

Consistency test of general relativity from large scale structure of the Universe

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We construct a consistency test of General Relativity (GR) on cosmological scales. This test enables us to distinguish between the two alternatives to explain the late-time accelerated expansion of the universe, that is, dark energy models based on GR and modified gravity models without dark energy. We derive the consistency relation in GR which is written only in terms of observables - the Hubble parameter, the density perturbations, the peculiar velocities and the lensing potential. The breakdown of this consistency relation implies that the Newton constant which governs large-scale structure is different from that in the background cosmology, which is a typical feature in modified gravity models. We propose a method to perform this test by reconstructing the weak lensing spectrum from measured density perturbations and peculiar velocities. This reconstruction relies on Poisson's equation in GR to convert the density perturbations to the lensing potential. Hence any inconsistency between the reconstructed lensing spectrum and the measured lensing spectrum indicates the failure of GR on cosmological scales. The difficulties in performing this test using actual observations are discussed.

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I. INTRODUCTION

In 1998, astronomers discovered that the expansion of the Universe was accelerating, not slowing down as expected [1, 2]. This late-time acceleration of the Universe is surely the most challenging problem in cosmology. Within the framework of General Relativity (GR), the acceleration originates from dark energy. The simplest option is the cosmological constant, first introduced by Einstein. However, in order to explain the current acceleration of the Universe, the required value of the cosmological constant must be incredibly small. Particle physics predicts the existence of vacuum energy, but it is typically many orders of magnitude larger than the observed values of the cosmological constant (for a review see Ref. [3]).

Alternatively, there could be no dark energy, but a large-distance modification to GR may account for the late-time acceleration of the universe [4, 5] (for a review see [6, 7]). In order to realize the late-time acceleration, it is necessary to modify GR at cosmological scales. It is really challenging to construct a consistent theory of modified gravity (MG) and progress in theoretical physics is required in order to have fully consistent models. Even if successful theoretical models for MG are constructed, we need to distinguish them from dark energy models in GR via observations. This is indeed possible if one can combine various data sets which measure not only the expansion history of the Universe but also the structure of the Universe [8]. However, it has been argued that by fine-tuning the properties of dark energy, it is always possible to mimic any MG model and hence it is impossible to distinguish between the two possibilities [9, 10].

In this paper, we argue that it is indeed *possible* to distinguish between MG models and dark energy models, and we propose a consistency test of GR which enables us to distinguish between these two possibilities (see also Ref. [11]). From large-scale structure in the Universe, we can measure three quantities. One is the density perturbation which can be measured from the galaxy distribution and cluster abundance. The second is the peculiar velocity which can be measured from the redshift distortions of the galaxy power spectrum and the internal dynamics of clusters and galaxies. Finally, we can measure the lensing potential from weak lensing of galaxies and clusters. We construct a consistency equation in GR which can be written only in terms of these observables. In this way, it is possible to avoid using specific MG models or parametrising MG models in order to test GR on cosmological scales. Then we discuss how we can realize this test using actual observations in the future. The most difficult part of the consistency test is that all the quantities should be measured at the same time and at the same location. However, weak lensing only measures the integrated effects of the lensing potential at different redshifts and there is no simple way to reconstruct the lensing potential at a given redshift. In this paper, we propose a novel way to overcome this problem by reconstructing the lensing power spectrum from measured matter perturbations at given redshifts.

Before going into the construction of the consistency test, we first summarize the assumptions that we make in this paper. First of all, we assume the cosmological principle and describe our background universe as an isotropic and homogeneous Friedmann-Robertson-Walker universe. Thus we do not consider the possibility of an inhomogeneous universe which has been extensively studied in the literature as a possible explanation for the late-time accelerated expansion of the Universe (see [12] for a review). The cosmological principle can be tested from observations [13, 14, 15, 16] and we assume that

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we can use this to exclude the inhomogeneous universe. Next, we assume that the energy-momentum tensor for dark matter and baryons is conserved. In MG models, we try to explain the late-time accelerated expansion of the Universe without introducing any exotic fluids, so this is a reasonable assumption. In dark energy models, dark matter can be coupled to dark energy (see for example [17]). However, coupling between baryons and dark energy is strongly constrained by local experiments. Thus only the conservation of the energy-momentum tensor for dark matter can be broken. This creates a difference between peculiar velocities of dark matter and baryons, which can be tested from observations. This test will be investigated in a forthcoming paper [18] and we assume that we can use this to exclude the possibility of the breakdown of energy-momentum conservation for dark matter. The final assumption is that MG models are metric theories and there exists a Newtonian limit of the theory. This is necessary to reproduce the very tight constraints on deviations from GR at solar system scales. With this assumption, on subhorizon scales, only the time-time component and the anisotropic part of the space-space component of the modified Einstein equations for perturbations are important. From the Bianchi identity and the conservation of the energy-momentum tensor, other components of the modified Einstein equations should be trivially satisfied.

