} \\
&+ \frac{4W_0\tau_3\left(Aae^{-a\tau_3} - Bbe^{-b\tau_3}\right)}{\mathcal{V}^2} + \frac{3\xi}{4\,g_s^{3/2}\,\mathcal{V}^3}\,,
\end{aligned} \tag{3.4}$$

and:

$$\begin{aligned}
V_{\mathcal{O}(\mathcal{V}^{-3-p})} = &- e^{-2\pi\tau_1}\left\{ -\frac{16C_1\sqrt{\tau_3}e^{-2b\tau_3}}{3\mathcal{V}}[Z^2 a - ZB(a-b)] \right. \\
&+ \frac{4C_1W_0 e^{-b\tau_3}}{\mathcal{V}^2}[2\pi\,Z\tau_1 + Za\tau_3 - Bb(a-b)\tau_3] \\
&\left. + n\left[\frac{-16Bb^2\sqrt{\tau_3}e^{-2b\tau_3}Z}{3\mathcal{V}} + \frac{4W_0 Bbe^{-b\tau_3}}{\mathcal{V}^2}(b\tau_3 + c\tau_1)\right]\right\}\,.
\end{aligned} \tag{3.5}$$

In Eq. (3.4) we kept only the leading order terms in the volume expansion, in particular we consistently neglected terms proportional to $\mathcal{V}^{-n}$ for $n > 3$ and terms proportional to $e^{-2\pi\tau_1}$. In Eq. (3.5) terms proportional to $e^{-2\pi n\tau_1}$ for $n > 1$ were left out as they are subdominant in the regime where the effective field theory is under control. This is equivalent to expanding the double exponentials of Eq. (3.2) as products of single exponentials. Moreover, following [25], we also introduced new parameters to simplify the expressions:

$$Z \equiv B\,b - A\,a\,e^{-(a-b)\tau_3} \qquad \text{and} \qquad n \equiv C_2 - C_1\,. \tag{3.6}$$

The leading contribution, Eq. (3.4), is the standard LVS potential obtained from a racetrack superpotential. Notice that this term carries no information about the poly-instantons in W. Minimisation with respect to the volume and the blow-up modulus provides a minimum at:

$$\langle\tau_3\rangle^{3/2} = \frac{3\xi}{32\alpha\gamma f_1(1-f_1)g_s^{3/2}}\,, \qquad \langle\mathcal{V}\rangle = f_2\,e^{b\langle\tau_3\rangle}\,, \tag{3.7}$$

where we have defined:

$$f_1 \equiv \frac{(a-b)\,\tau_3 Bb + Z(1 - a\,\tau_3)}{4\,(a-b)\,\tau_3 Bb + Z(1 - 4a\,\tau_3)}\,, \qquad \text{and} \qquad f_2 \equiv \frac{3\alpha\gamma W_0\sqrt{\langle\tau_3\rangle}}{Z}f_1\,. \tag{3.8}$$

We stress that the result of Eq. (3.7) involves no approximations and is therefore exact. In particular the function f_1 generalises the definition of f_{corr} in [25]. The equation that determines the VEV of τ_3 can be solved iteratively, giving a more accurate result than the leading order estimate obtained by setting $f_1 = 1/4$. Notice that the volume is stabilised at exponentially large values, leading to a LVS.

As we have already mentioned, the leading order potential, Eq. (3.4), depends on τ_1 only through $\mathcal{V}$, therefore after stabilising τ_3 and $\mathcal{V}$ at their minima, Eq. (3.7), there is one flat direction left in the (τ_1, τ_2) plane. This flat direction is then lifted by the subleading effects due to poly-instantons encoded in Eq. (3.5). Notice that unlike $V_{\mathcal{O}(\mathcal{V}^{-3})}$ which depends only on τ_3 and $\mathcal{V}$, $V_{\mathcal{O}(\mathcal{V}^{-3-p})}$ depends on all the three directions in moduli space. The approach of using subleading non-perturbative corrections to the superpotential as in [25] provides an alternative for generating a minimum for the fibre modulus, with respect to using string loop contributions to the Kähler potential as originally explored in [33].

Setting the blow-up and volume moduli at their minima, Eq. (3.7), we can rewrite the subleading part of the resulting scalar potential in Eq. (3.5) as:

$$V_{\mathcal{O}(\mathcal{V}^{-3-p})} = -\frac{F_0}{\langle\mathcal{V}\rangle^3}\left(2\pi\,\tau_1 - p\,b\,\langle\tau_3\rangle\right)e^{-2\pi\,\tau_1}\,, \tag{3.9}$$

where $F_0 \equiv 4\,f_2\,W_0\,r_1$ and p is defined in terms of the other parameters as:

$$p \equiv -\frac{r_1}{r_3}\,, \tag{3.10}$$

with:

$$r_1 \equiv C_1 Z + n\,b\,B\,, \tag{3.11}$$

$$r_3 \equiv (1 - 4f_1)\left[\frac{a}{b}r_1 - (a - b)\,B\,(C_1 + n)\right]\,. \tag{3.12}$$

The minimum of $V_{\mathcal{O}(\mathcal{V}^{-3-p})}$ for fixed τ_3 and $\mathcal{V}$ can be shown to lie at:

$$2\pi\,\langle\tau_1\rangle = p\,b\,\langle\tau_3\rangle + 1\,. \tag{3.13}$$

By tuning the parameters in K and W, it is possible to obtain p positive and of order unity so that the minimum (3.13) is within the regime of validity of the effective field theory. As we have already said, ref. [25] obtained a very anisotropic configuration with TeV-scale strings corresponding to volumes of the order $\mathcal{V} \sim 10^{30}$. Here we shall instead be interested in a higher fundamental gravity scale which is suitable for cosmological purposes, and so we shall choose our underlying parameters in order to obtain smaller volumes of the order $\mathcal{V} \sim 10^3$.

