# Non-linear isocurvature perturbations and non-Gaussianities

David Langlois[a], Filippo Vernizzi[b] and David Wands[c]

[a] *APC (CNRS-Université Paris 7), 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France*

[b] *Abdus Salam ICTP, Strada Costiera 11, 34014 Trieste, Italy*

[c] *Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth, PO1 2EG, United Kingdom*

**Abstract**

We study non-linear primordial adiabatic and isocurvature perturbations and their non-Gaussianity. After giving a general formulation in the context of an extended $\delta N$ formalism, we analyse in detail two illustrative examples. The first is a mixed curvaton-inflaton scenario in which fluctuations of both the inflaton and a curvaton (a light isocurvature field during inflation) contribute to the primordial density perturbation. The second example is that of double inflation involving two decoupled massive scalar fields during inflation. In the mixed curvaton-inflaton scenario we find that the bispectrum of primordial isocurvature perturbations may be large and comparable to the bispectrum of adiabatic curvature perturbations.

## 1 Introduction

With recent cosmic microwave background (CMB) anisotropy data due to the WMAP satellite and the further improved data expected from the Planck satellite, our knowledge of primordial cosmological perturbations is becoming more and more precise. This influx of data has stimulated the study of models whose predictions differ from the simplest models of single field slow-roll inflation. To discriminate between these models, a particularly important observable is the amplitude (and the shape) of non-Gaussianity of the CMB anisotropies. Another crucial property, potentially observable in the CMB data, would be the presence of a primordial isocurvature (or entropy) component as it would require a multi-field scenario for the origin of the primordial fluctuations.

The purpose of this paper is to investigate the impact of non-adiabatic fluctuations during inflation on the predicted non-Gaussianity of primordial density perturbations, including primordial isocurvature matter perturbations as well as adiabatic modes which would contribute to the bispectrum and higher-order correlations in the CMB anisotropies. Isocurvature perturbations could have large departures from Gaussianity while remaining sub-dominant in the linear perturbation spectrum [1, 2].

To study primordial non-Gaussianity, one needs to study relativistic cosmological perturbations beyond linear order and there has been considerable progress in this field in recent years. On scales larger than the Hubble radius, the non-linear evolution of perturbations generated during inflation is compactly described in terms of the perturbed expansion from an initial hypersurface (usually taken at Hubble crossing during inflation) up to a final uniform-density hypersurface (usually during the radiation-dominated era) – the so-called $\delta N$-formalism [3]. This is particularly useful for evaluating the primordial non-Gaussianity generated on large scales [4].

As we show in this paper, one can easily extend the $\delta N$-formalism to describe the non-Gaussianities of the non-linearly evolved primordial perturbations including isocurvature fluctuations. In order to illustrate our general, but formal, result, we study two emblematic examples of multi-field scenarios, which can generate isocurvature fluctuations in addition to the usual adiabatic fluctuations. The first example is the curvaton scenario [1, 5]. Previous works, e.g. [6, 7, 8], have investigated non-Gaussianity and isocurvature perturbations in this scenario, but in our case, we do not assume that the contribution of inflaton fluctuations to the CMB anisotropies is negligible. In this so-called mixed inflaton-curvaton setup [9, 10], the isocurvature mode is not necessarily constrained by the data to be zero, in contrast with the conclusion of [8]. Our second example is a model of double inflation [11] with two uncoupled massive scalar fields that drive in turn inflation. In contrast with the previous example, the final isocurvature perturbation depends on both scalar field fluctuations during inflation, but it can still be determined analytically at second order.

The adiabatic and isocurvature perturbations we refer to above correspond to the primordial adiabatic and isocurvature perturbations defined during the standard radiation era, i.e., after inflation and *after* the curvaton decay, if any. These perturbations can be related, but are not equivalent, to the *instantaneous* adiabatic and isocurvature (or entropy) field perturbations which can be defined during inflation by decomposing the perturbations along the directions, respectively, parallel and orthogonal to the inflationary trajectory in field space (see [12, 13] in the linear case and [14, 15] in the non-linear case). The instantaneous isocurvature perturbation during inflation is not necessarily converted into an isocurvature perturbation after inflation. However, even if the isocurvature perturbation during inflation does not survive, it can have a strong impact on the resulting primordial adiabatic perturbation and its non-Gaussianity, as illustrated, for instance, recently in the context of multi-field Dirac-Born-Infeld inflation [16].

The outline of the paper is the following. In the next section, we introduce the non-linear definitions of the primordial adiabatic and isocurvature perturbations and show how they are related to the primordial scalar field fluctuations in a very general multi-field inflation framework. The following section is devoted to the study of the mixed curvaton-inflaton scenario. We then consider, in the fourth section, the case of double inflation with two decoupled massive scalar fields. We discuss our results in the final section. In Appendix A we give some details of the calculations of section IV, while in Appendix B we review the decomposition into the adiabatic and entropy components of the field perturbations and their equations of motion at second order, and we compute their 3-point correlation functions. Finally, in the last appendix we give general expressions obtained using the

$\delta N$-formalism for the primordial power spectra and bispectra at leading order.

While this paper was being written up, similar results have been obtained by Kawasaki *et al.* who use the $\delta N$-approach to calculate primordial non-Gaussianity of isocurvature perturbations and in particular axion isocurvature perturbations [17] and baryon isocurvature perturbations [18].

# 2 Adiabatic and isocurvature perturbations

A powerful technique to compute the non-linear primordial perturbations on large scales is the $\delta N$-formalism [3, 4]. The idea is to use solutions to the homogeneous FRW cosmology in order to calculate the integrated expansion on large scales from some initial state to a final state of fixed energy density.

The $\delta N$-formalism is closely related to the notion of non-linear curvature perturbation on uniform density hypersurfaces, which can be defined in a geometrical and covariant way as shown in [19, 20]. Indeed, in the case of a perfect fluid characterized by the energy density $\rho$, the pressure $P$ and the four-velocity $u^a$, the conservation law for the energy-momentum tensor,

$$\nabla_a T^a_{\ b} = 0, \qquad T_{ab} = (\rho + P)\, u_a u_b + P g_{ab}, \tag{1}$$

implies that the covector

$$\zeta_a \equiv \nabla_a \alpha - \frac{\dot{\alpha}}{\dot{\rho}} \nabla_a \rho \tag{2}$$

satisfies the relation

$$\dot{\zeta}_a \equiv \mathcal{L}_u \zeta_a = -\frac{\Theta}{3(\rho + p)} \left( \nabla_a p - \frac{\dot{p}}{\dot{\rho}} \nabla_a \rho \right) , \tag{3}$$

where we have defined

$$\Theta = \nabla_a u^a, \quad \alpha = \frac{1}{3} \int d\tau\, \Theta , \tag{4}$$

and where a dot denotes a Lie derivative along $u^a$, which is equivalent to an ordinary derivative for *scalar* quantities (e.g. $\dot{\rho} \equiv u^a \nabla_a \rho$). This result is valid for any spacetime geometry and does not depend on Einstein's equations. In the cosmological context, $\alpha$ can be interpreted as a non-linear generalization, according to an observer following the fluid, of the number of e-folds of the scale factor.

The covector $\zeta_a$ can be defined for the global cosmological fluid or for any of the individual cosmological fluids (the case of interacting fluids is discussed in [21]). Using the non-linear conservation equation

$$\dot{\rho} = -3\dot{\alpha}(\rho + P) , \tag{5}$$

which follows from $u^b \nabla_a T^a_{\ b} = 0$, one can re-express $\zeta_a$ in the form

$$\zeta_a = \nabla_a \alpha + \frac{\nabla_a \rho}{3(\rho + P)} . \tag{6}$$

If $w \equiv P/\rho$ is constant, the above covector is a total gradient and can be written as

$$\zeta_a = \nabla_a \left[ \alpha + \frac{1}{3(1+w)} \ln \rho \right] . \tag{7}$$

On scales larger than the Hubble radius, the above definitions are equivalent to the non-linear curvature perturbation on uniform density hypersurfaces as defined in [22, 23],

$$\zeta = \delta N - \int_{\bar\rho}^{\rho} H \frac{d\tilde\rho}{\dot{\tilde\rho}} = \delta N + \frac{1}{3}\int_{\bar\rho}^{\rho} \frac{d\tilde\rho}{(1+w)\tilde\rho} \, , \tag{8}$$

where $N = \alpha$ and $H = \dot\alpha = \dot a/a$ is the Hubble rate of the Friedmann metric $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$. The above equation is simply the integrated version of (2), or of (6).

In the following, we will be mainly interested in non-linear isocurvature, or entropy, perturbations. For simplicity, we will consider only cold dark matter (CDM) isocurvature perturbations and assume that the Universe, in the standard eras, is filled with only two fluids: the radiation fluid and the CDM fluid. Our analysis can be easily extended to other types of isocurvature perturbations.

It will be useful to distinguish the non-linear curvature perturbation $\zeta$ of the total fluid, which describes the primordial *adiabatic* perturbation, from the non-linear perturbations $\zeta_{\rm r}$ and $\zeta_{\rm m}$ describing respectively the radiation fluid ($w_r = 1/3$) and the cold dark matter (CDM) fluid ($w_m = 0$), which are given, according to our definitions (7) or (8), by

$$\zeta_{\rm r} = \delta N + \frac{1}{4}\ln\left(\frac{\rho_{\rm r}}{\bar\rho_{\rm r}}\right) \, , \tag{9}$$

$$\zeta_{\rm m} = \delta N + \frac{1}{3}\ln\left(\frac{\rho_{\rm m}}{\bar\rho_{\rm m}}\right) \, , \tag{10}$$

where a bar denotes a homogeneous quantity.

In the radiation dominated era, the adiabatic perturbation coincides with $\zeta_{\rm r}$, whereas the CDM isocurvature perturbation is characterized by the non-linear perturbation

$$\mathcal{S}_{\rm m} = 3(\zeta_{\rm m} - \zeta_{\rm r}) = \ln\left(\frac{\rho_{\rm m}}{\bar\rho_{\rm m}}\right) - \frac{3}{4}\ln\left(\frac{\rho_{\rm r}}{\bar\rho_{\rm r}}\right) \, , \tag{11}$$

which can be expanded in terms of the density contrasts $\delta_{\rm r} = \delta\rho_{\rm r}/\bar\rho_{\rm r}$ and $\delta_{\rm m} = \delta\rho_{\rm m}/\bar\rho_{\rm m}$,

$$\mathcal{S}_{\rm m} = \delta_{\rm m} - \frac{3}{4}\delta_{\rm r} - \frac{1}{2}\delta_{\rm m}^2 + \frac{3}{8}\delta_{\rm r}^2 + \dots \tag{12}$$

Note that these expressions are independent of the hypersurface on which the density perturbations are defined.