The structure of this paper is as follows. In section II, we summarize the basic equations for background cosmology and perturbations in GR and MG. In section III, the consistency test of GR is constructed, written only in terms of observables. We provide a set of consistency equations depending on the assumptions for the properties of dark energy in GR. In section IV, we propose a method to perform the test by reconstructing the weak lensing power spectrum from measured matter perturbations. Section V is devoted to conclusions.

II. BACKGROUND AND PERTURBATIONS

In GR, the Friedman equation determines the expansion rate of the Universe

$$H^2 = \frac{8\pi G}{3}\rho_t^{GR}, \quad \rho_t^{GR} = \rho_m + \rho_{de}, \quad (1)$$

where ρ_m is the matter energy density which include both dark matter, and baryon and the dark energy density ρ_{de} accounts for the late-time acceleration of the Universe.

On the other hand, in MG models, the expansion is determined by a modified Friedman equation $H^2 = F(8\pi G\rho_m)$ where F is a function determined by the underlying modified theory of gravity. We can always rewrite this modified Friedman equation as

$$H^2 = \frac{8\pi\bar{G}_{eff}(t)}{3}\rho_t^{MG}, \quad \rho_t^{MG} = \rho_m. \quad (2)$$

For a given MG model, it is always possible to find a dark energy model which gives exactly the same ex-

pansion history, i.e. the same H . This gives the relation between total energy densities in two models as $\rho_t^{GR}/\rho_t^{MG} = \bar{G}_{eff}(t)/G$.

We use the Newtonian gauge to describe the metric perturbations

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 + 2\Phi)a^2\delta_{ij}dx^i dx^j. \quad (3)$$

The density fluctuation is defined by $\delta = (\rho - \bar{\rho})/\bar{\rho}$ for each component of matter where the quantities with bar denotes the background quantity. The divergence of the peculiar velocity is $\theta = \nabla_j T_0^j/(\bar{\rho} + \bar{P})$. In GR, the perturbed Einstein equations at sub-horizon scales give

$$k^2\Phi = 4\pi G a^2 \rho_t^{GR} \delta_t^{GR}, \quad (4)$$

$$k^2(\Phi + \Psi) = -12\pi G a^2 (1 + w_t) \rho_t^{GR} \sigma_t^{GR}, \quad (5)$$

where σ is an anisotropic stress. The quantity with the subscript t is the total contribution from matter perturbations

$$\rho_t \delta_t = \rho_m \delta_m + \rho_{de} \delta_{de}, \quad (6)$$

$$(\rho_t + P_t)\theta_t = (\rho_{de} + P_{de})\theta_{de} + \rho_m \theta_m, \quad (7)$$

$$(\rho_t + p_t)\sigma_t = (\rho_{de} + p_{de})\sigma_{de}. \quad (8)$$

As explained in the introduction, we assume that there is no interaction between dark energy and dark matter/baryons. Then conservation of the energy-momentum tensor gives

$$\dot{\delta}_m = -\frac{\theta_m}{a}, \quad \dot{\theta}_m = -H\theta_m + \frac{k^2\Psi}{a}, \quad (9)$$

and

$$\dot{\delta}_{de} = -(1 + w_{de})\frac{\theta_{de}}{a} - 3H\frac{\delta P_{de}}{\rho_{de}} + 3Hw_{de}\delta_{de}, \quad (10)$$

$$\dot{\theta}_{de} = -H(1 - 3w_{de})\theta_{de} - \frac{\dot{w}_{de}}{1 + w_{de}}\theta_{de} + \frac{k^2}{a} \left(\frac{1}{\rho_{de}} \frac{1}{1 + w_{de}} \delta P_{de} - \sigma_{de} + \Psi \right), \quad (11)$$

where δP_{de} is the pressure perturbation of the dark energy fluid.

Assuming a Newtonian limit in the MG model, the modified Einstein equations yield the following equations [11]

$$k^2\Phi = 4\pi G_{eff}(a, \mathbf{k}) a^2 \rho_t^{MG} \delta_t^{MG}, \quad (12)$$

$$\Phi = -\eta(a, \mathbf{k})\Psi, \quad (13)$$

where $G_{eff}(a, \mathbf{k})$ and $\eta(a, \mathbf{k})$ should be determined by the modified theory of gravity. In the MG model, we have only dark matter ($\rho_t = \rho_m$, $\delta_t = \delta_m$) and the conservation equations are given by Eq. (9). All other components of modified Einstein equations should be satisfied trivially thanks to the Bianchi identity.