3.2 Inflationary potential

Let us now study the inflationary dynamics working in the single field approximation. We shall set the blow-up mode τ_3 and the volume $\mathcal{V}$ at their τ_1-independent VEVs: $\tau_3 = \langle\tau_3\rangle$ and $\mathcal{V} = \langle\mathcal{V}\rangle$, and displace τ_1 far from its minimum. We shall then investigate if the fibre modulus can drive a long enough period of inflation while rolling down its potential. Notice

that the approximation of stable τ_3 and $\mathcal{V}$ during inflation is justified by the mass hierarchy found in [25]:

$$m_{\tau_1} \sim \frac{M_p}{\mathcal{V}^{(3+p)/2}}, \qquad m_{\mathcal{V}} \sim \frac{M_p}{\mathcal{V}^{3/2}}, \qquad m_{\tau_3} \sim \frac{M_p}{\mathcal{V}}, \tag{3.14}$$

when all the moduli sit at their minima. In what follows we shall set $p \simeq \mathcal{O}(1)$, implying that for large volume $m_{\tau_3} \gg m_{\mathcal{V}} \gg m_{\tau_1}$. Moreover, as we shall see later on, the Hubble parameter during inflation scales with the volume as $H \sim m_{\tau_1}$. Hence both τ_3 and $\mathcal{V}$ are heavy during inflation since $m_{\tau_3} \gg m_{\mathcal{V}} \gg H$, justifying our single field approximation. We point out that the actual mass of τ_1 far from its minimum is much smaller than m_{τ_1} due to the rapid exponential suppression of its potential. Hence if we displace this field far from its minimum, it is naturally lighter than H, and can therefore drive inflation.

The effective potential for the inflaton field τ_1 can be written as:

$$V_{\text{inf}} = V_{\text{up}} + V_{\text{fib}}, \tag{3.15}$$

where V_{fib} is the scalar potential (3.9) generated by poly-instanton effects, while V_{up} is the uplift term. Regardless of the nature of the uplifting, we shall assume that it gives rise to a τ_1-independent constant once the volume is fixed. This constant is obtained by requiring $V_{\text{up}} = -V_{\text{fib}}(\langle \tau_1 \rangle)$ since we are tuning the uplifting in order to ensure the vanishing of the scalar potential at leading $\mathcal{V}^{-3}$ order. Notice that V_{up} will slightly modify the position of $\langle \mathcal{V} \rangle$ but we shall neglect this small effect since it only affects the overall inflationary scale.

Let us then rewrite the full inflationary potential as:

$$V_{\text{inf}} = \frac{F_{\text{poly}}}{\langle \mathcal{V} \rangle^{3+p}} \left[1 - (1 + 2\pi\hat{\tau}_1) e^{-2\pi\hat{\tau}_1} \right], \quad \text{with} \quad F_{\text{poly}} = F_0 f_2^p e^{-1}, \tag{3.16}$$

where we defined $\hat{\tau}_1 \equiv \tau_1 - \langle \tau_1 \rangle$ as the shift of the inflaton from its minimum. Notice that the inflationary potential scales with the volume as $1/\mathcal{V}^{3+p}$, and so we can get different values for the scale of inflation by varying the parameter p. This will be important in what follows.

Let us now canonically normalise the inflaton field by recalling that the kinetic Lagrangian is given by:

$$\mathcal{L}_{\text{kin}} = K_{i\bar{j}} \partial_\mu T_i \partial^\mu \bar{T}_j = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_i \partial \tau_j} (\partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu b_i \partial^\mu b_j), \tag{3.17}$$

where in the second equality we rewrote the complexified Kähler moduli in terms of its real and imaginary components ($T_i = \tau_i + ib_i$) and used the fact that the Kähler potential is independent of the axionic fields. The Kähler metric turns out to be:

$$K_{i\bar{j}}^0 = \begin{pmatrix} \frac{1}{4\tau_1^2} & \frac{\tau_3^{3/2}}{4\tau_1^{3/2}\tau_2^2} & -\frac{3\sqrt{\tau_3}}{8\tau_1^{3/2}\tau_2} \\ \frac{\tau_3^{3/2}}{4\tau_1^{3/2}\tau_2^2} & \frac{1}{2\tau_2^2} & -\frac{3\sqrt{\tau_3}}{4\sqrt{\tau_1}\tau_2^2} \\ -\frac{3\sqrt{\tau_3}}{8\tau_1^{3/2}\tau_2} & -\frac{3\sqrt{\tau_3}}{4\sqrt{\tau_1}\tau_2^2} & \frac{3}{8\sqrt{\tau_1}\tau_2\sqrt{\tau_3}} \end{pmatrix}, \tag{3.18}$$

where we kept only the leading order term in the volume expansion for each entry. Treating both the volume and the blow-up cycle as stable during inflation, the kinetic term for the fibre modulus simplifies to:

$$\mathcal{L}_{\text{kin}} = \frac{3}{8\tau_1^2} \partial_\mu \tau_1 \partial^\mu \tau_1. \tag{3.19}$$