Since our goal is to relate the perturbations in the radiation era to the perturbations produced during an inflationary era, it is important to generalize Eq. (8) for scalar fields. In this case a convenient description is in terms of the (relative) comoving curvature perturbation,[1]

$$\mathcal{R}_A = \delta N - \int_{\bar\varphi_A}^{\varphi_A} H \frac{d\tilde\varphi_A}{\dot{\tilde\varphi}_A} \, , \tag{13}$$

which is the curvature perturbation on constant $\varphi_A$ hypersurface. In slow-roll inflation, the initial state of the system – when the cosmological perturbations are produced – is defined only by the scalar field values, $\varphi_{A*}$, on an initial spatially-flat hypersurface, where with a star we denote that

[1]Note that the convention adopted here is that $\mathcal{R}_A$ has the same sign as $\zeta_A$, such that, in the single field case, $\mathcal{R} = \zeta$ on large scales.

we evaluate the quantity at Hubble crossing $k = aH$. One can then calculate the number of e-folds, or integrated expansion, $N^{(\varphi_A)}$, from this initial state to a "final" hypersurface characterized by the "final" scalar field amplitudes $\varphi_A$. By choosing the final hypersurface to be of uniform $A$-field, one can write $\mathcal{R}_A$ as a perturbative expansion in terms of the initial field fluctuations $\delta\varphi_{A*}$, whose correlation properties must be known. Equation (13) thus becomes

$$\mathcal{R}_A = \delta N^{(\varphi_A)} = N^{(\varphi_A)}_{,A}\delta\varphi_{A*} + \frac{1}{2}N^{(\varphi_A)}_{,AB}\delta\varphi_{A*}\delta\varphi_{B*} + \dots \,, \tag{14}$$

where $N^{(\varphi_A)}_{,A} = \partial N^{(\varphi_A)}/\partial\varphi_{A*}$, etc. This is a particular application of the $\delta N$-formalism that generalizes the usual expansion of $N$ defined on a final *total* uniform density hypersurface. Note that when there are several scalar fields, $\mathcal{R}_A$ can be different from the relative curvature perturbation on uniform density hypersurfaces $\zeta_A$. Indeed, the uniform density and uniform field hypersurfaces do not always coincide even on large scales [24].

However, the *total* comoving and uniform density hypersurfaces coincide on large scales at second [25] and non-linear order [22, 15] and $\zeta$ is generally used to describe the adiabatic perturbation also for scalar fields. The curvature perturbation on uniform density hypersurfaces $\zeta$ will be given now as the standard perturbative expansion of $N$ defined on a final uniform density hypersurface. Thus one can rewrite $\zeta$ in terms of the expansion [4]

$$\zeta = \delta N = N_{,A}\delta\varphi_{A*} + \frac{1}{2}N_{,AB}\delta\varphi_{A*}\delta\varphi_{B*} + \dots \,. \tag{15}$$

Similarly, the non-linear isocurvature perturbation (11) can be given in terms of the difference in the non-linear expansion, $\mathcal{S}_{\rm m} = 3\delta(\Delta N)$, where $\Delta N \equiv N^{(\rm m)} - N^{(\rm r)}$, between final hypersurfaces of uniform matter density and uniform radiation density

$$\mathcal{S}_{\rm m} = 3\left(\delta N^{(\rm m)} - \delta N^{(\rm r)}\right) = 3\Delta N_{,A}\delta\varphi_{A*} + \frac{3}{2}\Delta N_{,AB}\delta\varphi_{A*}\delta\varphi_{B*} + \dots \,. \tag{16}$$

In the following sections we apply these definitions to two examples: the curvaton model and double inflation. Although the previous expressions hold non-linearly, we will concentrate on a second order expansion, which is expected to give the leading order terms for the 3-point correlation properties. We will assume that the initial field perturbations (on scales close to the horizon scale during inflation) are independent, Gaussian random fields. Thus any non-Gaussianity of the curvature perturbations will arise from the non-linear terms in Eqs. (15) and (16). This is a good approximation for weakly-coupled scalar fields (with canonical kinetic terms) during slow-roll inflation [26] but may break-down for scalar fields with non-standard kinetic terms.

# 3 Mixed inflaton and curvaton perturbations

As a first application of the general formalism presented in the previous section, we consider a curvaton scenario [5], or more precisely a mixed inflaton and curvaton scenario [9, 10] as we will take into account both the perturbations generated by the inflaton field driving inflation and the curvaton. The curvaton is a weakly coupled scalar field, $\chi$, which is light relative to the Hubble rate during inflation, and hence acquires an almost scale-invariant spectrum and effectively Gaussian distribution of perturbations, $\delta\chi$, during inflation. After inflation the Hubble rate drops and eventually the curvaton becomes non-relativistic so that its energy density grows relative to radiation, until it contributes a significant fraction of the total energy density, $\Omega_\chi \equiv \bar{\rho}_\chi/\bar{\rho}$, before it decays. Hence the initial curvaton field perturbations on large scales can give rise to a primordial density perturbation after it decays.

The non-relativistic curvaton (mass $m \gg H$), before it decays, can be described by a pressureless, non-interacting fluid with energy density

$$\rho_\chi = m^2 \chi^2 \ , \tag{17}$$

where $\chi$ is the rms amplitude of the curvaton field, which oscillates on a timescale $m^{-1}$ much less than the Hubble time $H^{-1}$. Making use of Eq. (10) for the oscillating curvaton to rewrite its local density in terms of its homogeneous value and the inhomogeneous expansion perturbation, $\delta N$, we have

$$\rho_\chi = \bar{\rho}_\chi e^{3(\zeta_\chi - \delta N)} \ . \tag{18}$$

In the post-inflation era where the curvaton is still subdominant, the spatially flat hypersurfaces are characterized by $\delta N = \zeta_{\rm inf}$, where $\zeta_{\rm inf}$ corresponds to the adiabatic perturbation generated by the inflaton fluctuations. On such a hypersurface, the curvaton energy density can be written as

$$\bar{\rho}_\chi e^{3(\zeta_\chi - \zeta_{\rm inf})} = \bar{\rho}_\chi e^{\mathcal{S}_\chi} = m^2 \left(\bar{\chi} + \delta\chi\right)^2 \ . \tag{19}$$

where $\mathcal{S}_\chi \equiv 3(\zeta_\chi - \zeta_{\rm inf})$ is the entropy perturbation of the curvaton.

As long as the curvaton is subdominant and weakly-coupled, so that we may neglect self-interactions, the evolution equation for $\chi$ is linear, and the field perturbation $\delta\chi$ obeys the same evolution equation on super-Hubble scales as the background expectation value $\bar{\chi}$. In this case it is well known that the ratio $\delta\chi/\bar{\chi}$ remains unchanged as long as the curvaton is subdominant [11]. This result holds also at second order in the perturbation $\delta\chi$, as shown in [15] and in Appendix B, where we have written the evolution equation of an entropy field perturbation. Thus, expanding Eq. (19) at second order we obtain

$$\mathcal{S}_\chi = 2\frac{\delta\chi_*}{\bar{\chi}_*} - \left(\frac{\delta\chi_*}{\bar{\chi}_*}\right)^2 \ . \tag{20}$$

Note that we will assume that the initial curvaton field perturbations, $\delta\chi_*$, are strictly Gaussian, as would be expected for a weakly coupled field.

The precise density perturbation produced after the curvaton decays can be calculated numerically [27, 28, 23], but it can also be estimated analytically using the sudden-decay approximation [6], which assumes that the curvaton decays suddenly on a spatial hypersurface of uniform total energy density. Any initial inflaton perturbation gives rise to a perturbation in the radiation energy density *before* the curvaton decay, which we denote by $\rho_R$. Similarly to Eqs. (9) and (10) we can write

$$\rho_R = \bar{\rho}_R e^{4(\zeta_{\rm inf} - \delta N)} \ , \qquad \rho_\chi = \bar{\rho}_\chi e^{3(\zeta_\chi - \delta N)} \ . \tag{21}$$

On the decay hypersurface characterized by $\rho_\chi + \rho_R = \bar{\rho}_{\rm r}$ and thus $\delta N = \zeta_{\rm r}$ where $\zeta_{\rm r}$ is the total curvature perturbation *after* the decay, we find [23]

$$\Omega_{\chi,{\rm decay}} e^{3(\zeta_\chi - \zeta_{\rm r})} + (1 - \Omega_{\chi,{\rm decay}}) e^{4(\zeta_{\rm inf} - \zeta_{\rm r})} = 1 \ , \tag{22}$$

where $\Omega_{\chi,\mathrm{decay}} \equiv \bar{\rho}_\chi/(\bar{\rho}_\chi + \bar{\rho}_R)$.