III. CONSISTENCY TEST OF GR

First, let us consider simple dark energy models like quintessence, where we can safely ignore clustering of dark energy, at least under horizon scales. In this case, the metric perturbations are given by

$$k^2\Phi^{GR} = 4\pi G a^2 \rho_m^{GR} \delta_m^{GR}, \quad \Psi^{GR} = -\Phi^{GR}, \quad (14)$$

in the GR dark energy model. In reality there is no direct measurement of the curvature perturbation Φ . The weak gravitational lensing provides the combination $\Phi_- \equiv (\Phi - \Psi)/2$ which determines the geodesics of photons. Then we can define a consistency parameter as

$$\alpha^{(1)}(a, \mathbf{k}) = \frac{k^2\Phi_-}{4\pi G a^2 \rho_m \delta_m}, \quad (15)$$

where G is the Newton constant measured by local experiments. If GR holds, $\alpha^{(1)} = 1$ (see also Ref. [19] for an early attempt to test GR from large-scale structure and gravitational lensing). We emphasize that all quantities in the consistency relation are observables and we do not need to assume any theory. It is easy to see that unless $(1 + \eta^{-1})G_{eff}/2G = 1$, we have $\alpha^{(1)} \neq 1$ in MG. We should emphasize that this test can allow a special class of MG with $(1 + \eta^{-1})G_{eff}/2G = 1$ to pass the GR consistency test $\alpha^{(1)} = 1$. However, if $G_{eff}/G \neq 1$, then $\eta \neq 1$, which means that the GR relation $\Phi = -\Psi$ is modified. In order to check this, we need one more equation which relates the Newtonian potential Ψ to matter-energy perturbations. Since energy-momentum is conserved, the continuity equation for matter is not altered and Eq. (9) gives $k^2\Psi = d(a\theta_m)/dt$. Then we define a consistency parameter for $\eta \neq 1$ as

$$\tilde{\alpha}^{(1)}(a, \mathbf{k}) = -\frac{\Phi}{\Psi} = -\frac{8\pi G a^2 \rho_m \delta_m}{3 a^2 H^2 V_m}, \quad (16)$$

where $V_m = 2(d\theta_m/dt + H\theta_m)/(3aH^2)$. V_m can be constructed by direct measurements of the peculiar velocity and acceleration of matter. We should emphasize that only energy-momentum conservation for dark matter is used. Thus even if there is a peculiar velocity of dark energy θ_{de} , the reconstruction of the Newtonian potential is not affected. If $\alpha^{(1)} = 1$ and $\tilde{\alpha}^{(1)} = 1$, we can conclude that there is no modification of GR and dark energy has no clustering.

However, even if one finds $\alpha^{(1)} \neq 1$ or $\tilde{\alpha}^{(1)} \neq 1$, this does not give definite evidence for the breakdown of GR. In general, dark energy can have non-trivial clustering. In fact, the conservation equations for dark energy perturbations are not closed and the pressure perturbation δP_{de} and the anisotropic stress σ_{de} should be determined by the microphysics of the dark energy fluid. Given that we do not know the origin of dark energy, we can treat these two functions as arbitrary functions. Then it is always possible to mimic Φ and Ψ in a MG model by tuning δP_{de} and σ_{de} in a dark energy model in GR, because $\delta\rho_{de}$ mimics G_{eff} and σ_{de} mimics η [9]. This fact

can be clearly seen by rewriting Poisson's equation in GR with clustering DE as

$$\begin{aligned} k^2\Phi &= 4\pi G a^2 (\rho_m \delta_m + \rho_{de} \delta_{de}) \\ &= 4\pi G a^2 \left(1 + \frac{\rho_{de} \delta_{de}}{\rho_m \delta_m}\right) \rho_m \delta_m. \end{aligned} \quad (17)$$

Then, if we only observe dark matter fluctuations δ_m , we cannot distinguish between clustering DE and modified gravity as the clustering DE mimics the modification of Newton's constant

$$G_{eff}(a, \mathbf{k}) = G \left(1 + \frac{\rho_{de} \delta_{de}}{\rho_m \delta_m}\right). \quad (18)$$

Thus if we find $\alpha^{(1)} \neq 1$ and $\tilde{\alpha}^{(1)} \neq 1$, there are still two possibilities. One is MG and the other is dark energy model with non-trivial clustering.