It then follows that the canonically normalised field is defined as:

$$\phi \equiv \frac{\sqrt{3}}{2} \ln \tau_1 \qquad \text{or} \qquad \tau_1 \equiv e^{2\phi/\sqrt{3}}. \tag{3.20}$$

These definitions imply that the shift $\hat{\phi}$ of the canonically normalised inflaton from its minimum takes the form:

$$\hat{\tau}_1 = \langle \tau_1 \rangle \left(e^{\frac{2}{\sqrt{3}}\hat{\phi}} - 1 \right), \tag{3.21}$$

and so the full inflationary potential (3.16) can be rewritten as:

$$V_{\text{inf}} = \frac{F_{\text{poly}}}{\langle \mathcal{V} \rangle^{3+p}} \left[1 - e^{-c\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}} - 1 \right)} \left(1 + c \left(e^{\frac{2\hat{\phi}}{\sqrt{3}}} - 1 \right) \right) \right], \tag{3.22}$$

where $c = 2\pi \langle \tau_1 \rangle$. This potential is sketched in Figure 1. The relation (3.21) suggests the following field redefinition:

$$\hat{\psi} \equiv \frac{\sqrt{3}}{2} \left(e^{\frac{2}{\sqrt{3}}\hat{\phi}} - 1 \right) \qquad \Leftrightarrow \qquad \hat{\tau}_1 = \frac{2}{\sqrt{3}} \langle \tau_1 \rangle \, \hat{\psi}, \tag{3.23}$$

which allows us to write (3.22) in a very compact form as:

$$V_{\text{inf}} \simeq \frac{F_{\text{poly}}}{\langle \mathcal{V} \rangle^{3+p}} \left(1 - \kappa \, \hat{\psi} \, e^{-\kappa \hat{\psi}} \right), \tag{3.24}$$

where $\kappa = \frac{2p}{\sqrt{3}} \ln \mathcal{V}$ and we have approximated $c = p \ln \mathcal{V} + 1 - p \ln f_2 \simeq p \ln \mathcal{V}$. We stress that we shall obtain a model of small field inflation where the inflaton travels a sub-Planckian distance in field space during inflation: $\Delta \hat{\phi} \simeq 0.5 M_p$. Therefore we can Taylor expand the exponent in (3.23) finding that the field $\hat{\psi}$ gives the leading order approximation of the canonically normalised inflaton $\hat{\phi}$ since:

$$\hat{\psi} \equiv \frac{\sqrt{3}}{2} \left(e^{\frac{2}{\sqrt{3}}\hat{\phi}} - 1 \right) \simeq \hat{\phi} + \frac{1}{\sqrt{3}} \hat{\phi}^2 + \dots. \tag{3.25}$$

Thus the compact expression (3.24) gives a rather accurate qualitative description of the inflationary dynamics when we just identify $\hat{\psi}$ with $\hat{\phi}$.

The aim of this work is to investigate whether the dynamics of the scalar field $\hat{\phi}$ rolling down the potential (3.22) is suitable for generating a prolonged period of inflation. With this in mind we compute the slow-roll parameters for the canonically normalised fibre modulus:

$$\epsilon = \frac{1}{2V_{\text{inf}}^2} \left(\frac{\partial V_{\text{inf}}}{\partial \phi} \right)^2 \qquad \text{and} \qquad \eta = \frac{1}{V_{\text{inf}}} \frac{\partial^2 V_{\text{inf}}}{\partial \phi^2}. \tag{3.26}$$

The slow-roll parameters turn out to be (for simplicity we express them in terms of $\hat{\psi}$):

$$\epsilon = \frac{2c^2}{3} \frac{\left(\frac{2c}{\sqrt{3}}\hat{\psi} \right)^2 \left(\frac{2}{\sqrt{3}}\hat{\psi} + 1 \right)^2}{\left(e^{\frac{2c}{\sqrt{3}}\hat{\psi}} - 1 - \frac{2c}{\sqrt{3}}\hat{\psi} \right)^2} \simeq \kappa^4 \, e^{-2\kappa\hat{\psi}}, \tag{3.27}$$

and:

$$\eta = -\frac{4c}{3} \frac{\left(\frac{2}{\sqrt{3}}\hat{\psi}+1\right)\left[\left(\frac{2c}{\sqrt{3}}\hat{\psi}\right)^2 + (c-2)\frac{2c}{\sqrt{3}}\hat{\psi} - c\right]}{e^{\frac{2c}{\sqrt{3}}\hat{\psi}} - 1 - \frac{2c}{\sqrt{3}}\hat{\psi}} \simeq -\kappa^3 \, e^{-\kappa\hat{\psi}} \, . \tag{3.28}$$