Expanding Eq. (22) at first order we obtain

$$\zeta_{\mathrm{r}} = r\zeta_\chi + (1-r)\zeta_{\mathrm{inf}}\,, \tag{23}$$

where $r \equiv 3\Omega_{\chi,\mathrm{decay}}/(4-\Omega)_{\chi,\mathrm{decay}}$. Up to second-order we obtain

$$\zeta_{\mathrm{r}} = r\zeta_\chi + (1-r)\zeta_{\mathrm{inf}} + \frac{r(1-r)(3+r)}{2}\left(\zeta_\chi - \zeta_{\mathrm{inf}}\right)^2 = \zeta_{\mathrm{inf}} + \frac{r}{3}\mathcal{S}_\chi + \frac{r(1-r)(3+r)}{18}\mathcal{S}_\chi^2\,. \tag{24}$$

The entropy perturbation (20) contains a linear part $S_G$ which is Gaussian and a second order part which is quadratic in $S_G$:

$$\mathcal{S}_\chi = S_G - \frac{1}{4}S_G^2\,, \quad \text{where} \quad S_G \equiv 2\frac{\delta\chi_*}{\bar{\chi}_*}\,. \tag{25}$$

Substituting in (24) we then have

$$\zeta_{\mathrm{r}} = \zeta_{\mathrm{inf}} + \frac{r}{3}S_G + \frac{r}{18}\left(\frac{3}{2} - 2r - r^2\right)S_G^2\,. \tag{26}$$

Keeping only the linear part of the above relation, one finds that the power spectrum for the primordial adiabatic perturbation $\zeta_{\mathrm{r}}$ can be expressed as

$$\mathcal{P}_{\zeta_{\mathrm{r}}} = \mathcal{P}_{\zeta_{\mathrm{inf}}} + \frac{r^2}{9}\mathcal{P}_{S_G}\,, \tag{27}$$

where the entropy power spectrum amplitude is given by

$$\mathcal{P}_{S_G} = \frac{4}{\chi_*^2}\left(\frac{H_*}{2\pi}\right)^2\,. \tag{28}$$

In the case of single field inflation we have

$$\mathcal{P}_{\zeta_{\mathrm{inf}}} = \frac{1}{2M_P^2\epsilon_*}\left(\frac{H_*}{2\pi}\right)^2\,, \tag{29}$$

where $\epsilon_* \equiv -\dot{H}_*/H_*^2$ is the usual slow-roll parameter during inflation and $M_P^2 = (8\pi G)^{-1}$ is the reduced Planck mass. In order to compare the relative contributions of the inflaton and of the curvaton in the final power spectrum (27), it is useful to introduce the dimensionless parameter [29]

$$\lambda \equiv \frac{8}{9}r^2\epsilon_*\left(\frac{M_P}{\chi_*}\right)^2 \tag{30}$$

so that $\mathcal{P}_{\zeta_{\mathrm{r}}} = (1+\lambda)\mathcal{P}_{\zeta_{\mathrm{inf}}}$. If $\lambda \gg 1$, one recovers the standard curvaton scenario where the inflaton perturbations can be ignored: since $r$ and $\epsilon_*$ are bounded by 1, this requires $\chi_* \ll M_P$. A value of $\lambda$ of order 1 or smaller is possible if $r$ or $\epsilon_*$ are sufficiently small and/or $\chi_*$ is of the order of $M_P$. In the present work, we will always assume $\chi_* \ll M_P$. If this is not the case the curvaton starts to oscillate at about the same time as it decays and cannot be described as a dust field (see [9] for details).

In slow-roll inflation the 3-point function of the inflaton perturbations, $\zeta_{\rm inf}$, is suppressed by slow-roll parameters [30, 31] and large non-Gaussianities can arise only from the curvaton contribution. Indeed, the 3-point function of $\zeta_{\rm r}$ yields (see also [32] for a similar analysis)

$$\langle \zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\zeta_{\rm r}(\vec{k}_3)\rangle = (2\pi)^3\delta(\Sigma_i \vec{k}_i) b_{NL}^{\zeta\zeta\zeta}\left[P_{\zeta_{\rm r}}(k_1)P_{\zeta_{\rm r}}(k_2) + {\rm perms}\right] , \tag{31}$$

where $P_{\zeta_{\rm r}}(k) = 2\pi^2 \mathcal{P}_{\zeta_{\rm r}}(k)/k^3$ and $b_{NL}^{\zeta\zeta\zeta}$ is a non-linear parameter given in this case by

$$b_{NL}^{\zeta\zeta\zeta} = \frac{1}{r}\frac{\left(\frac{3}{2} - 2r - r^2\right)}{(1+\lambda^{-1})^2} , \tag{32}$$

as follows from Eq. (26). Non-Gaussianities are thus significant when the curvaton decays well before it dominates, $r \ll 1$.

When $\lambda \gg 1$ and the perturbations from inflation are negligible, one recovers the standard curvaton result [33] and $b_{NL}^{\zeta\zeta\zeta}$ is proportional to the much used local non-linear parameter $f_{NL}$ defined by $\zeta_{\rm r} = \zeta_G + (3/5) f_{NL}\zeta_G^2$, i.e., $b_{NL}^{\zeta\zeta\zeta} = (6/5) f_{NL}$. However, in general $b_{NL}^{\zeta\zeta\zeta}$ is different from $f_{NL}$. Indeed, for other values of $\lambda$, although only the curvaton contributes to the 3-point function, the 2-point function depends also on the initial inflaton fluctuation, which is a Gaussian random field, independent of the curvaton fluctuation. This differs from the original definition of $f_{NL}$ where only one Gaussian random field is present [34].

It is instructive to see how (32) depends on the curvaton expectation value during inflation, $\chi_*$. Substituting the relation $r \sim (\chi_*/M_P)^2/\sqrt{\Gamma_\chi/m_\chi}$ (valid in the limit $r \ll 1$) [35], where $\Gamma_\chi$ is the decay rate of the curvaton, into the definition (30), one sees that $\lambda$ is proportional to $\chi_*^2$, like $r$. One then finds that $b_{NL}^{\zeta\zeta\zeta}$ given in (32) reaches its maximal value $b_{NL}^{\zeta\zeta\zeta}({\rm max}) \sim \epsilon_*/\sqrt{\Gamma_\chi/m_\chi}$ for $\lambda \sim 1$, i.e., for $\chi_* \sim \sqrt{\Gamma_\chi/(m_\chi\epsilon_*)}M_P$. A significant non-Gaussianity is thus possible if $\epsilon_* \gg \sqrt{\Gamma_\chi/m_\chi}$. Note also that when $r$ becomes small $b_{NL}^{\zeta\zeta\zeta}$ does not grow indefinitely as one would naively expect by considering $f_{NL} \simeq 5/(4r)$. Finally, in the limit $r \ll 1$ and $\lambda \ll 1$, where the inflaton contribution dominates the power spectrum, the expression (32) simplifies into

$$b_{NL}^{\zeta\zeta\zeta} \simeq \frac{3}{2}\frac{\lambda^2}{r} \sim \frac{\epsilon_*^2 m_\chi^{3/2}}{\Gamma_\chi^{3/2}}\frac{\chi_*^2}{M_P^2} , \qquad (\lambda \ll 1, \quad r \ll 1) . \tag{33}$$

After the analysis of the non-Gaussianities for the *adiabatic* perturbation, which essentially agrees with the discussion given in [32], let us now turn to entropy perturbations between CDM and radiation,

$$\mathcal{S}_{\rm m} = 3\left(\zeta_{\rm m} - \zeta_{\rm r}\right) , \tag{34}$$

which could be generated *in the radiation era after the curvaton decay.* If all the particle species are in full thermal equilibrium after the curvaton decays, with vanishing chemical potentials, then the primordial density perturbation must be adiabatic [6, 36] and we have $\zeta_{\rm m} = \zeta_{\rm r}$ and hence $\mathcal{S}_{\rm m} = 0$. However, if CDM remains decoupled from (part of) the radiation, the curvaton isocurvature perturbation may be converted into a residual isocurvature perturbation after the curvaton decays. We now consider two possibilities leading to a non-trivial isocurvature perturbation [6].

## 3.1 CDM created *before* curvaton decay

If the CDM is created before the curvaton decay, then $\zeta_{\rm m} = \zeta_{\rm inf}$, which generates

$$\mathcal{S}_{\rm m} = 3\left(\zeta_{\rm inf} - \zeta_{\rm r}\right) = -rS_G - \frac{r}{6}\left(\frac{3}{2} - 2r - r^2\right)S_G^2 . \tag{35}$$

This implies that the ratio between the isocurvature and adiabatic power spectra is given by

$$\frac{\mathcal{P}_{\mathcal{S}_{\rm m}}}{\mathcal{P}_{\zeta_{\rm r}}} = \frac{9}{1+\lambda^{-1}} \,. \tag{36}$$

This quantity is constrained to be small by the CMB data. In the case where the curvaton dominates the final $\zeta$, i.e. $\lambda \gg 1$, this scenario is thus ruled out and this case is often disregarded in the literature [7]. However, if the inflaton contribution is sufficiently important, $\lambda \ll 1$, such an entropy contribution is allowed. More specifically, the observational constraint on $\alpha$, defined by $\mathcal{P}_{\mathcal{S}_{\rm m}}/\mathcal{P}_{\zeta_{\rm r}} \equiv \alpha/(1-\alpha)$, is currently $\alpha_0 < 0.067$ at 95% CL [37]. The subscript 0 refers to the case where the entropy and adiabatic fluctuations are *un-correlated*, which is appropriate here when $\lambda \ll 1$. The non-Gaussianity of $\zeta$ is described by Eq. (33). Thus, it can become significant without violating the current bound on the presence of isocurvature component in the power spectrum.

The amount of non-Gaussianity in the temperature fluctuations of the CMB anisotropies will depend both on $\zeta_{\rm r}$ and $\mathcal{S}_{\rm m}$. Thus, a complete study of these anisotropies would require the knowledge of the 3-point correlation properties of both variables. One can generalize Eq. (31) and define

$$\langle X(\vec{k}_1)Y(\vec{k}_2)Z(\vec{k}_3)\rangle = (2\pi)^3\delta(\Sigma_i\vec{k}_i)b_{NL}^{XYZ}\left[P_{\zeta_{\rm r}}(k_1)P_{\zeta_{\rm r}}(k_2) + \text{perms}\right] \,, \tag{37}$$

where $X, Y, Z$ can be $\zeta_{\rm r}$ or $\mathcal{S}_{\rm m}$. In this case all the 3-point functions have the same amplitude (up to numerical factors of $-3$), $b_{NL}^{\mathcal{SSS}} = -3b_{NL}^{\mathcal{SS}\zeta} = 9b_{NL}^{\mathcal{S}\zeta\zeta} = -27b_{NL}^{\zeta\zeta\zeta}$, and equally contribute to the non-Gaussianity of the CMB temperature anisotropies.

