CHARACTERIZATION OF RAIN FIELDS FOR UK SATELLITE NETWORKS

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ABSTRACT

High frequency satellite links are significantly affected by rain-induced attenuation in the satellite footprint [1]. The detailed planning and performance prediction of satellite systems networks (a group of links) requires a detailed understanding of the space-time characteristics of rain fields, [1][2][3][4]. This paper presents results of key empirical properties of rainfall rate relevant to developing a UK space-time stochastic model. In particular, we assess the impact of integration time and spatial scales on the space-time correlation of rain fields, the probability of rain occurrence and the point statistics of rainfall rate.

1. Introduction

Rainfall involves the interplay of many complex physical processes in the atmosphere [5]. Our main interest is in any communication system operating at frequencies above 10 GHz that is significantly affected by rain due to large radio propagation losses resulting in network-level or link unavailability situations [6]. In order to cope with rain induced effect using so-called fade mitigation techniques, it is necessary to gain an accurate knowledge of the dynamic statistical characteristics and variability of rainfall rate which is the primary natural cause of microwave frequency attenuation.

Many interesting studies on the modelling of rainfall rate or equivalent have already been carried out. For example, in 1981, Maseng and Bakken proposed the stochastic-dynamic time-series model for rain attenuation, [7]. Menabde in [8] used a discrete random cascade to generate a field with the desired statistical structure. In [2] Bell proposed an alternative approach achieving the desired spatial and time correlation structure and a log-normally distributed rain field. In this paper, we present experimental results obtained from UK radar data that are compatible with [2]. The main novelty is that we provide detailed empirical results showing how key properties of rain actually scale with varying spatial and temporal resolutions. For this we studied rain fields with space resolutions range between 1 and 256 km and time resolutions ranging from 5 minutes to 640 minutes.

The experimental input data used here are 5 mins radar maps collected by the UK NIMROD radar network. The NIMROD dataset consists of a series of radar-derived rain-rate maps on a 1-km grid. The complete maps have size 2175x1725 km covering the whole UK. In this paper, we only present processing from a 256x256 km area centred at 50.9 degree latitude and -1.7214 degree longitude in the vicinity of Portsmouth, UK. The rest of this paper is organized as follows: in Section 2, we present the background information on the space-time stochastic modelling of rain fields. The experimental results of the properties of rain, including the probability of rain, spatial/temporal correlation of rain as well as the statistics of rain are presented in Section 3. Section 4 gives the conclusion and further work. Further experimental results are given in the appendix.

2. Characterisation of Rain Attenuation Fields

The average rainfall over an area *A* during a time period *T* can be expressed as:

$$R = \frac{1}{T}\int_0^T dt \frac{1}{A}\int_0^A r(\boldsymbol{x},t)d\boldsymbol{x} \tag{1}$$

where $r(\boldsymbol{x},t)$ denoted the point rainfall rate in mm/h at location $\boldsymbol{x}$ and time t on a 2D Cartesian grid. The interval T is the integration time or time resolution while the area $A = L \times L$ with L being the spatial resolution. In practice, each pixel of a NIMROD radar map consists of a Cartesian grid of such space-time averaged values. Each new rain field map is produced every T seconds. In this paper, we analyse rainfall rate radar data collected in 2008. Although, it is well known (e.g. see [6]) that

longer durations ought to be looked at in order to capture seasonal and yearly cycles, the results presented is still significant especially for spatial characteristics.
The change of time scale, T or spatial scale L is particularly relevant to network system studies. In particular, one might be particularly interested in shorter time scales commensurate with typical network durations (e.g. 1 second). Thus time downscaling (i.e. time interpolation) is very important since typical radar maps are produced with long integration times (the NIMROD system has a 5 mins sampling period). Spatial scaling is less of an issue since for example the NIMROD data achieve a 1 km resolution already. A priori, this seems more than adequate for prediction of rain-affected satellite networks with fade mitigation techniques. Following the lead in [2], the following parameters are key to the development a reasonable space-time model of rainfall rate.