Both ϵ and η are naturally exponentially small in the region $\kappa\hat{\psi} \gg 1$. Due to the large parameter $\kappa = \frac{2p}{\sqrt{3}}\ln\mathcal{V} \gg 1$, the slow-roll conditions are satisfied for small shifts $\hat{\psi} < 1$. This implies that we are dealing with a model of small field inflation and justifies our leading order approximation $\hat{\psi} \simeq \hat{\phi}$. Moreover our model is characterised by the interesting relation:

$$\epsilon \simeq \left(\frac{\eta}{\kappa}\right)^2 , \tag{3.29}$$

which implies the following hierarchy throughout all the inflationary region:

$$\epsilon \ll |\eta| \ll 1 \, . \tag{3.30}$$

Requiring that $|\eta|$ is at most a few percents leads to values of ϵ within the interval $10^{-7} < \epsilon < 10^{-5}$. Notice that a negative η is a generic feature of our potential showing that the inflating region is tachyonic and therefore unstable. Hence the field $\hat{\phi}$ drives inflation as it slowly rolls towards its minimum.

3.3 Inflationary observables

In order to solve the basic problems of standard Big-Bang cosmology, the inflationary trajectory must also give rise to a sufficient number of e-foldings:

$$N_e \equiv \int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*} \frac{1}{\sqrt{2\epsilon}}d\hat{\phi} \simeq \int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*} \frac{\kappa}{|\eta|}d\hat{\phi} \simeq \frac{1}{\kappa^2}\int_{\hat{\phi}_{\rm end}}^{\hat{\phi}_*} e^{\kappa\hat{\phi}}d\hat{\phi} \simeq \frac{1}{\kappa^3}\left(e^{\kappa\hat{\phi}_*} - e^{\kappa\hat{\phi}_{\rm end}}\right) . \tag{3.31}$$

In our numerical analysis we shall consider the end of inflation as taking place at the point $\hat{\phi}_{\rm end}$ where $\epsilon \simeq 1$. We stress that there is nothing special in this choice since our final results for the cosmological observables are not sensitive to the exact point where inflation ends. From the expression (3.27) we can then see that $\hat{\phi}_{\rm end}$ is, in practice, just a function of κ since it is rather insensitive to the other underlying parameters. In turn, from (3.31), we see that the number of e-foldings N_e is a function of κ and the point of horizon exit $\hat{\phi}_*$.

However, the exact number of e-foldings depends on the inflationary scale $M_{\rm inf} = V_{\rm inf}^{1/4}(\hat{\phi}_{\rm end})$ and the reheating temperature $T_{\rm rh}$. In fact, under the general assumption that inflation is followed first by a matter-dominated reheating epoch, and then by a radiation-dominated epoch with initial temperature $T_{\rm rh}$, we have [37]:

$$N_e \simeq 62 + \ln\left(\frac{M_{\rm inf}}{10^{16}\,{\rm GeV}}\right) - \frac{1}{3}\ln\left(\frac{M_{\rm inf}}{T_{\rm rh}}\right) . \tag{3.32}$$

As can be seen from (3.22), $M_{\rm inf}$ depends in general on $\mathcal{V}$, p and other parameters via the combination $F_{\rm poly}$ of Eq. (3.16). For fixed $F_{\rm poly}$, we can consider $M_{\rm inf}$ as depending on just κ given that the inflationary scale is rather insensitive to the actual value of p which we shall always set $p \simeq \mathcal{O}(1)$. As we shall argue later on, $T_{\rm rh}$ is also a function of κ. Hence by equating (3.31) with (3.32), we obtain an equation in two unknowns: $\hat{\phi}_*$ and κ (or $\mathcal{V}$).

We can find a unique solution for each value of F_{poly} by noticing that the requirement of generating the correct amount of density perturbations gives a second equation in $\hat{\phi}_*$ and κ. In fact, the COBE normalisation can be written in terms of the inflaton potential as [3]:

$$A_{\text{COBE}} \equiv \frac{g_s}{8\pi} \left(\frac{V_{\text{inf}}^{3/2}}{V_{\text{inf}}'} \right)^2 \Bigg|_{\hat{\phi}=\hat{\phi}_*} \simeq 2.7 \cdot 10^{-7} \,. \qquad (3.33)$$

We solved numerically these two equations for $p \simeq \mathcal{O}(1)$ and different values of F_{poly}. We found solutions with an internal volume large enough to trust the effective field theory, $\mathcal{V} \sim 10^3$, corresponding to $\kappa \simeq 8$, for $F_{\text{poly}} \sim \mathcal{O}(10)$ which gives also $\hat{\phi}_* \simeq 0.9$. In turn, these results lead to $\hat{\phi}_{\text{end}} \simeq 0.35$, $N_e \simeq 54$ and $M_{\text{inf}} \simeq M_p/\mathcal{V} \simeq 10^{15}$ GeV. Notice that these numerical results confirm our initial statement that we are dealing with a model of small field inflation since $\Delta\hat{\phi} = \hat{\phi}_* - \hat{\phi}_{\text{end}} \simeq 0.45$ in Planck units.

Since our model has a preference for moderately small volumes it seems hard to simultaneously accommodate GUT-scale inflation and TeV-scale SUSY which would require $\mathcal{V} \sim 10^{15}$. This is the infamous tension between the scale of inflation and the scale of SUSY breaking [23] which afflicts most of the models of string inflation. Given that in this work we are interested only in inflation, we shall not attempt to address this issue (see [19, 36] for possible solutions).