## 3.2 CDM created *from* curvaton decay

The second possibility leading to non-trivial isocurvature perturbation is when the local matter density is produced solely from the local curvaton density (for instance, some fraction of the curvaton decays to produce CDM particles or the out-of-equilibrium curvaton decay generates the primordial baryon asymmetry). Then we expect the matter density to be directly proportional to the curvaton density on the decay hypersurface

$$\bar{\rho}_{\rm cdm} e^{3(\zeta_{\rm m}-\zeta_{\rm r})} = c\bar{\rho}_\chi e^{3(\zeta_\chi-\zeta_{\rm r})} \,, \tag{38}$$

where $c = (\bar{\rho}_m/\bar{\rho}_\chi) \ll 1$, and hence to all orders $\zeta_{\rm m} = \zeta_\chi$. The matter isocurvature perturbation (34) is then given by

$$\mathcal{S}_{\rm m} = 3(\zeta_\chi - \zeta_{\rm r}) = \mathcal{S}_\chi + 3(\zeta_{\rm inf} - \zeta_{\rm r}) = (1-r)\left(S_G - \frac{3+6r+2r^2}{12}S_G^2\right) \,. \tag{39}$$

This implies that the ratio between the isocurvature and adiabatic power spectra is

$$\frac{\mathcal{P}_{\mathcal{S}_{\rm m}}}{\mathcal{P}_{\zeta_{\rm r}}} = \frac{9(1-r)^2}{r^2(1+\lambda^{-1})} \,. \tag{40}$$

This quantity can be small, as required by observations, in two limiting cases. Either $r$ is very close to 1 or $\lambda$ is very small. In the pure curvaton model ($\lambda \gg 1$), $r$ is constrained to be very close to 1,

$$9(1-r)^2 \simeq \alpha_{-1} < 0.0037 \quad (95\%\ \text{CL}), \tag{41}$$

using the constraint given in [37] for the totally *anti-correlated* case,[2] and the non-linearity parameters $b_{NL}^{XYZ}$ defined in Eq. (37) involving entropy perturbations are suppressed by factors of $(1-r)$ with respect to $b_{NL}^{\zeta\zeta\zeta}$, itself of order unity.

If the inflaton dominates the linear perturbations, i.e. $\lambda \ll 1$, then the ratio (40) can be small even if the curvaton decays long before it dominates. For non-Gaussianity, the small $r$ limit appears interesting because the amplitudes of the 3-point correlation functions $b_{NL}^{XYZ}$ are related by

$$b_{NL}^{\mathcal{SSS}} \sim \frac{3}{r} b_{NL}^{\mathcal{SS}\zeta} \sim \frac{9}{r^2} b_{NL}^{\mathcal{S}\zeta\zeta} \sim \frac{27}{r^3} b_{NL}^{\zeta\zeta\zeta} , \tag{42}$$

which shows that the amplitude of $b_{NL}^{\mathcal{SSS}}$ can be much larger than that of $b_{NL}^{\zeta\zeta\zeta}$ in this small $r$ limit.

However this situation is viable only if $\lambda$ satisfies the constraint

$$\frac{\mathcal{P}_{\mathcal{S}_{\rm m}}}{\mathcal{P}_{\zeta_{\rm r}}} \simeq \frac{9\lambda}{r^2} \simeq \alpha_0 < 0.067 \quad (95\% \text{ CL}) \qquad (r \ll 1, \quad \lambda \ll 1) , \tag{43}$$

using the constraint given in [37] for the *un-correlated* case. In this case $b_{NL}^{\zeta\zeta\zeta}$ is given by its limit in Eq. (33) and the expression for the non-Gaussianity of the isocurvature component,

$$b_{NL}^{\mathcal{SSS}} = -\frac{27}{2}(3+6r+2r^2)\frac{(1-r)^3}{r^4(1+\lambda^{-1})^2} \tag{44}$$

reduces to the simpler form

$$b_{NL}^{\mathcal{SSS}} \simeq -\frac{1}{2}\left(\frac{9\lambda}{r^2}\right)^2 \simeq -32\epsilon_*^2\left(\frac{M_P}{\chi_*}\right)^4 , \tag{45}$$

where one recognizes the square of the isocurvature/adiabatic ratio $\mathcal{P}_{\mathcal{S}_{\rm m}}/\mathcal{P}_{\zeta_{\rm r}}$, which must be very small because of the constraint (43).

Thus, even if the non-Gaussianity of the isocurvature component is much bigger than that of the adiabatic component, the constraint on the amplitude of the linear isocurvature perturbations (43) also constrains the magnitude of the bispectrum of isocurvature perturbations to be small. This follows simply from the fact that in this case the second order part of the matter isocurvature perturbation, $\mathcal{S}_{\rm m}$ in Eq. (39) is of order $\mathcal{S}_{\rm m}^2$, and if the linear part is constrained, then so is the non-linear part.

# 4 Double quadratic inflation

In this section, we consider double quadratic inflation [11], i.e., an inflationary phase driven by two massive and minimally coupled scalar fields described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(\partial_\mu\chi)^2 - V(\phi,\chi) , \qquad V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 . \tag{46}$$

The adiabatic curvature perturbations generated by this model have been computed in [11, 38, 39] and their non-Gaussianity in [40, 41, 42] (see also [43]). This model can also generate isocurvature

[2] Reference [37] uses the same convention as ours for the sign of the adiabatic and isocurvature perturbations. However, the cross-correlation is defined with opposite sign to $\langle\zeta_{\rm r}\mathcal{S}_{\rm m}\rangle$, so that in the pure curvaton case adiabatic and isocurvature perturbations are referred to as being *anti-correlated*.

perturbations. At linear order these have been computed in [24] and it was first noticed in [44] that they can be correlated with the adiabatic ones. Here we extend these results at second order and we compute the 3-point correlation properties of the isocurvature perturbations and their correlation with the adiabatic perturbations.

We start by computing the adiabatic curvature perturbation $\zeta$ at second order, using the $\delta N$-formalism. The expression of the number of e-folds in terms of the scalar fields can be obtained from the slow-roll equations of motion, which read

$$3H^2 M_P^2 = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} m_\chi^2 \chi^2 , \quad 3H\dot{\phi} + m_\phi^2 \phi = 0 , \quad 3H\dot{\chi} + m_\chi^2 \chi = 0 , \tag{47}$$

and imply $dN/dt = H = -(\phi\dot{\phi} + \chi\dot{\chi})/(2M_P^2)$. For a given scale, the number of e-folds between horizon crossing $t_*$ and some subsequent time $t$ is given by the expression

$$N = \frac{1}{4M_P^2} \left( \phi_*^2 + \chi_*^2 - \phi^2 - \chi^2 \right) , \tag{48}$$

where $\phi = \phi(t)$ and $\chi = \chi(t)$. Furthermore, from the last two slow-roll equations in (47) one can derive

$$\left( \frac{\phi}{\phi_*} \right)^{R^2} = \frac{\chi}{\chi_*} , \tag{49}$$

where $R \equiv m_\chi / m_\phi$ and without loss of generality we take $R \geq 1$.

To compute $\zeta$, we choose as final hypersurface at time $t$ a uniform density hypersurface defined by the condition

$$R^2 \chi^2 + \phi^2 = C . \tag{50}$$

Then, Eqs. (49) and (50) uniquely fix the relation between $(\phi, \chi)$ and $(\phi_*, \chi_*)$. Indeed, by combining these two equations we can derive

$$R^2 \left( \frac{\phi}{\phi_*} \right)^{2R^2} \chi_*^2 + \phi^2 = C, \qquad R^2 \chi^2 + \left( \frac{\chi}{\chi_*} \right)^{\frac{2}{R^2}} \phi_*^2 = C , \tag{51}$$

which can be used to find the derivatives of $\phi$ and $\chi$ with respect to $\phi_*$ and $\chi_*$. These relations can then be employed to compute the first and second derivatives of $N$ with respect to $\phi_*$ and $\chi_*$ by differentiating Eq. (48). The calculation is reported in Appendix A. By using Eq. (15) up to second order in $\delta\phi_*$ and $\delta\chi_*$, one obtains an expression for $\zeta$,

$$\begin{aligned} \zeta \ &= \ \frac{1}{2M_P^2} \left( \bar{\phi}_* \delta\phi_* + \bar{\chi}_* \delta\chi_* + \frac{1}{2} \delta\phi_*^2 + \frac{1}{2} \delta\chi_*^2 \right) \\ &\quad + \frac{(R^2 - 1)}{2M_P^2} \bar{g} \left[ \frac{\delta\chi_*}{\bar{\chi}_*} \left( 1 - \frac{\delta\chi_*}{2\bar{\chi}_*} \right) - R^2 \frac{\delta\phi_*}{\bar{\phi}_*} \left( 1 - \frac{\delta\phi_*}{2\bar{\phi}_*} \right) + \frac{\bar{\chi}_* \bar{g}_{,\chi_*}}{2\bar{g}} \left( \frac{\delta\chi_*}{\bar{\chi}_*} - R^2 \frac{\delta\phi_*}{\bar{\phi}_*} \right)^2 \right] , \end{aligned} \tag{52}$$

where $g = g(\phi, \chi)$ is defined as $g = \phi^2 \chi^2 / (R^4 \chi^2 + \phi^2)$. This relation holds until the end of slow-roll inflation.

We will consider the following scenario. Inflation is initially driven by the heavy field, $\chi$, which slow-rolls down the potential. Later, the heavy field becomes subdominant and then starts oscillating, while the light field, $\phi$, drives inflation. In the last stage of slow-roll inflation, when $\bar{\chi} \ll M_P$, the coefficients $\bar{g}/M_P^2$ and $\bar{\chi}_* \bar{g}_{,\chi_*}/M_P^2$ are very small and the second line of Eq. (52) becomes negligible.

The curvature perturbations $\zeta$ thus becomes effectively constant, and since its value is unaffected by the subsequent stages of inflation and reheating, one finds the expression

$$\zeta_{\rm r} = \frac{1}{2M_P^2}\left(\bar{\phi}_*\delta\phi_* + \bar{\chi}_*\delta\chi_* + \frac{1}{2}\delta\phi_*^2 + \frac{1}{2}\delta\chi_*^2\right) \tag{53}$$

for the adiabatic perturbation during the radiation era.