The probability P_0 of rain occurrence in geographical area A representing the spot-beam or footprint of the satellite network is the first main parameter. There is no real physical way of determining such a probability which thus will be measured at different spatial and time scales. We will see in the next section that a good curve fit is:

$$P_0 = a - f exp(bx^c + d) \tag{2}$$

where *a*, *b*, *c* and *f* are experimental constants and x denotes either L or T if we consider spatial or time scaling of P_0, respectively. The second important characteristic is the spatial correlation function (equal to the inverse Fourier transform of the spectrum):

$$\rho = cov(R_1, R_2)/\sigma_1\sigma_2 \tag{3}$$

where R_1 and R_2 are the point rainfall rates (millimetre/hour) for two locations 1 and 2, respectively, of interest and r is the cross-correlation factor between R_1 and R_2. cov( ) and σ are covariance and variance, respectively. In our case, we have measured the spatial correlation function of rainfall rate assuming that the field is space homogeneous and isotropic. In such a case, the correlation function of rainfall rate only depends on the separation distance between the selected points. However we want to measure how this changes with different spatial scales.
Rain events are also highly variable in time, so the third characteristic must be the time correlation:

$$\rho = cov(R_{t_1}, R_{t_2})/\sigma_1\sigma_2 \tag{4}$$

where R_{t1} and R_{t2} are the point rainfall rates (millimetre/hour) at two different times 1 and 2, respectively. The correlation level is changing with different scales, typically, it goes up with the increase of scale because the variance will be compensated and become smaller at larger scales; and then the value of correlation will go up.
A broad agreement is that point rainfall rate (when it is raining, $R > 0$) is well modelled as a lognormal random variable, [1], [2] and [6] with probability density function (PDF):

$$f(R) = \frac{1}{\sqrt{2\pi}\sigma R} exp\left(-\frac{1}{2}\left(\frac{lnR - \mu}{\sigma}\right)^2\right) \tag{5}$$

We will test the applicability of this statistical model for different space and time scales using the method described in [9]. The details are given in section 3.3.

3. Experimental Results

3.1. Scaling Probability of Rain

The probability of rain occurrence P_0(for which $R > 0$) represents equally well the probability of rain at one location over a long period of time, or, the expected fraction of the rainy area $P_0 = A_{rainy}/A$ that one can expect in a satellite network. In this paper, we processed one full month of radar data from January 2008. Experimental results showing the impact of spatial and time resolutions are given in Table1 and Table 2 bearing in mind that the original scales of the experimental data was 1 km and 5 mins.

L (km)	1	2	4	8	16	32	64	128	256
P_0 (%)	13.15	15.95	20.95	29.51	43.30	66.34	77.66	92.07	99.95

Table 1: Probability of rain for different spatial resolutions L (km) for UK in 2008 (T=5mins)

T (mins)	5	10	20	40	80	160	320	640
P_0 (%)	13.15	13.15	17.46	22.80	29.78	38.64	49.96	62.43

Table 2: Probability of rain for different temporal resolutions T (mins) for UK in 2008 (L=1km)

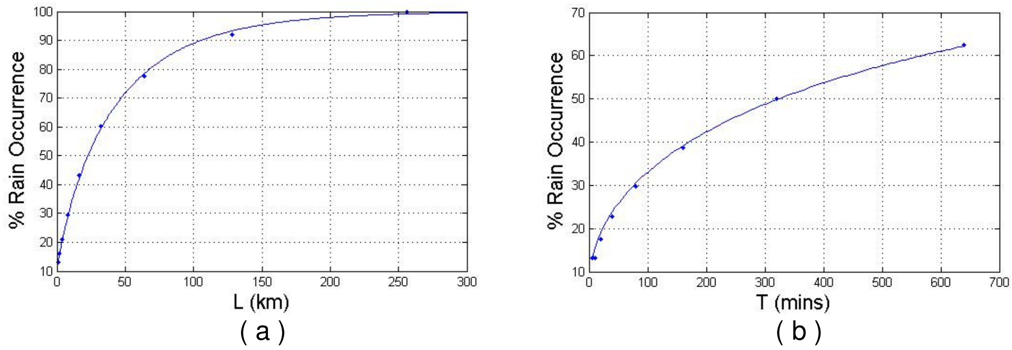


Fig.1: Scaling of probability of rain occurrence for increasing resolution in UK (Jan 2008): (a) is for different space scales *L* (km) and (b) is for varying time scales T (mins).

Fig.1 depicts how the probability of rain changes with resolution both in space and time domain. Clearly the probability of rain occurrence increases with increasing scales. The dots represent the data based on NIMROD measurement and the curves are the fitted lines. It is evident (and logical) that the larger the scale the higher the probability of rain. The mathematical equations for the fitted curves are given as follows.