Let us now turn to the study of the observational footprints of our model. The spectral index and the tensor-to-scalar ratio are defined by:

$$n_s = 1 + 2\eta_* - 6\epsilon_* \qquad \text{and} \qquad r = 16\epsilon_* \,, \qquad (3.34)$$

and they take the values:

$$n_s \simeq 0.96 \qquad \text{and} \qquad r \simeq 10^{-5} \,. \qquad (3.35)$$

Notice that since $\eta < 0$ this inflationary model necessarily leads to the observationally preferred value $n_s < 1$. The smallness of ϵ implies the absence of observable primordial tensor modes in agreement with the fact that the inflaton range during inflation is sub-Planckian [41]. In section 3.5, after discussing the effect of loop corrections on the inflationary potential, we shall present three different numerical fits for the underlying parameters which lead to these successful predictions in agreement with current data.

3.4 Reheating

The study of reheating for models of closed string inflation has already been performed in [38, 39]. After the end of inflation, the inflaton behaves as a classical condensate which oscillates coherently around its minimum. Due to the steepness of the potential, these oscillations end very rapidly because of the non-perturbative production of inflaton quanta [38]. Subsequently reheating takes place via the perturbative decay of inflaton particles into visible sector degrees of freedom localised on D7-branes wrapping an additional blow-up mode [39].

The gravitational coupling of the inflaton to all the other particles in the model can be read off from the moduli dependence of the kinetic and mass terms of open string modes by expanding the moduli around their VEVs and then expressing them in terms of the

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[3]We included a prefactor of $g_s/8\pi$ for the correct normalisation of the potential in Einstein frame [27].

canonically normalised fields [40]. Following this procedure, the inflaton coupling to visible sector gauge bosons scales with the overall volume as [25]:

$$g \simeq \frac{1}{M_p \mathcal{V}^p} \,. \tag{3.36}$$

Notice that this coupling is much weaker than gravitational, reflecting the fact that the direction τ_1 is not lifted by the leading order stabilising dynamics. Thus the width of the inflaton decay into visible sector degrees of freedom behaves as:

$$\Gamma \simeq g^2 m_{\tau_1}^3 \simeq \frac{M_p}{\mathcal{V}^{(9+7p)/2}} \,, \tag{3.37}$$

and so the corresponding reheating temperature is given by:

$$T_{\rm rh} \simeq \sqrt{\Gamma M_p} \simeq \frac{M_p}{\mathcal{V}^{(9+7p)/4}} \,. \tag{3.38}$$

For $p \simeq \mathcal{O}(1)$ and $\mathcal{V} \sim 10^3$, this expression gives $T_{\rm rh} \simeq 10^6$ GeV which is much higher than the Big-Bang Nucleosynthesis temperature $T_{\rm BBN} \simeq 1$ MeV, and moreover it allows electro-weak baryogenesis. Notice that the inflaton dumps all its energy into visible degrees of freedom since its decay to hidden sector particles is kinematically forbidden. In fact, the condensing field theory on τ_3 develops a mass gap and all the particles acquire a mass of the order the scale of strong dynamics $\Lambda \sim M_p/\mathcal{V}^{5/6} \gg m_{\tau_1}$ [39].

3.5 Loop corrections

In the discussion of the structure of the scalar potential for the fibre modulus we have so far neglected the effect of g_s corrections to the Kähler potential. In this section we shall therefore study their behaviour showing that they are naturally subleading with respect to the poly-instanton corrections both around the minimum and, more importantly, in all the inflationary region. Thus our model features a nice solution of the η-problem.

First of all, a key observation is that the presence of open string loop corrections would definitely dominate over the tiny poly-instanton effects, and generate a potential for τ_1 which is also able to give rise to slow-roll inflation due to the 'extended no-scale structure' as studied in [15]. We shall however forbid the presence of these g_s corrections to K by not wrapping any D7-brane either on τ_1 or on any four-cycle intersecting the K3 or T^4 fibre (i.e. τ_2 in our case). In this way there is no open string localised on τ_1, and so we do not expect any τ_1-dependent open string loop correction to K.

However, closed string loops might still introduce a dependence on the fibre modulus since they correspond to loops of bulk Kaluza-Klein modes which cannot be avoided by construction. Hence it is crucial to study the behaviour of these g_s effects and their relative strength with respect to poly-instanton corrections. Following [22], the dependence of these corrections on the Kähler moduli can be estimated by using the one-loop Coleman-Weinberg potential [42]:

$$\delta V_{(g_s)} \simeq \Lambda^4 \, \mathrm{STr}(M^0) + \Lambda^2 \, \mathrm{STr}(M^2) \,, \tag{3.39}$$

where the first contribution vanishes since supersymmetry implies $\mathrm{STr}(M^0) = 0$. The cut-off Λ can be taken as the scale at which the 4D effective field theory description ceases to be

valid. Given the hierarchy of scales described in [25] this should be M_{KK}^{6D}, the scale above which the theory becomes 6D:

$$\Lambda \equiv M_{KK}^{6D} \simeq \frac{M_s}{t_1^{1/2}} \simeq \frac{\sqrt{\tau_1}}{\mathcal{V}} M_p. \qquad (3.40)$$