Let us now focus on the isocurvature perturbation which can be produced, after inflation, in this type of model. To determine it, it is convenient to use the relative comoving curvature perturbations $\mathcal{R}_\phi$ and $\mathcal{R}_\chi$. According to Eq. (13), they are given by

$$\mathcal{R}_\phi = \delta N - \int_{\bar{\phi}}^{\phi} H\frac{d\tilde{\phi}}{\dot{\tilde{\phi}}}\,, \qquad \mathcal{R}_\chi = \delta N - \int_{\bar{\chi}}^{\chi} H\frac{d\tilde{\chi}}{\dot{\tilde{\chi}}}\,. \tag{54}$$

The light field, $\phi$, remains in slow-roll all the time during inflation.[3] In order to compute $\mathcal{R}_\phi$ at second order we can use Eq. (14) for $\varphi_A = \phi$, and expand $N^{(\phi)}$, i.e., the number of e-folds from an initial flat hypersurface at $t_*$ to a final uniform field $\phi$ hypersurface, up to second order in $\delta\phi_*$ and $\delta\chi_*$. To compute $N^{(\phi)}$ we substitute Eq. (49) in Eq. (48) and impose that the final value of $\phi$ is a constant, $\phi = C_\phi$. Using this condition and differentiating $N^{(\phi)}$ with respect to the initial field values (see Appendix A), Eq. (14) then yields

$$\mathcal{R}_\phi = \zeta_r + \frac{\bar{\chi}^2}{2M_P^2}\left(R^2\frac{\delta\phi_*}{\bar{\phi}_*} - \frac{\delta\chi_*}{\bar{\chi}_*} - \frac{2R^4+R^2}{2}\frac{\delta\phi_*^2}{\bar{\phi}_*^2} - \frac{1}{2}\frac{\delta\chi_*^2}{\bar{\chi}_*^2} + 2R^2\frac{\delta\phi_*}{\bar{\phi}_*}\frac{\delta\chi_*}{\bar{\chi}_*}\right)\,, \tag{55}$$

where the explicit expression for $\zeta_r$ is given in (53). At the end of inflation $\phi$ dominates and reheats the universe. When $\phi$ becomes dominant its comoving and uniform energy density curvature perturbations are the same on large scales, $\zeta_\phi = \mathcal{R}_\phi$. Furthermore, when $\bar{\chi}^2 \ll M_P^2$, the second term on the right hand side of Eq. (55) becomes negligible, and $\mathcal{R}_\phi = \zeta_{\rm r}$.

The evolution of the perturbation of the heavy field $\chi$ is more complicated. During the $\phi$-dominated slow-roll phase, a calculation similar to the one for $\phi$ yields, replacing $\phi$ by $\chi$ and $R^2$ by $R^{-2}$ in Eq. (55),

$$\mathcal{R}_\chi|_{\rm slow-roll} = \zeta_r + \frac{3H^2}{m_\phi^2}\left(\frac{1}{R^2}\frac{\delta\chi_*}{\bar{\chi}_*} - \frac{\delta\phi_*}{\bar{\phi}_*} - \frac{2+R^2}{2R^4}\frac{\delta\chi_*^2}{\bar{\chi}_*^2} - \frac{1}{2}\frac{\delta\phi_*^2}{\bar{\phi}_*^2} + \frac{2}{R^2}\frac{\delta\phi_*}{\bar{\phi}_*}\frac{\delta\chi_*}{\bar{\chi}_*}\right)\,, \tag{56}$$

where we have used that $\phi$ dominates the Universe and thus $H^2 = m_\phi^2\bar{\phi}^2/(6M_P^2)$. This expression is valid when $\chi$ is subdominant and in slow-roll but cannot be used when $\chi$ oscillates. It is convenient to use Eq. (54) to rewrite Eq. (56) in terms of the field fluctuation $\delta\chi$ on a constant total energy density hypersurface ($\delta N = \zeta$),

$$\mathcal{R}_\chi|_{\rm slow-roll} = \zeta - \int_{\bar{\chi}}^{\chi} H\frac{d\tilde{\chi}}{\dot{\tilde{\chi}}} = \zeta + \frac{3H^2}{m_\chi^2}\int_{\bar{\chi}}^{\chi}\frac{d\tilde{\chi}}{\tilde{\chi}}\,, \tag{57}$$

where we have used the property that $H$, which depends only on the slow-rolling $\phi$, is (spatially) constant on a constant total energy density hypersurface, since the latter coincides with a constant

[3] We assume that there is no intermediate dust-like phase between the heavy field dominated inflation and the light field dominated inflation.

$\phi$ hypersurface when $\chi$ is subdominant. This yields the non-linear relation between the isocurvature perturbation during slow-roll and the local value of the heavy field,

$$\chi = \bar{\chi} e^{\frac{m_\chi^2}{3H^2}(\mathcal{R}_\chi|_{\rm slow-roll} - \zeta)} \,, \tag{58}$$

which expanded up to second order yields

$$\mathcal{R}_\chi|_{\rm slow-roll} = \zeta + \frac{3H^2}{m_\chi^2}\left(\frac{\delta\chi}{\bar{\chi}} - \frac{1}{2}\frac{\delta\chi^2}{\bar{\chi}^2}\right) \,. \tag{59}$$

As for the curvaton, when $\chi$ oscillates we can describe it as a non-relativistic fluid and use Eq. (18). Expanding this equation up to second order in the field fluctuation $\delta\chi$, we obtain the curvature perturbation $\zeta_\chi$ during the oscillations,

$$\zeta_\chi|_{\rm osc} = \zeta_{\rm r} + \frac{2}{3}\left(\frac{\delta\chi}{\bar{\chi}} - \frac{1}{2}\frac{\delta\chi^2}{\bar{\chi}^2}\right) \,. \tag{60}$$

Now we can use the constancy of $\delta\chi/\bar{\chi}$ valid up to second order to match Eqs. (59) and (60) and express $\zeta_\chi|_{\rm osc}$ in terms of $\mathcal{R}_\chi|_{\rm slow-roll}$. Using Eq. (56), we find that the value of $\zeta_\chi$ after inflation is

$$\zeta_\chi = \zeta_{\rm r} + \frac{2}{3}R^2\left(\frac{1}{R^2}\frac{\delta\chi_*}{\bar{\chi}_*} - \frac{\delta\phi_*}{\bar{\phi}_*} - \frac{2+R^2}{2R^4}\frac{\delta\chi_*^2}{\bar{\chi}_*^2} - \frac{1}{2}\frac{\delta\phi_*^2}{\bar{\phi}_*^2} + \frac{2}{R^2}\frac{\delta\phi_*}{\bar{\phi}_*}\frac{\delta\chi_*}{\bar{\chi}_*}\right) \,. \tag{61}$$

We assume that the light field decays into radiation which dominates the Universe after inflation, and that the heavy field decays into CDM when it oscillates. Then

$$\mathcal{S}_{\rm m} = 3(\zeta_\chi - \zeta_{\rm r}) \,, \tag{62}$$

and

$$\mathcal{S}_{\rm m} = 2R^2\left(\frac{1}{R^2}\frac{\delta\chi_*}{\bar{\chi}_*} - \frac{\delta\phi_*}{\bar{\phi}_*} - \frac{2+R^2}{2R^4}\frac{\delta\chi_*^2}{\bar{\chi}_*^2} - \frac{1}{2}\frac{\delta\phi_*^2}{\bar{\phi}_*^2} + \frac{2}{R^2}\frac{\delta\phi_*}{\bar{\phi}_*}\frac{\delta\chi_*}{\bar{\chi}_*}\right) \,. \tag{63}$$

This equation generalizes at second order the results of [24].

At this point it is useful to express the final curvature and entropy perturbations in terms of the instantaneous adiabatic and entropy perturbations during inflation (more precisely when the scale of interest exits the Hubble radius). The decomposition of two scalar field perturbations in terms of (instantaneous) adiabatic and entropy perturbations has been introduced at linear order in [12] and generalized at non-linear order in [15]. The general definitions are recalled in Appendix B. Here we give the expressions for the adiabatic perturbation $\delta\sigma$ and the entropy perturbation $\delta s$ for the particular case of double quadratic inflation. To simplify the notation, let us define

$$c_\theta = \cos\theta = -\bar{\phi}/\xi \,, \qquad s_\theta = \sin\theta = -R^2\bar{\chi}/\xi \,, \tag{64}$$

with $\xi = (\bar{\phi}^2 + R^4\bar{\chi}^2)^{1/2}$. The angle $\theta$ is simply the angle between the instantaneous direction of the field trajectory and the $\phi$-axis. At first order in perturbations we have

$$\delta\sigma^{(1)} = c_\theta\delta\phi + s_\theta\delta\chi \,, \qquad \delta s^{(1)} = c_\theta\delta\chi - s_\theta\delta\phi \,, \tag{65}$$

while the second order expression are given by

$$\delta\sigma = \delta\sigma^{(1)} - \frac{R^2{c_\theta}^2 + {s_\theta}^2}{2\xi}\delta s^{(1)}\delta s^{(1)} \,, \tag{66}$$

$$\delta s = \delta s^{(1)} + \frac{R^2{c_\theta}^2 + {s_\theta}^2}{\xi}\delta s^{(1)}\delta\sigma^{(1)} + \frac{(R^2-1)c_\theta s_\theta}{2\xi}\delta\sigma^{(1)2} \,. \tag{67}$$

Note that at second order the definition of $\delta\sigma$ contains first order perturbations of $\delta s^{(1)}$ and vice verse. Indeed, as explained in [15], the adiabatic and entropy field decomposition is local and second order fluctuations will be sensitive to first order fluctuations of the angle $\theta$, which can be re-expressed in terms of the field fluctuations $\delta\sigma^{(1)}$ and $\delta s^{(1)}$.

Using these definitions and evaluating Eq. (52) at $t = t_*$, we can rewrite $\zeta_*$, i.e. the curvature perturbation on uniform density hypersurfaces at Hubble crossing,

$$\begin{aligned}\zeta_* \;=\; & -\frac{1}{2M_P^2}\Big[\left(c_{\theta*}^2 + R^{-2}s_{\theta*}^2\right)\xi_*\delta\sigma_* - \frac{1}{2}\left(1 - R^{-2}(1-R^2)^2 c_{\theta*}^2 s_{\theta*}^2\right)\delta\sigma_*^2 \\ & + (R^2-1)c_{\theta*}s_{\theta*}(c_{\theta*}^2 + R^{-2}s_{\theta*}^2)\delta\sigma_*\delta s_*\Big] \;. \end{aligned} \tag{68}$$

The second order expression for $\zeta_*$ contains also the first order entropy field perturbation. Indeed, this is the case also for its general slow-roll form (107) given in Appendix B, derived in [15].