For space domain (result (a)), with *L* in km:

$$P_0 = 100 - 13.15\exp\left(-0.042L^{0.85} + 1.9298\right)\ \% \quad (6)$$

For time domain (result (b)), with *T* in mins:

$$P_0 = 100 - 13.15\exp\left(-0.0256T^{0.55} + 1.9498\right)\ \% \quad (7)$$

The fitted equations give a reasonable estimate of the measured data (see Fig.1).

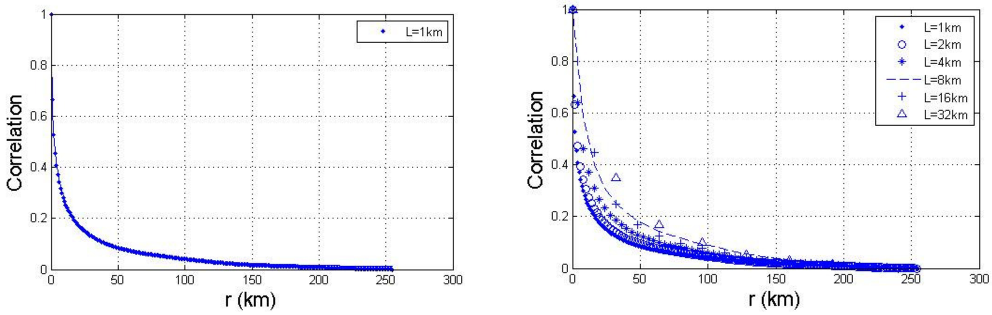
3.2. Scaling of Correlation of Rainfall Rate

The space-time correlation function is an important element for developing the rain attenuation model. The spatial and temporal correlation of rainfall rate is in particular critical for statistical simulation models of rain probability both in space and time domains. In this paper, one month of rainfall rate radar data (Jan 2008) were processed to study the correlation of rain in the UK at different scales.

Fig2 (a) is the comparison of spatial correlations of 1-km gridded rain rate. The dots and curve are the measured results and the solid line is the fitted line. For our work, the general empirical equation for the spatial correlation function for the UK is given by:

$$\rho(r) = \left(\frac{a}{exp(br^c + a)}\right)^n \quad (8)$$

where r stands for the distance in km, a, b, c and n are parameters to be determined from the data. We note that the correlation function falls off very quickly with distance and that the fitted curve is very accurate throughout the whole range of distances.



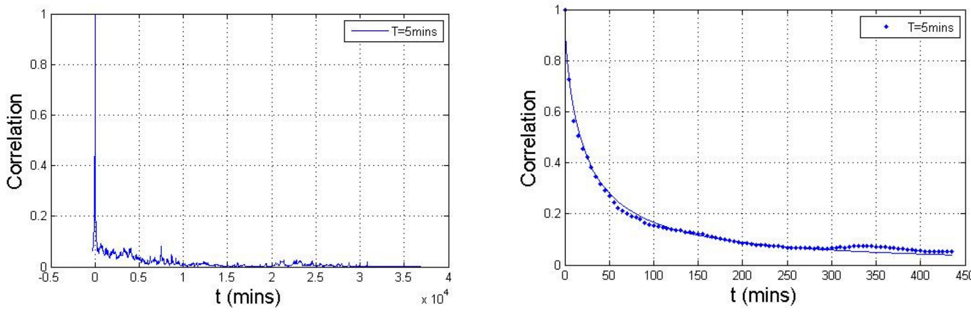
(a) (b)

Fig.2: The spatial correlation functions of rainfall rate: (a) spatial correlations with 1 km resolution (the curve is the fitted line) and (b) spatial correlations for different space scales.

Fig.2(b) shows the correlation functions for different spatial resolutions ranging from L=1km to 32km. This clearly indicates that the spatial correlation function is dependent on the actual spatial resolution. The fitted values of the parameters in Eq.(3) for each resolution are listed in Table 3, (see also the fitted curves for different resolutions in Appendix A). The results for the time correlation of rainfall rate are depicted in Fig.3 and Fig.4, respectively. Fig.3(a) shows that the correlation of rainfall rate is highly peaked. It drops down quickly over short time lags (labelled t); then the correlation drops more gently from value 0.1 down to zero over time lags up to about 10000 mins (=7 days). Fig.3(b) shows the zoomed in short-lag correlation function. The dots and curve are the measured data and fitted line, respectively. The fitted curve based on Eq.(3) agrees quite well with the measured results at small time lags up to 300 minutes. This fact is also true at others time scales (see the fitted curves in Appendix B).