However, this argument is clearly based on the assumption that we only observe the clustering of dark matter [11]. Once the dark energy clusters like dark matter, both perturbations participate in forming halos and thus galaxies. Thus the galaxy distribution should trace the total matter perturbations, which are the sum of the dark matter and dark energy perturbations. In the same way, the cluster abundance is sensitive to the total matter perturbations [11]. When we measure the total matter perturbations from the galaxy distribution, bias between the galaxy density fluctuations and the total matter perturbations becomes the main issue. In this paper, we assume that bias can be measured independently by higher order statistics or other methods. We will come back to this issue in the conclusions.

Assuming we can measure the total density perturbations from observations, it becomes possible to improve the consistency test and we can distinguish clustering dark energy and modified gravity. Including the dark energy perturbations, Poisson's equation is now written as

$$k^2\Phi^{GR} = 4\pi G a^2 \rho_t^{GR} \delta_t^{GR}, \quad (19)$$

in the GR dark energy model, and

$$k^2\Phi^{MG} = 4\pi G_{eff}(a, \mathbf{k}) a^2 \rho_t^{MG} \delta_t^{MG}, \quad (20)$$

in MG models. Let us consider two models that give the same $\alpha^{(1)}(a, \mathbf{k}) \neq 1$. In these two models, $\Phi^{GR} = \Phi^{MG}$. Then the relation between total density perturbations in the two models is

$$\frac{\delta_t^{GR}}{\delta_t^{MG}} = \frac{G_{eff} \rho_t^{MG}}{G \rho_t^{GR}} = \frac{G_{eff}(a, \mathbf{k})}{\bar{G}_{eff}(a)}. \quad (21)$$

Here $\bar{G}_{eff}(a)$ is the effective Newton constant determined by the background Friedman equation (2) and $G_{eff}(a, \mathbf{k})$ is the effective Newton constant determined by the Poisson equation (12). In general these two effective Newton constants are different, as is seen in explicit examples like

the DGP [4] and $f(R)$ gravity models [5]. We expect that this is a very general feature of MG.

Then we can define the consistency parameter as

$$\alpha^{(2)}(a, \mathbf{k}) = \frac{k^2 \Phi}{4\pi G a^2 \rho_t \delta_t}. \quad (22)$$

In GR, $\alpha^{(2)} = 1$. The total energy density ρ_t is reconstructed from H using the Friedman equation in GR, Eq. (1). In this case, we cannot assume the GR relation $\Phi = -\Psi$ and we should combine weak lensing that measures Φ_- and peculiar velocity that determines Ψ . Then the consistency parameter is given by

$$\alpha^{(2)}(a, \mathbf{k}) = \frac{4k^2}{3a^2} \left(\frac{\Phi_- - \frac{3}{4}a^2 H^2 V_m}{H^2 \delta_t} \right). \quad (23)$$

Again all the quantities in this equation can be measured from observations and we do not need to assume any theory to perform the test. From Eq. (21) and using the fact that in clustering DE and MG, Φ_- , V_m and H are the same, we find

$$\alpha^{(2)}(a, \mathbf{k}) = \frac{G_{eff}(a, \mathbf{k})}{G_{eff}(a)}, \quad (24)$$

in MG models. This test (23) is essentially the same as the consistency relation proposed in [11]. Here, we have shown that this consistency relation is a test of the difference between the Newton constants for the background cosmology and for sub-horizon perturbations.

IV. CONSISTENCY TEST FROM PROJECTED POWER SPECTRUM

All observables in the consistency test should be measured at the same time and at the same location. However, in reality, there is no simple method to compare the statistical observables Φ_- , δ_t and V_m in the consistency equation Eq. (23). This is because weak lensing is an integrated effect of the lensing potential determined by Φ_- along the line of sight and it is impossible to reconstruct the lensing potential at a given redshift. In this paper, we propose a novel way to accomplish the test using the projected power spectrum (see [20] for a review of the projected power spectrum).

The projected angular power spectra $C_\ell^{XX'}$ of any pair of perturbations X and X' are given by

$$C_\ell^{XX'} = \frac{2\pi^2}{\ell^3} \int dD D W^X(D) W^{X'}(D) \frac{9}{25} \Delta_{\zeta\zeta}, \quad (25)$$

where D is the comoving distance and we used the Limber approximation which is valid for large ℓ . $\Delta_{\zeta\zeta}(a_0, k)$ is the rms amplitude of curvature fluctuations on comoving hypersurfaces at some time given by $a = a_0$ during the matter-dominated era. The window function $W^X(D)$ is determined by the property of the quantity X as is shown below.