The entropy field perturbation sources $\zeta$ at first and second order (see Eq. (102) in Appendix B). Using Eqs. (53) and (68), the final value of $\zeta$ is thus

$$\zeta_{\rm r} = \zeta_* + \frac{1-R^{-2}}{2M_P^2}\xi_* c_{\theta*}s_{\theta*}\left(\delta s_* - \frac{R^2 c_{\theta*}^4 - s_{\theta*}^4}{2c_{\theta*}s_{\theta*}\xi_*}\delta s_*^2\right) \;. \tag{69}$$

Furthermore, we can rewrite the expression for $\mathcal{S}_{\rm m}$, Eq. (63), in terms of the adiabatic and entropy field perturbations. This reads

$$\mathcal{S}_{\rm m} = -\frac{2R^2}{c_{\theta*}s_{\theta*}\xi_*}\left(\delta s_* + \frac{1 + 2c_{\theta*}^2 + (R^2-1)c_{\theta*}^4}{2c_{\theta*}s_{\theta*}\xi_*}\delta s_*^2\right) \;. \tag{70}$$

Neglecting slow-roll corrections, the field perturbations $\delta\sigma_*$ and $\delta s_*$ are random fields with 2-point functions given by

$$\langle\delta\sigma_*(\vec{k})\delta\sigma_*(\vec{k}')\rangle = \langle\delta s_*(\vec{k})\delta s_*(\vec{k}')\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')\frac{H_*^2}{2k^3} \;, \qquad \langle\delta\sigma_*(\vec{k})\delta s_*(\vec{k}')\rangle = 0 \;. \tag{71}$$

At lowest order in slow-roll, these fields are Gaussian. However, their 3-point correlation functions are non vanishing and have been computed in Appendix B. From the expression of $\zeta_*$, Eq. (68), and from the general definition of $\zeta$ given in Appendix B, Eq. (107), one can see that there are second order corrections proportional to $\delta\sigma_*^2$ and $\delta\sigma_*\delta s_*$ so that $\zeta_*$ has not exactly the same correlation properties as $\delta\sigma_*$. However, these contributions are *generically* (for any slow-roll model) of the same order in slow-roll as the 3-point function of $\delta\sigma_*$ so that at lowest order in slow-roll $\zeta_* \propto \delta\sigma_*$ is a Gaussian random field.

A more convenient parametrization often used in the literature [11] is to rewrite the background values of the scalar fields in polar coordinates,

$$\bar{\phi} = 2M_P\sqrt{N_{\rm e} - N}\cos\alpha \;, \qquad \bar{\chi} = 2M_P\sqrt{N_{\rm e} - N}\sin\alpha \;. \tag{72}$$

In terms of the angle $\alpha$ and of $N_{\rm e} - N_*$, the number of e-folds from Hubble crossing to the end of inflation, the power spectrum of $\zeta_*$ is, using the linear term in Eq. (68),

$$\mathcal{P}_{\zeta_*} = (N_{\rm e} - N_*)\frac{(1 + R^2\tan^2\alpha_*)^2}{(1+\tan^2\alpha_*)(1 + R^4\tan^2\alpha_*)}\left(\frac{H_*}{2\pi M_P}\right)^2 \;. \tag{73}$$

Furthermore, instead of using $\delta s_*$, we find it useful to rewrite Eqs. (69) and (70) in terms of $S_*$, defined as having the same power spectrum of $\zeta_*$, i.e.,

$$S_* \equiv -\frac{1}{2M_P^2}(c_{\theta *}^2 + R^{-2} s_{\theta *}^2)\xi_* \delta s_* \,, \qquad \mathcal{P}_{S_*} = \mathcal{P}_{\zeta_*} \,, \tag{74}$$

in analogy with the linear term of Eq. (68). At leading order in slow-roll $S_*$ is a Gaussian random field uncorrelated with $\zeta_*$. Finally, using this parametrization we obtain

$$\zeta_{\rm r} = \zeta_* + \frac{(1-R^2)\tan\alpha_*}{1+R^2\tan^2\alpha_*}\left[S_* + \frac{\eta_{\phi\phi}}{2}\frac{1-R^6\tan^4\alpha_*}{\tan\alpha_*(1+R^4\tan^2\alpha_*)}S_*^2\right] \,, \tag{75}$$

and

$$\mathcal{S}_{\rm m} = 2\eta_{\phi\phi}\frac{1+R^4\tan^2\alpha_*}{\tan\alpha_*}\left[S_* - \frac{\eta_{\phi\phi}}{2}\frac{2+R^2+4R^4\tan^2\alpha_* + R^8\tan^4\alpha_*}{R^2\tan\alpha_*(1+R^4\tan^2\alpha_*)}S_*^2\right] \,, \tag{76}$$

where $\eta_{\phi\phi}$ is a slow-roll parameter,

$$\eta_{\phi\phi} = \frac{1+\tan^2\alpha_*}{2(N_{\rm e}-N_*)(1+R^2\tan^2\alpha_*)} \,. \tag{77}$$

As discussed in [44], adiabatic and entropy perturbations can be correlated at linear order but the correlation can be neglected when $R^2\tan\alpha_* \ll 1$ or $\tan\alpha_* \gg 1$. Indeed, in this case Eqs. (75) and (76) yields, at linear order, $\zeta_{\rm r} = \zeta_*$ and $\mathcal{S}_{\rm m} \propto S_*$. However, these equations show that adiabatic and entropy perturbations are always correlated at second order, even when they are uncorrelated at linear order. In this particular model of two quadratic potential that we could treat analytically, non-linear terms are small, being suppressed by slow-roll. This leads to small non-Gaussianities in the adiabatic perturbation (cf. Ref. [40]) and also in the entropy perturbation. However, we do not expect this to be a generic feature of all inflation models. In particular, the coefficients in front of the $S_*^2$ terms in Eqs. (75) and (76) may be much larger in other models, which can lead to a non-vanishing 3-point correlation between the adiabatic and entropy perturbations.

# 5 Conclusions

We have calculated the second-order primordial curvature and isocurvature perturbations from two models of inflation in the early universe. In the first example of a mixed curvaton-inflaton model we assume the curvaton is an isocurvature field completely decoupled from the inflaton field driving inflation. In the second, double quadratic inflation model the two massive fields driving inflation are gravitationally coupled during slow-roll.

The field perturbations at Hubble-exit during slow-roll inflation are effectively independent Gaussian random fields; their cross-correlation and non-linearities are suppressed by slow-roll parameters. However the coupled evolution on large scales after Hubble-exit can lead to cross-correlations at linear order [44, 12] and we have calculated the correlations that arise at second-order. This can lead to non-vanishing bispectra for the primordial curvature and isocurvature perturbations and their cross-correlations.

In both cases we find that the non-linear primordial curvature and isocurvature perturbations (15) and (16) can be given in terms of the adiabatic and entropy field perturbations at horizon-exit during inflation,

$$\zeta_{\rm r} = N_{,\sigma}\delta\sigma_* + N_{,s}\delta s_* + \frac{1}{2}N_{,ss}\delta s_*^2 + \left[\frac{1}{2}N_{,\sigma\sigma}\delta\sigma_*^2 + N_{,s\sigma}\delta s_*\delta\sigma_*\right] , \qquad (78)$$

$$\frac{1}{3}\mathcal{S}_{\rm m} = \Delta N_{,s}\delta s_* + \frac{1}{2}\Delta N_{,ss}\delta s_*^2 , \qquad (79)$$

where $N$ describes the expansion to a surface of uniform radiation density in the radiation dominated era and $\Delta N$ describes the difference between the expansion to hypersurfaces of uniform radiation density and uniform matter density. Note that $\Delta N$ vanishes for adiabatic perturbations, i.e., when $\delta s_* = 0$.

In the mixed curvaton-inflaton model we identify the inflaton field with the adiabatic perturbations during inflation, $\delta\sigma_*$, and the curvaton field with entropy field perturbations, $\delta s_*$. The bracketed terms in Eq. (78) can be neglected at leading order in a slow-roll approximation. It is well known that $\zeta$ can have a significant non-Gaussianity when $r \ll 1$ in the curvaton scenario (i.e., when $\lambda$, the ratio between curvaton and inflaton contributions to the curvature power spectrum, is large). However, in this case the primordial isocurvature perturbations are constrained to be very small [7, 8]. We have shown that it is possible for a residual isocurvature perturbation to have a bispectrum which is much larger than that of the adiabatic perturbation if $\lambda$ is small, i.e., if the inflaton perturbation dominates the primordial curvature perturbation at first order, and if the CDM is produced by the curvaton decay. However observational constraints on the power spectrum of isocurvature perturbations also constrains the bispectra to be small in this case. The most interesting situation is the scenario where the CDM is created before the curvaton decay, which is viable if the inflaton contribution dominates the linear power spectrum. In this case, it is possible to obtain a strong non-Gaussianity if $\epsilon_* \gg \sqrt{\Gamma_\chi/m_\chi}$ and we have found that the non-Gaussianity of the adiabatic component and of the isocurvature component are of the same order of magnitude.

In double quadratic inflation the two canonical fields, $\phi$ and $\chi$, are coupled gravitationally and the adiabatic and entropy field perturbations, $\delta\sigma_*$ and $\delta s_*$, are in general a linear combination of the two canonical fields. We have obtained explicit expressions at second order relating the initial and final adiabatic and isocurvature perturbations. In this simple case of two uncoupled fields with quadratic potentials we find that the non-linearities are small, but this need not be the case in other models. Indeed, as shown in Appendix B, in general two field slow-roll inflation we expect that the terms in square brackets in Eq. (78) can be neglected at leading order in slow-roll. However, the remaining non-linear terms due to initial entropy perturbation need not be suppressed. It would be interesting to investigate non-Gaussianity of isocurvature perturbations in more general models.