The supertrace can instead be approximated as $\mathrm{STr}(M^2) \simeq m_{3/2}^2 \simeq (W_0 M_p/\mathcal{V})^2$, and so the expression (3.39) takes the final form:

$$\delta V_{(g_s)} \simeq (g_s C_{\mathrm{loop}})^2 W_0^2 \frac{\tau_1}{\mathcal{V}^4}, \qquad (3.41)$$

where C_{loop} is an unknown function of the complex structure moduli which we had to introduce by hand since our argument does not allow us to constrain the dependence of the one-loop potential on the U-moduli [4]. However, this is not a problem since the complex structure moduli are flux-stabilised at tree-level, and so we can consider C_{loop} just as a constant parameter. This parameter is squared due to the extended no-scale structure [22] which implies the vanishing of the leading contribution proportional to C_{loop} (this corresponds to the vanishing of the $\mathcal{O}(\Lambda^4)$ term in the Coleman-Weinberg potential).

Therefore the loop-corrected potential for the canonically normalised inflaton becomes:

$$V = \frac{F_{\mathrm{poly}}}{\langle \mathcal{V} \rangle^{3+p}} \left[1 - e^{-c\left(e^{\frac{2}{\sqrt{3}}\hat{\phi}}-1\right)} \left(1 + c \left(e^{\frac{2\hat{\phi}}{\sqrt{3}}} - 1 \right) \right) \right] + \frac{F_{\mathrm{loop}}}{\langle \mathcal{V} \rangle^4} e^{\frac{2}{\sqrt{3}}\hat{\phi}}, \qquad (3.42)$$

where we recall that $c = 2\pi \langle \tau_1 \rangle$, and we have defined:

$$F_{\mathrm{loop}} = \frac{c}{2\pi} \left(g_s C_{\mathrm{loop}} W_0 \right)^2. \qquad (3.43)$$

As can be seen from Figure 1, the effect of the one-loop correction is to lift the inflationary region and to generate an inflection point at $\hat{\phi} = \hat{\phi}_{\mathrm{ip}}$ where $\partial^2 V/\partial \hat{\phi}^2 = 0$. The position of the inflection point as a function of the ratio

$$R \equiv F_{\mathrm{loop}}/F_{\mathrm{poly}} \qquad (3.44)$$

can be approximated by [5]:

$$e^{\kappa \hat{\psi}_{\mathrm{ip}}} \simeq \frac{\kappa^3 \langle \mathcal{V} \rangle^{1-p}}{R}, \qquad (3.45)$$

showing correctly that $\hat{\phi}_{\mathrm{ip}}$ gets larger for smaller values of R, i.e. when the loops get weaker than the poly-instanton effects. The position of the inflection point is also dependent on the value of the overall volume via the parameter p, showing that the loop corrections get volume suppressed for $p < 1$. We shall however focus on models where $p \lesssim 1$ since values of p much smaller than unity would both render the inflaton heavier and bring $\langle \tau_1 \rangle$ in a regime where we no longer trust the effective field theory.

For $\hat{\phi} > \hat{\phi}_{\mathrm{ip}}$ the slow-roll parameter η becomes positive while ϵ stays negligibly small. Hence if inflation started at $\hat{\phi}_* > \hat{\phi}_{\mathrm{ip}}$, we would obtain a spectral index $n_s > 1$ which is

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[4]The factor of g_s can be derived by demanding that the one-loop corrected K scales as g_s^2 in string frame.

[5]We again use $\hat{\psi}$ instead of $\hat{\phi}$ for simplicity. Recall that at leading order the two quantities coincide (see Eq. (3.25)).
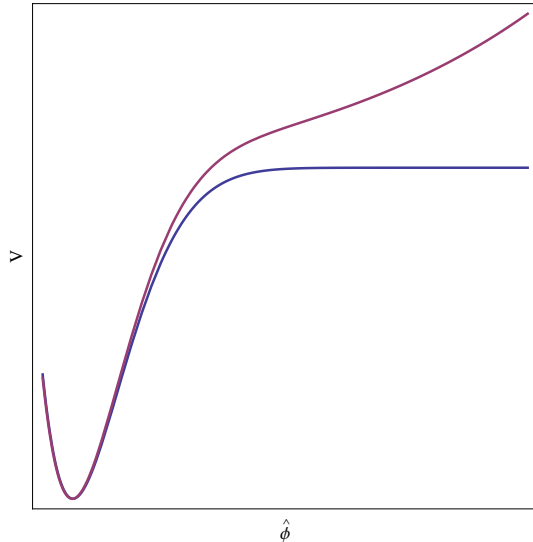
Figure 1. Sketch of the inflationary potential for an illustrative choice of the underlying parameters. In blue: the potential considering only the contribution from poly-instantons; in magenta: the potential including loop corrections.

incompatible with observations. We need therefore to focus on cases where the ratio R is small enough to have the inflection point lying outside the inflationary region, i.e. $\hat{\phi}_{\rm ip} > \hat{\phi}_*$. In this way, we can realise inflation with a potential generated by the poly-instantons in a region in which loop effects are negligible.