The deflection angle \mathbf{d} due to gravitational lensing is defined by the gradient field of the lensing potential, $\mathbf{d} = \nabla\phi$, where

$$\phi = -2 \int dD \frac{D_s - D}{D D_s} \Phi_-, \quad (26)$$

and D_s denotes the comoving distance to the source galaxies distributed on the thin redshift shell labeled by s . Then the window function for ϕ is

$$W^{s\phi}(D) = -2 \mathcal{G}_{\Phi_-}(a, k) \frac{(D_s - D)}{D_s D}, \quad (27)$$

where the growth function $\mathcal{G}_{\Phi_-}(a, k)$ is given by $\mathcal{G}_{\Phi_-}(a, k) = \Phi_-(a, k)/\Phi_-(a_0, k)$. The angular power spectrum of the deflection angle is given by $C_\ell^{s\phi\phi} = \ell(\ell+1)C_\ell^{s\phi\phi}$, and $C_\ell^{s\phi\phi}$ is related to the convergence power spectrum $C_\ell^{s\kappa\kappa}$ as $C_\ell^{s\kappa\kappa} = \ell(\ell+1)C_\ell^{s\phi\phi}/4$.

The density perturbations are measured at a given redshift labeled by i at the comoving distance D_i from the observer. In the approximation of the quasi-static evolution of perturbations, the projected angular power spectrum can be written in discretized form at the given redshift bin i as

$$C_\ell^{iXX'} = \frac{2\pi^3}{\ell^3} \Delta D_i D_i W^X(D_i) W^{X'}(D_i) \frac{9}{25} \Delta_{\zeta\zeta}. \quad (28)$$

The galaxy density fluctuation δ_g is a biased tracer of the total density perturbation δ_t , which includes the possible presence of dark energy clustering: $\delta_g = b\delta_t$. The window function for the δ_g component is given by

$$W^{i\delta_g}(D_i) = \frac{2}{3} \mathcal{G}_{\delta_t}(a_i, k) \frac{dz_i}{dD} n_i b_i \frac{l^2}{\Omega_m H_0^2 D_i^2}, \quad (29)$$

where n_i is the number density of galaxies at $z = z_i$, $n_g(z_i)$ and b_i is $b(z_i)$. The growth function \mathcal{G}_{δ_t} is given by

$$\mathcal{G}_{\delta_t}(a_i, k) = \frac{a_i \Phi(a_i, k)}{\Phi(a_0, k)} \quad (30)$$

if there is no clustering of dark energy ($\delta_t = \delta_m$) and

$$\mathcal{G}_{\delta_t}(a_i, k) = \frac{\Omega_m H_0^2 \Phi(a_i, k)}{a_i^2 H^2(a_i) \Phi(a_0, k)}, \quad (31)$$

if there is clustering of dark energy. Both growth functions are normalized to a_0 during the matter-dominated era.

The redshift-space power spectrum of galaxies is anisotropic due to the peculiar velocities of galaxies. We use an approximation that peculiar velocities of galaxies trace that of matter θ_m despite the existence of dark energy peculiar velocity θ_{de} [11]. This allows a statistical measurement of the peculiar velocities θ_m (see for example [21]). It is possible to extract the power spectrum of θ_m independently of galaxy bias [22]. The derivative of

θ_m appearing in V_m can be derived from a direct measurement of the acceleration field [23], or estimated from measured peculiar velocities at different time slicings [24]. Here we assume that it is built from one of these methods. Then the window function for V_m in each bin i is

$$W^i V_m(D_i) = \frac{2}{3} \mathcal{G}_{V_m}(a_i, k) \frac{dz_i}{dD_i} n_i \frac{\ell^2}{\Omega_m H_0^2 D_i^2}, \quad (32)$$

where the growth function $\mathcal{G}_{V_m}(a_i, k)$ is given by

$$\mathcal{G}_{V_m}(a_i, k) = \frac{\Omega_m H_0^2 \Psi(a_i, k)}{a_i^2 H^2(a_i) \Psi(a_0, k)}. \quad (33)$$

The galaxy power spectra $C_\ell^{i \delta_g \delta_g}$ and $C_\ell^{i V_m V_m}$ can be compared directly at each ℓ because these quantities are measured at the same time for a given measured comoving distance D_i , and at the same scale $k = \ell/D_i$ in the Limber approximation. However, we can observe only the integrated effect of the lensing potential and it is impossible to measure Φ_- at a given time and location. Thus we propose a new way to perform the consistency test. We statistically reconstruct the lensing power spectrum $\tilde{C}_\ell^{s dd}$ by replacing the lensing potential Φ_- with the measured density perturbations δ_t and peculiar velocities V_m at given redshifts. In the reconstruction, we use Poisson's equation in GR to relate δ_t to the 3D curvature perturbations Φ . Thus if there is inconsistency between $C_\ell^{s dd}$ and $\tilde{C}_\ell^{s dd}$, it indicates a modification of the GR Poisson equation, and hence the breakdown of GR in structure formation.