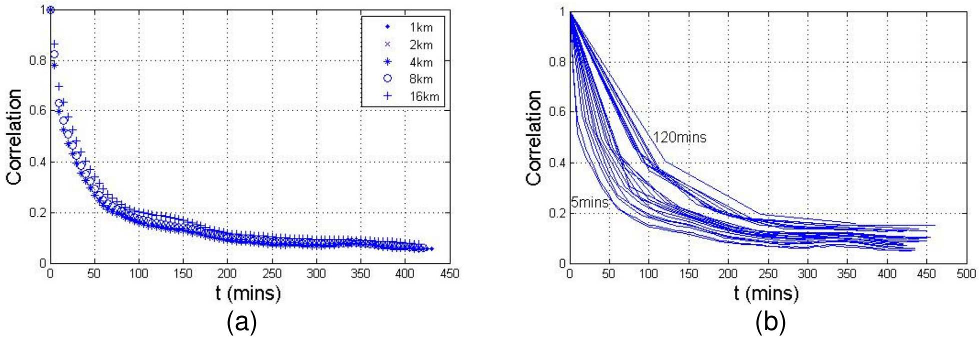
L (km)	1	2	4	8	16	32
a	9.4	10.8	14	16.8	19.9	29
b	1.0	1.1	0.9	0.8	0.7	0.3
c	0.5	0.5	0.6	0.7	0.8	1.2
n	4.5	4.3	4.0	3.1	3.0	2.0

Table 3: Experimental values of correlation function parameters for each spatial resolution



(a) (b)

Fig.3: (a) is the temporal correlation of rain with T=5mins resolution and, (b) is the zoomed-in correlation function for short-time lags.



(a) (b)

Fig.4: Time correlation functions of rainfall rate for different resolutions. (a) different spatial scales and, (b) is for the different temporal scales.

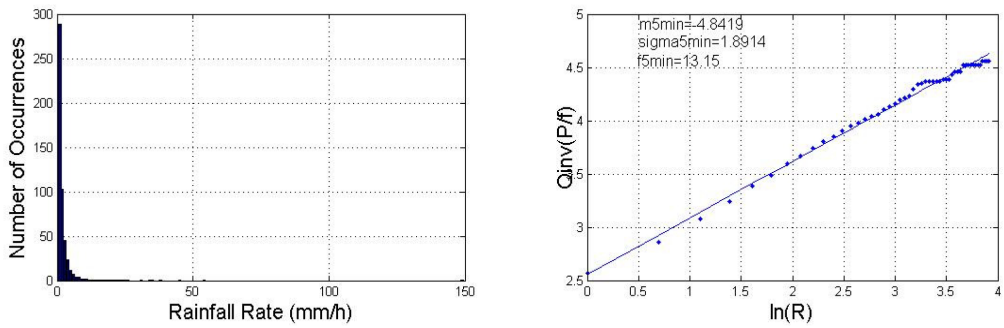
Fig.4(a) exhibits the interesting fact that the temporal correlation is not greatly affected by the spatial resolution. This might indicate that rainfall rate fields might have a separable space-time correlation function. In Fig.4(b), we however can find that the time correlation function of rainfall rate changes significantly with time resolution (i.e. integration time) between 5 minutes and 120 minutes. We note that for time lags greater than about 300 mins, irrespective of the time resolution, the time correlation falls off very slowly and is below 0.2. Table 4 lists the value of parameters for different time scales ranging from 5 minutes to 30 minutes.

T (mins)	**5**	**10**	**15**	**20**	**25**	**30**
a	19.5	19.5	22.5	23	20	24
b	1.1	1.3	1.4	1.5	1.65	1.67
c	0.74	0.72	0.7	0.7	0.65	0.63
n	1.8	1.8	1.8	1.7	1.7	1.9

Table 4: Experimental values of correlation parameters for each temporal resolution.

3.3. Scaling of the Statistics of Rain

The statistics of rainfall rate for 20 locations have been chosen in order to obtain average statistics representative of typical rain conditions for southern UK regions. Through summing up the histograms for each location in the selected area, an ordered and cumulative distribution of the measured occurrences of different intensities can be generated.