In Figure 2 we plot the slow-roll parameters and the spectral index for different indicative values of R. Given that the numerical study of the potential generated by poly-instanton effects gives $\hat{\phi}_* \simeq 0.9$ for $N_e \simeq 54$, Figure 2 reveals that the effect of loop corrections is negligible for $R \lesssim 10^{-3}$ (corresponding to $\hat{\phi}_{\rm ip} \gtrsim \hat{\phi}_*$).
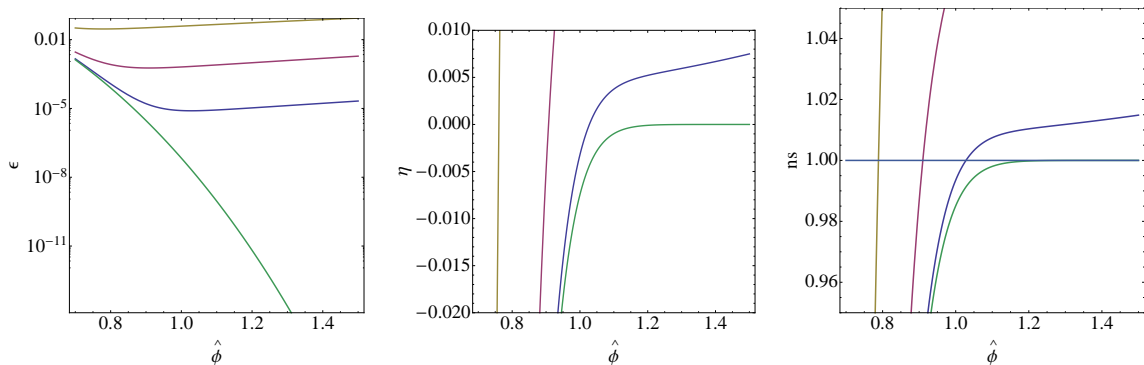

Figure 2. Slow-roll parameters and spectral index for different indicative values of R setting $p = 1$: blue $R = 10^{-3}$, magenta $R = 10^{-2}$, yellow $R = 10^{-1}$, green $R = 0$.

From the definitions of $F_{\rm poly}$, Eq. (3.16), and $F_{\rm loop}$, Eq. (3.43), it follows that R is

naturally very small, being proportional to the small parameter $(g_s C_{\text{loop}})^2 \ll 1$:

$$R \equiv \frac{F_{\text{loop}}}{F_{\text{poly}}} = (g_s C_{\text{loop}})^2 \left(\frac{W_0\, c}{8\pi f_2^{p+1} r_1\, e^{-1}} \right) \ll 1 \,, \tag{3.46}$$

since we expect all the parameters in the second parenthesis to be $\mathcal{O}(1-10)$. Thus this analysis shows that the loop corrections to the potential are naturally a subleading effect that does not affect the inflationary dynamics.

We present now three illustrative parameter fits which we found numerically. All of them satisfy all our phenomenological and theoretical constraints and give rise to a viable inflationary model. Table 1 displays the underlying parameters, Table 2 the compactification properties, and Table 3 the inflationary footprints.

	W_0	A	B	a	b	C_1	n	ξ	g_s	α	β
$\mathcal{P}_1$	14	11	8	$2\pi/4$	$2\pi/5$	4	-2.308	1.5	0.15	$\sqrt{1.5}$	$\sqrt{1.5}$
$\mathcal{P}_2$	6	12	6	$2\pi/4$	$2\pi/5$	3	-1.198	1.5	0.15	$\sqrt{1.5}$	$\sqrt{1.5}$
$\mathcal{P}_3$	5	30	10	$2\pi/4$	$2\pi/5$	9	-3.090	1.5	0.2	0.717	1

Table 1. Sampling of points in parameter space.

	$\langle \tau_3 \rangle$	$\langle \mathcal{V} \rangle$	$\langle \tau_1 \rangle$	p	Z	r_1	R/C_{loop}^2
$\mathcal{P}_1$	4.57	1480	1.07	1	5.95	0.58	0.017
$\mathcal{P}_2$	4.68	1300	1.00	0.9	3.21	0.60	0.013
$\mathcal{P}_3$	5.70	1419	1.07	0.8	4.69	3.42	0.035

Table 2. Minima for the Kähler moduli generated for the microscopic parameters of Table 1.

	N_e	n_s	r	A_{COBE}	$M_{\text{inf}}(\text{GeV})$
$\mathcal{P}_1$	54.2	0.959	9.56×10^{-6}	2.63×10^{-7}	1.81×10^{15}
$\mathcal{P}_2$	54.2	0.959	9.56×10^{-6}	2.26×10^{-7}	1.75×10^{15}
$\mathcal{P}_3$	54.2	0.959	9.56×10^{-6}	2.11×10^{-7}	1.72×10^{15}

Table 3. Inflationary observables for the fits of Table 1.

Comparing the results of Tables 1, 2 and 3 we see that it is possible to generate inflation at the right scale for natural values of the microscopic parameters, with the fibre modulus stabilised in the geometric regime. Furthermore we see in Table 2 that the values of the ratio R turn out to be always smaller than 10^{-3} for $C_{loop} \sim 0.1$, justifying the fact that we neglected string loop effects.