We illustrate how this idea works in the following. The continuous contributions from the lensing potential in Eq. (25) can be discretized

$$C_\ell^{s dd} = \frac{2\pi^2}{\ell} \sum_{i=1}^n \Delta D_i D_i \frac{4(D_s - D_i)^2}{D_s^2 D_i^2} \Delta_{\Phi_- \Phi_-}(a_i, k). \quad (34)$$

The first test is to assume that there are no dark energy perturbations. Then $\Delta_{\Phi_- \Phi_-}(a_i, k)$ can be replaced by the measured matter perturbations from the galaxy distribution

$$\Delta_{\Phi_- \Phi_-}^i = \frac{9}{8\pi^2 \ell} \frac{D_i^3}{\Delta D_i} \left(\frac{dz}{dD} n_i b_i \right)^{-2} \frac{\Omega_m^2 H_0^4}{a_i^2} C_\ell^{i \delta_g \delta_g}. \quad (35)$$

Substituting this into Eq. (34), we derive the *reconstructed* power spectrum

$$\begin{aligned} \tilde{C}_\ell^{s dd}(\alpha^{(1)}) &= \frac{9}{\ell^2} \sum_{i=1}^n \frac{D_i^2 (D_s - D_i)^2}{D_s^2} \left(\frac{dz}{dD} n_i b_i \right)^{-2} \\ &\times \frac{\Omega_m^2 H_0^4}{a_i^2} C_\ell^{i \delta_g \delta_g}. \end{aligned} \quad (36)$$

In the upper panel of Fig. 1, we demonstrate how accurately this reconstruction can be done, using LCDM with the cosmological parameters $\Omega_m = 0.25$, $\Omega_b = 0.05$, $H_0 = 72$ km/sec/Mpc, and a flat model with no DE

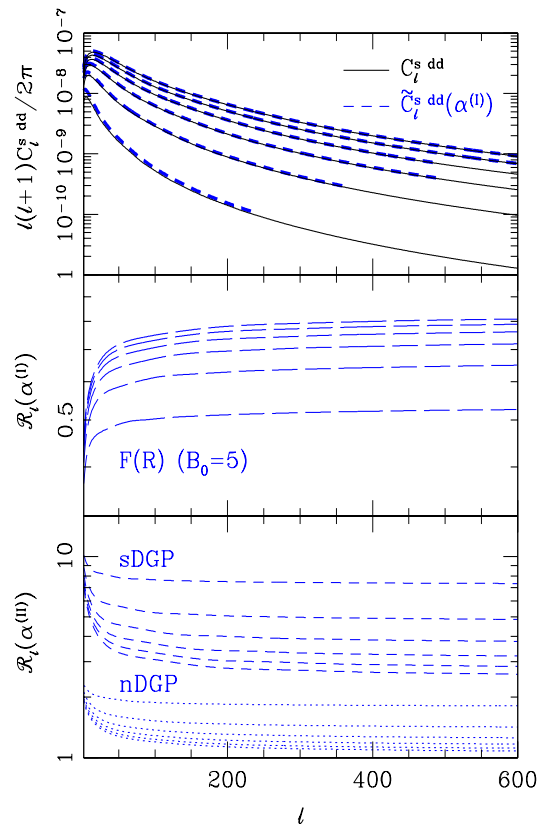


FIG. 1: *Upper panel:* The precision test between $C_\ell^{s dd}$ (solid curves) and $\tilde{C}_\ell^{s dd}$ with a redshift spacing $\Delta z_i = 0.1$ (dash curves) for 6 different z_s running from 0.25 (bottom) to 2.75 (top). We restricted the reconstructed power spectra to scales where the non-linear effects give less than 5% changes on the power spectra. *Middle panel:* The first consistency test for GR model (solid curves) and MG model -f(R) gravity [25] (long dash curves). *Bottom panel:* The second consistency test for GR model (solid curves), sDGP model [26, 27] (dash curves), and nDGP model [28, 29] (dotted curves).