The bispectra for primordial curvature and isocurvature perturbations and their cross-correlations are presented for a general two-field model in Appendix C, and given at leading order in a slow-roll expansion. These show that the non-zero primordial bispectra (in both curvature and isocurvature perturbations and their cross-correlations) arise due to entropy field perturbations at Hubble-exit. Our results for the primordial curvature perturbation are consistent with the non-linear $\delta N$-formalism [4], derived in the large scale limit where the separate universes approach [45] is used to evaluate the perturbed expansion using the homogeneous background solutions. In single-field slow-roll inflation the perturbations are adiabatic on large scales and the bispectrum is suppressed by slow-roll parameters [30, 31].

In multiple-field inflation there is the possibility of additional observational features which are absent in single-field models. We have shown that this could include the contribution of isocurvature field perturbations to the bispectra (3-point functions) as well as the power spectra (2-point functions). It is interesting to note that, in principle, the isocurvature perturbations might dominate the primordial bispectrum in, for example, the CMB temperature anisotropies while remaining sub-dominant in the power spectrum. The non-Gaussian primordial perturbations predicted from Gaussian field perturbations during inflation are of a specific local form, but should be distinguishable from the local non-Gaussianity of the primordial curvature described by conventional $f_{NL}$ parameter. The bispectrum of the isocurvature perturbations can be characterized by a new non-linearity parameter, and the cross-correlated bispectra yield additional parameters. But in curvaton models, for example, they are all determined by the single model parameter, $r$, and thus could provide a strong test of the curvaton scenario. The best constraints on specific models of non-Gaussianity are based on matched filtering techniques [46]. It will thus be important to develop optimized constraints for the non-Gaussianity of primordial isocurvature perturbations to obtain the optimal constraints on a wider range of theoretical models.

# Acknowledgments

We thank the organizers of the workshop "Astroparticle and Cosmology" at the Galileo Galilei Institute for Theoretical Physics, Florence, October 2006, where this work was initiated, and of the workshop "Non-Gaussianity from fundamental physics" at DAMTP, Cambridge, September 2008, where this work was completed, for their kind invitation. DL thanks the ICG for their hospitality. FV acknowledges the kind hospitality of APC, Paris, and ICG, Portsmouth, where part of this work was carried out. DW is supported by the STFC.

# A Derivatives of $N$, $N^{(\phi)}$ and $N^{(\chi)}$ in double inflation

The total number of e-folds in double inflation is given by

$$N = \frac{1}{4M_P^2}\left(\phi_*^2 + \chi_*^2 - \phi^2 - \chi^2\right) . \tag{80}$$

If $\phi$ and $\chi$ are the values of the fields on a final uniform total density hypersurface, $R^2\chi^2 + \phi^2 = C$, using Eq. (49) we obtain

$$R^2\left(\frac{\phi}{\phi_*}\right)^{2R^2}\chi_*^2 + \phi^2 = C, \qquad R^2\chi^2 + \left(\frac{\chi}{\chi_*}\right)^{\frac{2}{R^2}}\phi_*^2 = C , \tag{81}$$

which can be differentiate with respect to $\phi_*$ and $\chi_*$ to yield

$$\frac{\partial\phi}{\partial\phi_*} = \frac{R^4}{\phi_*\phi}g , \qquad \frac{\partial\phi}{\partial\chi_*} = -\frac{R^2}{\chi_*\phi}g , \qquad \frac{\partial\chi}{\partial\phi_*} = -\frac{R^2}{\phi_*\chi}g , \qquad \frac{\partial\chi}{\partial\chi_*} = \frac{1}{\chi_*\chi}g , \tag{82}$$

where $g = \phi^2\chi^2/(R^4\chi^2 + \phi^2)$. By differentiating $N$ in Eq. (80) we get

$$N_{,\phi_*} = \frac{\phi_*}{2M_P^2}\left[1 + (1 - R^2)R^2\frac{g}{\phi_*^2}\right] , \tag{83}$$

$$N_{,\chi_*} = \frac{\chi_*}{2M_P^2}\left[1 + (R^2 - 1)\frac{g}{\chi_*^2}\right] , \tag{84}$$

and

$$
\begin{aligned}
N_{,\phi_*\phi_*} &= \frac{1}{2M_P^2}\left[1+(1-R^2)R^2\left(\frac{g_{,\phi_*}}{\phi_*}-\frac{g}{\phi_*^2}\right)\right] , && (85)\\
N_{,\phi_*\chi_*} &= \frac{1}{2M_P^2}(1-R^2)R^2\frac{g_{,\chi_*}}{\phi_*} = N_{,\chi_*\phi_*} = \frac{1}{2M_P^2}(R^2-1)\frac{g_{,\phi_*}}{\chi_*} , && (86)\\
N_{,\chi_*\chi_*} &= \frac{1}{2M_P^2}\left[1+(R^2-1)\left(\frac{g_{,\chi_*}}{\chi_*}-\frac{g}{\chi_*^2}\right)\right] . && (87)
\end{aligned}
$$

If the final hypersurface is a uniform field $\phi$ hypersurface, $\phi = C_\phi$, the number of e-folds (80) reads,

$$
N^{(\phi)} = \frac{1}{4M_P^2}\left[\phi_*^2+\chi_*^2-C_\phi^2-\left(\frac{C_\phi}{\phi_*}\right)^{2R^2}\chi_*^2\right] , \tag{88}
$$

which can be differentiated to give

$$
\begin{aligned}
N^{(\phi)}_{,\phi_*} &= \frac{\phi_*}{2M_P^2}\left(1+R^2\frac{\chi^2}{\phi_*^2}\right) , && (89)\\
N^{(\phi)}_{,\chi_*} &= \frac{\chi_*}{2M_P^2}\left(1-\frac{\chi^2}{\chi_*^2}\right) , && (90)
\end{aligned}
$$

and

$$
\begin{aligned}
N^{(\phi)}_{,\phi_*\phi_*} &= \frac{1}{2M_P^2}\left(1-R^2\frac{\chi^2}{\phi_*^2}(1+2R^2)\right) , && (91)\\
N^{(\phi)}_{,\chi_*\phi_*} &= N^{(\phi)}_{,\phi_*\chi_*} = \frac{R^2}{M_P^2}\frac{\chi^2}{\phi_*\chi_*} , && (92)\\
N^{(\phi)}_{,\chi_*\chi_*} &= \frac{1}{2M_P^2}\left(1-\frac{\chi^2}{\chi_*^2}\right) , && (93)
\end{aligned}
$$

Similar expressions can be found if we consider a final uniform field $\chi$ hypersurface. In this case

$$
\begin{aligned}
N^{(\chi)}_{,\chi_*} &= \frac{\chi_*}{2M_P^2}\left(1+R^{-2}\frac{\phi^2}{\chi_*^2}\right) , && (94)\\
N^{(\chi)}_{,\phi_*} &= \frac{\phi_*}{2M_P^2}\left(1-\frac{\phi^2}{\phi_*^2}\right) , && (95)
\end{aligned}
$$

and

$$
\begin{aligned}
N^{(\chi)}_{,\chi_*\chi_*} &= \frac{1}{2M_P^2}\left(1-R^{-2}\frac{\phi^2}{\chi_*^2}(1+2R^{-2})\right) , && (96)\\
N^{(\chi)}_{,\phi_*\chi_*} &= N^{(\chi)}_{,\chi_*\phi_*} = \frac{R^{-2}}{M_P^2}\frac{\phi^2}{\chi_*\phi_*} , && (97)\\
N^{(\chi)}_{,\phi_*\phi_*} &= \frac{1}{2M_P^2}\left(1-\frac{\phi^2}{\phi_*^2}\right) . && (98)
\end{aligned}
$$

## B Adiabatic and entropy field decomposition

In this section we review the adiabatic and entropy decomposition approach at linear and second order, during slow-roll inflation. It is possible to make a rotation in field space to identify the instantaneous adiabatic and entropy field perturbations along and orthogonal to the field trajectory. We will use the results of [15] generalizing the work of [12] (see [14] for an equivalent approach). At linear order, the adiabatic and entropy field perturbations are defined, respectively, as

$$\delta\sigma^{(1)} = \cos\theta\delta\phi + \sin\theta\delta\chi \,, \qquad \delta s^{(1)} = -\sin\theta\delta\phi + \cos\theta\delta\chi \,, \tag{99}$$

where $\tan\theta = \dot\chi/\dot\phi$ and $\theta$ is the time-dependent angle of the instantaneous rotation. At second order, we define

$$\delta\sigma = \delta\sigma^{(1)} + \frac{\delta s\dot{\delta s}}{2\dot\sigma} \,, \qquad \delta s = \delta s^{(1)} - \frac{\delta\sigma}{\dot\sigma}\left(\dot{\delta s} + \frac{\dot\theta}{2}\delta\sigma\right) \,, \tag{100}$$

where $\dot\sigma^2 = \dot\phi^2 + \dot\chi^2 = -2\dot H M_P^2$.

The adiabatic curvature perturbation on uniform density hypersurfaces is defined at second order as

$$\zeta = -\frac{H}{\dot\sigma}\delta\sigma - \frac{\delta\sigma}{\dot\sigma}\left[-\left(\frac{H}{\dot\sigma}\delta\sigma\right)^{\cdot} + \frac{1}{2}\left(\frac{H}{\dot\sigma}\right)^{\cdot}\delta\sigma + \dot\theta\frac{H}{\dot\sigma}\delta s\right] \,, \tag{101}$$

where $\delta\sigma$ is evaluated on a uniform flat hypersurface. The evolution of $\zeta$ is sourced by first and second order perturbations of $\delta s$. On super-Hubble scales it reads[4]

$$\dot\zeta = -\frac{H}{\dot\sigma^2}\left[2\dot\theta\dot\sigma\delta s - (V_{,ss} + 4\dot\theta^2)\delta s^2 + \frac{V_{,\sigma}}{\dot\sigma}\delta s\dot{\delta s}\right] \,, \tag{102}$$

where $\dot\theta = -V_{,s}/\dot\sigma$, with $V_{,s} = -\sin\theta V_{,\phi} + \cos\theta V_{,\chi}$ and $V_{,ss} = V_{,\phi\phi}\sin^2\theta - 2V_{,\phi\chi}\cos\theta\sin\theta + V_{,\chi\chi}\cos^2\theta$. The entropy field perturbation $\delta s$ evolves independently on super-Hubble scales and its evolution equation reads,

$$\ddot{\delta s} + 3H\dot{\delta s} + (V_{,ss} + 3\dot\theta^2)\delta s = -\frac{\dot\theta}{\dot\sigma}\delta s^2 - \frac{2}{\dot\sigma}\left(\ddot\theta + \dot\theta\frac{V_{,\sigma}}{\dot\sigma} - \frac{3}{2}H\dot\theta\right)\delta s\dot{\delta s} - \left(\frac{1}{2}V_{,sss} - 5\frac{\dot\theta}{\dot\sigma}V_{,ss} - 9\frac{\dot\theta^3}{\dot\sigma}\right)\delta s^2 \,, \tag{103}$$

where $V_{,\sigma} = \cos\theta V_{,\phi} + \sin\theta V_{,\chi}$ and $V_{,sss} = -V_{,\phi\phi\phi}\sin^3\theta + 3V_{,\phi\phi\chi}\cos\theta\sin^2\theta - 3V_{,\phi\chi\chi}\cos^2\theta\sin\theta + V_{,\chi\chi\chi}\cos^3\theta$.