We stress that this was not possible in the model of [14] even if the inflationary potential was still generated by non-perturbative effects, and so had a shape very similar to (3.24). The reason is that in [14] the parameter κ scales with the volume as $\kappa \sim \mathcal{V}^{1/2}$, so it is large for large values of the volume. Instead, in our case it results much smaller since it scales with the volume as $\kappa \sim \ln \mathcal{V}$. This is due to the different topological origin of the inflaton

which in [14] was a blow-up mode whereas, in our case it is a fibre modulus. The fact that κ is smaller in our set-up, with respect to the model of [14], has a two-fold implication:

- Our potential is steeper than the one of [14] resulting in a larger value of the tensor-to-scalar ratio $r \sim 10^{-5}$ compared to the value $r \sim 10^{-10}$ of [14];

- Looking at the dependence of the slow-roll parameters (3.27) and (3.28) on κ, we realise that larger values of κ require larger values of $\hat{\phi}$ in order to satisfy the slow-roll conditions. Hence the inflationary region in our case is much closer to the minimum than in [14]. This is the reason why, contrary to [14], our model is not affected by the η-problem since we can avoid to experience regions in field space that are affected by string loop corrections.

In our set-up, our numerical analysis confirms that our potential gives rise to negligible tensor contributions to the CMB, and with a spectral index for scalar perturbations that is compatible with current observations. Notice that the fibre inflation model of [15] has also an inflationary potential with a shape very similar to (3.24). However in [15] the parameter κ is smaller and volume-independent: its value is $\kappa = 1/\sqrt{3}$. This gives a steeper potential with respect to our case, which is able to give rise to observable tensor modes.

4   Conclusions

In this paper we derived a new model of inflation driven by a Kähler modulus within the context of type IIB Calabi-Yau flux compactifications. This class of inflationary models features an interesting solution to the η-problem since they are characterised by the presence of several directions which are naturally flat. In fact, due to the no-scale structure of the Kähler potential, all the $h^{1,1}$ directions in moduli space are flat at tree-level. Then the leading order effect, that breaks the no-scale structure, is an α' correction which lifts only the volume mode. Hence all the other $(h^{1,1} - 1)$ directions in moduli space are flatter than the volume mode which sets the Hubble scale H. This implies that all these scalar fields are natural inflaton candidates since they turn out to be lighter than H. Moreover, the expansion of the prefactor e^K of the F-term scalar potential generates higher-dimensional operators which depend only on the volume mode, and so do not introduce any dangerously large correction to the inflaton mass.

An inflationary potential for these remaining flat directions can be generated at subleading order either by non-perturbative corrections to the superpotential, as in Kähler moduli inflation [14], or by perturbative string loop corrections to the Kähler potential, as in fibre inflation [15]. However, as noticed in [15], the model of [14] is plagued by the η-problem since loop corrections dominate over the non-perturbative potential and spoil inflation.

In this paper we proposed a new way to generate an inflationary potential via non-perturbative effects which does not lead to an η-problem. The new ingredient is the inclusion of poly-instanton corrections to the superpotential [24] for Calabi-Yau manifolds with a K3 or T^4 fibre over a $\mathbb{P}^1$ base [25, 26]. The main difference with the model of [14] is the topological nature of the inflaton field. In our case it is a fibre modulus, whereas in the case of [14] it was a blow-up mode. Due to this difference, our potential is not affected by the η-problem. In fact, in our case, the inflationary region is much closer to the minimum where the 'extended no-scale structure' [22] guarantees that string loop corrections are subdominant for natural values of the underlying parameters. Moreover, the different topology of the inflaton field is

reflected also in a steeper potential with respect to Kähler moduli inflation. We therefore obtain a larger value for the tensor-to-scalar ratio $r \sim 10^{-5}$ compared to the value $r \sim 10^{-10}$ of [14], even if it is still beyond the observational reach.

This small value of r corresponds to a high inflationary scale, $M_{\text{inf}} \simeq 10^{15}$ GeV, and a sub-Planckian motion of the inflaton in field space $\Delta\phi \simeq 0.5 M_p$. Horizon exit takes place around $N_e \simeq 54$ e-folding before the end of inflation where the spectral index assumes a value $n_s \simeq 0.96$ in perfect agreement with current observations. At the end of inflation, the visible sector degrees of freedom get excited by the gravitational decay of the inflaton field which leads to a reheating temperature of the order $T_{\text{rh}} \simeq 10^6$ GeV.

Finally it is worth emphasising that our analysis of the effect of string loop corrections on the inflationary potential relies on a low-energy estimate instead of a proper computation of string scattering amplitudes. Due to the relatively simple dependence of these loop corrections on the Kähler moduli, we believe the results we find here are likely to capture their leading behaviour. However, it would be very interesting to have more explicit calculations of g_s corrections to the Kähler potential for general Calabi-Yau manifolds.

Apart from inflation, we point out that our set-up possesses two crucial properties that can be applied to build models for quintessence. The first is that our potential is unaffected by loop corrections, and the second is that the coupling of the inflaton field to visible sector degrees of freedom is weaker than Planck strength. In this way one could avoid the stringent bounds coming from fifth-force experiments. Hence it would be interesting to investigate if our framework could also give rise to a successful quintessence model. We leave this study for future investigation.

Acknowledgments

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