(a) (b)

Fig.6: (a) is the histograms of rainfall rate and, (b) is the test for log-normality of the rainfall rate distribution of UK in January 2008: the time resolution is 5 mins.

Fig.6(a) gives the histogram of rainfall rate conditioned on actual occurrence of rain for rainfall rate ranging from 1 mm/h to 150 mm/h. Over the period of observation (one month in Jan 2008), it is clear that the rainfall rate is less than 20 mm/h and the probability of heavy rain events is extremely low. The complementary CDF (CCDF) of rainfall rate has then been determined. Using the technique described in [9], the experimental CCDF is then tested for its log-normality. This is shown in Fig.6(b). If the data is log-normal, then the transformed CCDF shows up as a straight line. This is clearly the case (see additional results in the appendix).

The general formula for the fitted line is given by $y = x/\sigma + \mu/\sigma$, where μ and σ are the mean and variance of the log-normal PDF given in Eq.(3). The values of μ and σ for different time scales (i.e. integration times) between 5 minutes and 640 minutes are listed in Table 4. Our results show that the distribution of rainfall rate remains log-normal (see the fitted line in Appendix C).

T (minutes)	μ	σ
5	-4.8419	1.8914
10	-4.5379	1.6444
20	-3.8791	1.5227
40	-3.9527	1.5260
80	-3.3412	1.3674
160	-3.6789	1.3208
320	-3.3449	1.1758
640	-7.5368	2.3348

Table 5: Experimental values of parameters of fitted line for each time resolutions

4. Conclusion and further work

In conclusion, empirical results characterizing the space-time nature of rainfall rate have been presented. We focussed on measuring how the probability of rain occurrence, the spatial and the time correlation functions and the first order log-normal statistics of rainfall rate change with varying spatial and temporal resolutions/scales. We provide empirical equations that can be used in later statistical or simulation studies. This will be applied to satellite network studies. Further work is required to ascertain the variability of these characteristics over large areas spanning different climatic regimes.

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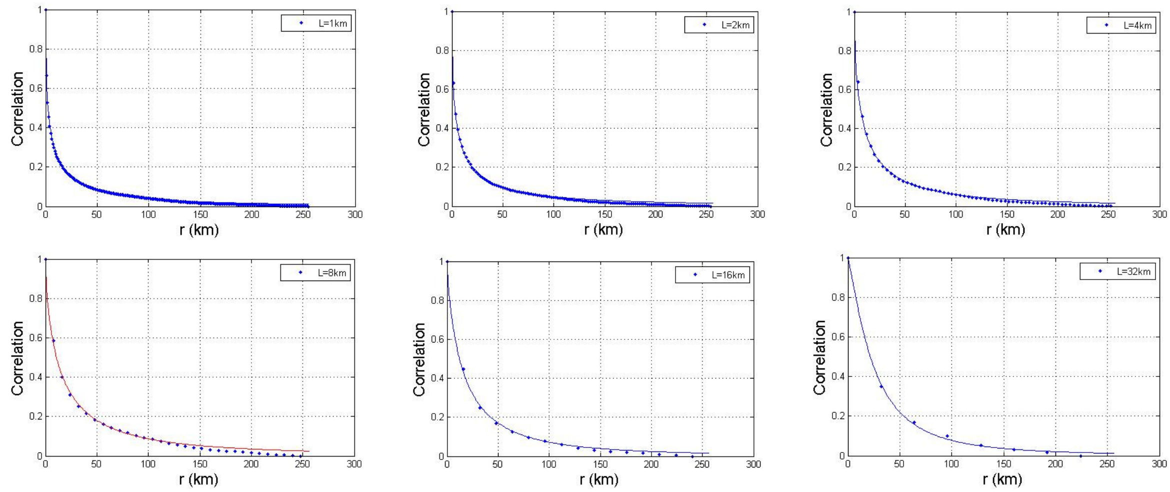
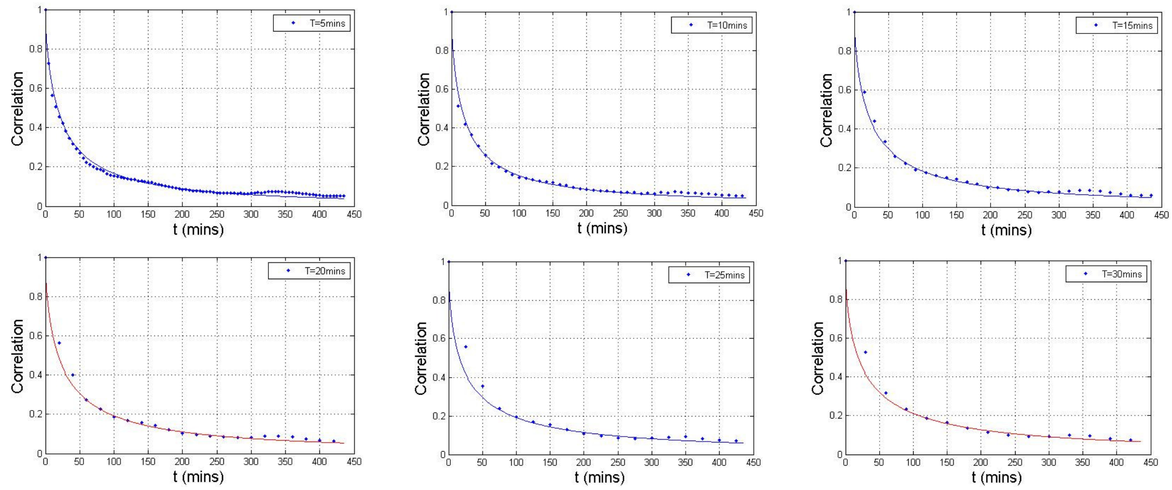
Appendix