clustering. The primordial perturbations are characterized by $\Delta_{\zeta\zeta}(a_0, k) = \delta_\zeta^2(a_0, k_n) (k/k_n)^{(n-1)} T^2(k)$, where $\delta_\zeta(a_0, k_n) = 4.52 \times 10^{-5}$, $n = 0.95$, $k_n = 0.05 \text{ Mpc}^{-1}$, and $T(k)$ denotes the transfer function. The bin number of source galaxy distributions is running from 1 to 6 between $z = 0.25$ and $z = 2.75$, with the spacing $\Delta z_s = 0.5$. For definiteness, we assume that the galaxy sets come from a net galaxy distribution of $n_g(z) \propto z^2 e^{-(z/1.5)^2}$. The biasing $b(z_i)$ is fixed to 1. We show the accuracy of this reconstruction with diverse binning spacing of $\Delta z_i = 0.02$ and 0.1.

We define an estimator corresponding to the test Eq. (15) as

$$\mathcal{R}_\ell(\alpha^{(1)}) = \frac{\tilde{C}_\ell^{s dd}(\alpha^{(1)})}{C_\ell^{s dd}}. \quad (37)$$

We can also define an estimator which provides a test of

anisotropic stress or $\eta = 1$ as

$$\mathcal{R}_\ell^i(\tilde{\alpha}^{(1)}) = \frac{\Omega_m^2 H_0^4 C_\ell^i \delta_g \delta_g}{a_i^6 H(a_i)^4 C_\ell^i V_m V_m}. \quad (38)$$

If the consistency conditions $\alpha^{(1)} = 1$ and $\tilde{\alpha}^{(1)} = 1$ are broken, these estimators deviate from 1. Then there are two possibilities left: MG or dark energy with clustering and/or anisotropic stress. In this case, we should reconstruct the lensing potential using δ_g and V_m . Then $\Delta_{\Phi_- \Phi_-}^i$ in the discretized approximation is given by

$$\Delta_{\Phi_- \Phi_-}^i = \frac{9}{32\pi^2 l} \frac{D_i^3}{\Delta D_i} \left(\frac{dz}{dD} n_i \right)^{-2} a_i^4 H^4(a_i) \quad (39)$$

$$\times \left[b_i^{-2} C_\ell^i \delta_g \delta_g + b_i^{-1} 2C_\ell^i \delta_g V_m + C_\ell^i V_m V_m \right].$$

The reconstructed deflection power spectra $\tilde{C}_\ell^{s dd}(\alpha^{(2)})$ is given by putting this into Eq. (34). Then the estimator for the final consistency test Eq. (23) is given by

$$\mathcal{R}_\ell(\alpha^{(2)}) = \frac{\tilde{C}_\ell^{s dd}(\alpha^{(2)})}{C_\ell^{s dd}}. \quad (40)$$

In the middle/bottom panel of Fig. 1, we show $\mathcal{R}_\ell(\alpha^{(2)})$ in a $f(R)$ gravity model and $\mathcal{R}_\ell(\alpha^{(2)})$ in the DGP model as examples for MG models. Clearly, the deviation from the GR consistency relation is larger than the errors coming from the reconstruction. Of course, in order to apply this test to real observations, there are numbers of problems we should solve, such as the precise measurements of redshifts, the reconstruction of V_m and the measurements of bias for δ_i . We should also refine our formulation including redshift distortions. However, it is encouraging that it is in principle possible to distinguish between two major possibilities for the late-time acceleration. It would be crucial to calculate the signal to noise ratio in future observations and optimize the detection of the deviation from the consistency condition.

V. SYSTEMATIC ERRORS IN CONSISTENCY TEST

The main challenge in realizing the consistency test is to understand bias and the distribution of galaxies. One possibility to measure bias is to use higher-order statistics proposed by Ref. [30] and applied to the 2dF [31]. Assuming a local bias $\delta_g = b_1 \delta_m + b_2 \delta_m^2/2$ and gaussian initial conditions, the measured bias in the 2df is $b = 1.04 \pm 0.11$. For the SDSS LRGs, Ref. [32] reported that the measured bias is $b = 1.87 \pm 0.07$ assuming a linear biasing. Ref. [33] gives a forecast for the measurements of bias from the bispectrum. Assuming primordial gaussian fluctuations, a 100 Gpc³ survey could measure b_1 with a few percent accuracy. We should note that there are additional systematic errors coming from the assumption on the local biasing. The other method is