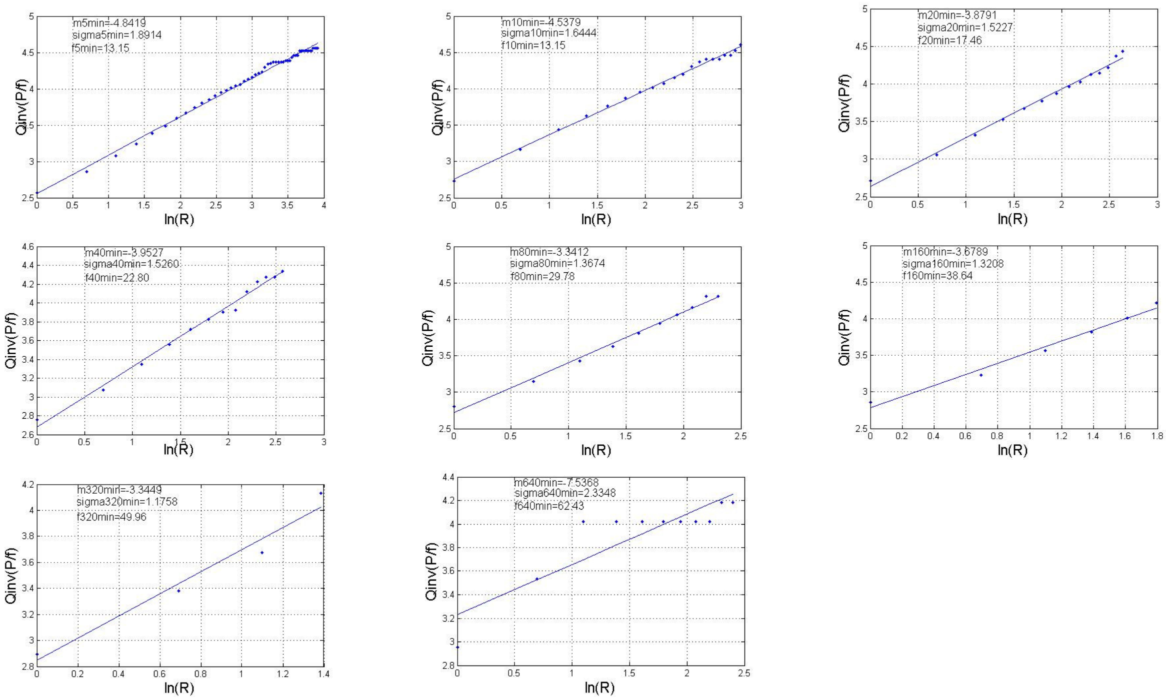
A: Fitted Correlation Function for Different Spatial Resolutions (1km to 32km)



B: Fitted Correlation Function for Different Temporal Resolutions (5mins to 30mins)



C: Rainfall Rate Distribution Test for Different Time Scales from 5mins to 640mins



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