to use the redshift distortions to extract the velocity divergent power spectrum $P_{\theta_m \theta_m}$ which we explain below [34]. In the first test, we assume that dark energy has no perturbations and the growth of structure is determined solely by the background cosmology. Then we can reconstruct the power spectrum of density perturbations δ_m using Eq. (9), from which we can measure bias by comparing it with the galaxy power spectrum $P_{\delta_g \delta_g}$. Ref. [34] reported that a full sky survey would measure bias at a sub-percent level at $z < 2$. This includes the systematic errors in the measurement of the velocity divergent power spectrum discussed below. In general, there are additional systematic errors because the assumptions made to measure bias in the above arguments should be modified depending on the nature of gravity. Thus it is very important to understand the effect of MG on bias [36]. One possibility is to extend the halo model approach [37] to MG models. Here the modified Newton constant affects the critical density δ_c at which the spherical over-density collapses. The study of spherically symmetric collapse in modified gravity has just begun [38] and it is necessary to understand the relation between $G_{eff}(a, \mathbf{k})$ and δ_c which eventually determines the halo bias. The uncertainty in galaxy redshift distributions also introduces inconsistency between the reconstructed lensing power spectrum and measured lensing spectrum. This is studied in detail in [34]. It is found that the uncertainty in redshift measurements give an extra bias to the measurements of density perturbations.

In Ref. [34], the errors in the reconstructed power spectrum coming from the uncertainty in the measurement of bias and redshift distributions are estimated as

$$\Delta \tilde{C}_\ell^{s dd} = \left\{ \sum_{i=1}^n \left[\frac{1}{b_i^2} F_\ell^i \left(2 \frac{\Delta b_i}{b_i} \right) \right]^2 \right\}^{1/2}, \quad (41)$$

where

$$F_\ell^i = \frac{9}{\ell^2} \frac{D_i^2 (D_s - D_i)^2}{D_s^2} \left(\frac{dz}{dD} n_i \right)^{-2} \frac{\Omega_m^2 H_0^4}{a_i^2} C_\ell^{i gg}. \quad (42)$$

An estimation of errors using the redshift distortion measurements of bias can be found in Fig. 5 of Ref. [34].

In the second test, it is required to measure the power spectrum of the velocity divergence. One of the most promising ways is to use the redshift distortions. Even though the clustering of galaxies in real space to have no preferred direction, galaxy maps in redshift space show an anisotropic distribution due to the peculiar velocities of galaxies. Thus the measurements of anisotropies allow us to extract the peculiar velocities. The errors on the reconstruction of $P_{\theta_m \theta_m}$ from redshift distortions are estimated in Ref. [35]. Although it is challenging to measure $P_{\theta_m \theta_m}$ from the redshift surveys, it is shown that future surveys such as Euclid can give the fractional errors on $P_{\theta_m \theta_m}$ with a few % to 10% accuracy depending on scales for $k < 0.1 \text{ Mpc}^{-1}$. This is one of the major systematic errors in the second test and we should probably wait for the surveys like SKA to achieve sub % accuracy.

VI. CONCLUSION

In this paper, we proposed a consistency test of general relativity by combining geometrical tests and structure formation tests using large-scale structure. From the geometrical test using supernovae, cosmic microwave background and baryon acoustic oscillations, we can reconstruct the Hubble parameter H . From structure formation, we measure the density perturbation δ_t , the peculiar velocity function V_m and the lensing potential Φ_- . We constructed a consistency test which is written only in terms of these observables, Eq. (23). The major advantage of this approach is that we do not need to assume any theory for modified gravity models to test GR on cosmological scales. This test is essentially the same as the consistency relation derived in [11]. We have shown that this test probes the difference between the Newton constants in the background cosmology and in large-scale structure. The main obstacle in realizing this test was that we should measure all the quantities in the consistency test at the same time and in the same location. This is not straightforward because weak lensing measures an integrated effect of the lensing potential at different redshifts and it is impossible to reconstruct Φ_- at a given redshift. In this paper, we proposed a way to overcome this problem by reconstructing the lensing potential from measured density perturbations and peculiar velocities. In the reconstruction, we use the Poisson equation

in GR to reconstruct Φ from the density perturbation. Thus any inconsistency between the reconstructed lensing power spectrum and the measured lensing spectrum indicates the failure of the Poisson equation in GR.

In summary, we have shown that, given the assumptions (i) the cosmological principle, (ii) energy-momentum conservation for dark matter and baryon, (iii) existence of a Newtonian limit in the MG model, we can test general relativity only by using the measured quantities from the background expansion history and large-scale structure of the Universe. We should also emphasize that there exist independent ways to test these assumptions from observations.

We summarized the major systematic errors in the consistency test in section V. Some of them have been studied in detail in Ref. [34], but clearly controlling these systematic errors are essential to perform the consistency test.

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