Given these evolution equations we expect that their solutions can be written as

$$\zeta = \zeta_* + T_\zeta^{(1)}\delta s_* + T_\zeta^{(2)}\delta s_*^2 \,, \tag{104}$$

$$\delta s = T_{\delta s}^{(1)}\delta s_* + T_{\delta s}^{(2)}\delta s_*^2 \,, \tag{105}$$

where $T^{(1)}_{\zeta,\delta s}$ and $T^{(2)}_{\zeta,\delta s}$ are the first and second order transfer functions, for $\zeta$ and $\delta s$, respectively, and $\zeta_*$, $\delta s_*$ are their initial conditions at Hubble exit. Equation (75) is an example of the general solution (104), while Eq. (76) is an example of (105) rewritten in terms of the CDM entropy perturbation $\mathcal{S}_{\rm m}$.

[4]This equation corresponds to Eq. (221) of [15]. Note however that in `v1` and `v2` of the `arXiv` and in the published version on JCAP, the last term inside the bracket is missing in this equation. We thank Sébastien Renaux-Petel and Gianmassimo Tasinato for pointing out this omission.

Let $\epsilon$ be the standard slow-roll parameter, and let us define the mass slow-roll parameters $\eta_{ij} = V_{,ij}/(3H^2)$ and $\eta_{\sigma s} = (\eta_{\chi\chi} - \eta_{\phi\phi})\cos\theta\sin\theta + \eta_{\phi\chi}(\cos^2\theta - \sin^2\theta)$, $\eta_{ss} = \eta_{\phi\phi}\sin^2\theta - 2\eta_{\phi\chi}\cos\theta\sin\theta + \eta_{\chi\chi}\cos^2\theta$. By using $\dot\theta = -H\eta_{\sigma s}$, which follows from the time derivative of $\tan\theta = \dot\chi/\dot\phi \simeq V_{,\chi}/V_{,\phi}$, and $\dot{\delta s} = -H\eta_{ss}\delta s$, one can rewrite the second order definitions of $\delta\sigma$ and $\delta s$, Eq. (100), and the definition of $\zeta$, Eq. (101), in terms of the slow-roll parameters,

$$\delta\sigma = \delta\sigma^{(1)} - \frac{1}{2\sqrt{2\epsilon}M_P}\,\eta_{ss}\delta s^2\ , \qquad \delta s = \delta s^{(1)} + \frac{1}{\sqrt{2\epsilon}M_P}\left(\eta_{ss}\delta s + \frac{\eta_{\sigma s}}{2}\delta\sigma\right)\delta\sigma\ , \tag{106}$$

and

$$\zeta = -\frac{1}{\sqrt{2\epsilon}M_P}\left\{\delta\sigma - \frac{1}{\sqrt{2\epsilon}M_P}\left[\left(\epsilon - \frac{\eta_{\sigma\sigma}}{2}\right)\delta\sigma^2 - \eta_{\sigma s}\delta\sigma\delta s\right]\right\}\ . \tag{107}$$

Note that in these three definitions, the non-linear terms on the right hand side are slow-roll suppressed with respect to the linear terms. If evaluated at Hubble crossing, when the slow-roll parameters are small, they lead to contributions to the intrinsic non-Gaussianities of $\delta\sigma_*$, $\delta s_*$, and $\zeta_*$ which are small.

For completeness, we compute here the 3-point correlation functions of $\delta\sigma_*$ and $\delta s_*$. From the results of [26], namely

$$\langle\delta\varphi_*^I(\vec k_1)\delta\varphi_*^J(\vec k_2)\delta\varphi_*^K(\vec k_3)\rangle = (2\pi)^3\delta(\Sigma_i\vec k_i)\frac{H_*^4}{16M_P^2}\sum_{\rm perms}\frac{\dot\varphi_*^I\delta^{JK}}{H_*\Pi_i k_i^3}\mathcal{M}(k_1,k_2,k_3)\ , \tag{108}$$

with

$$\mathcal{M}(k_1,k_2,k_3) \equiv -k_1k_2^2 - 4\frac{k_2^2k_3^2}{k_t} + \frac{1}{2}k_1^3 + \frac{k_2^2k_3^2}{k_t^2}(k_2-k_3), \quad k_t \equiv k_1+k_2+k_3\ , \tag{109}$$

one can compute the 3-point correlators of $\delta\sigma^{(1)}$ and $\delta s^{(1)}$, simply by using the change of basis in field space (99). We find that $\langle\delta\sigma_*^{(1)}\delta\sigma_*^{(1)}\delta\sigma_*^{(1)}\rangle$ is the same as for a single scalar field [30], whereas $\langle\delta\sigma_*^{(1)}\delta\sigma_*^{(1)}\delta s_*^{(1)}\rangle$ and $\langle\delta s_*^{(1)}\delta s_*^{(1)}\delta s_*^{(1)}\rangle$ vanish, simply because $\dot s = 0$. We also find

$$\langle\delta s_*^{(1)}(\vec k_1)\delta s_*^{(1)}(\vec k_2)\delta\sigma_*^{(1)}(\vec k_3)\rangle = (2\pi)^3\delta(\Sigma_i\vec k_i)\frac{\sqrt{\epsilon_*}H_*^4}{8\sqrt2 M_P}\left(\frac{k_3^3 - k_3(k_1^2+k_2^2)}{\Pi_i k_i^3} - 8\frac{k_1^2k_2^2}{k_t\Pi_i k_i^3}\right)\ , \tag{110}$$

and similar expressions for $\langle\delta s_*^{(1)}(\vec k_1)\delta\sigma_*^{(1)}(\vec k_2)\delta s_*^{(1)}(\vec k_3)\rangle$ and $\langle\delta\sigma_*^{(1)}(\vec k_1)\delta s_*^{(1)}(\vec k_2)\delta s_*^{(1)}(\vec k_3)\rangle$ by relabeling appropriately the $k_i$ appearing on the right hand side of (110).

Taking also into account the second order parts of the adiabatic and entropy perturbations given in Eq. (106), we eventually find for the full 3-point functions

$$\begin{aligned}
\langle\delta\sigma_*(\vec k_1)\delta\sigma_*(\vec k_2)\delta\sigma_*(\vec k_3)\rangle &= (2\pi)^3\delta(\Sigma_i\vec k_i)\frac{\sqrt{\epsilon_*}H_*^4}{8\sqrt2 M_P}\left(\frac{\Sigma_i k_i^3 - \Sigma_{i\neq j}k_ik_j^2}{\Pi_i k_i^3} - 8\frac{\Sigma_{i>j}k_i^2k_j^2}{k_t\Pi_i k_i^3}\right)\ , && (111)\\
\langle\delta\sigma_*(\vec k_1)\delta\sigma_*(\vec k_2)\delta s_*(\vec k_3)\rangle &= (2\pi)^3\delta(\Sigma_i\vec k_i)\frac{\eta_{\sigma s*}H_*^4}{8\sqrt{2\epsilon_*}M_P}\frac{1}{k_1^3k_2^3}\ , && (112)\\
\langle\delta s_*(\vec k_1)\delta s_*(\vec k_2)\delta\sigma_*(\vec k_3)\rangle &= (2\pi)^3\delta(\Sigma_i\vec k_i)\frac{H_*^4}{8\sqrt{2\epsilon_*}M_P}\left[\epsilon_*\left(\frac{k_3^3 - k_3(k_1^2+k_2^2)}{\Pi_i k_i^3} - 8\frac{k_1^2k_2^2}{k_t\Pi_i k_i^3}\right)\right.\\
&\quad\left. +\eta_{ss*}\left(\frac{-k_3^3+2k_1^3+2k_2^3}{\Pi_i k_i^3}\right)\right]\ , && (113)\\
\langle\delta s_*(\vec k_1)\delta s_*(\vec k_2)\delta s_*(\vec k_3)\rangle &= 0\ . && (114)
\end{aligned}$$

This shows that the intrinsic non-Gaussianities of $\delta\sigma_*$, $\delta s_*$ and, as a consequence of Eq. (107), of $\zeta_*$ are all small for slow-roll models.

## C Primordial power spectra and bispectra

At first-order the expressions in Eqs. (78) and (79) for the primordial curvature and isocurvature perturbations in terms of the field perturbations during inflation give the power spectra of the primordial perturbations at leading order

$$
\begin{aligned}
\langle\zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\rangle &= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\left\{N_{,\sigma}^2P_\sigma(k_1)+2N_{,\sigma}N_{,s}C_{s\sigma}(k_1)+N_{,s}^2P_s(k_1)\right\}\,, && (115)\\
\frac{1}{3}\langle\zeta_{\rm r}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\rangle &= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\left\{N_{,s}\Delta N_{,s}P_s(k_1)+N_{,\sigma}\Delta N_{,s}C_{s\sigma}(k_1)\right\}\,, && (116)\\
\frac{1}{9}\langle\mathcal{S}_{\rm m}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\rangle &= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\Delta N_{,s}^2P_s(k_1)\,, && (117)
\end{aligned}
$$

where at horizon-crossing during inflation we have

$$
\begin{aligned}
\langle\delta\sigma_*(\vec{k}_1)\delta\sigma_*(\vec{k}_2)\rangle &= (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_\sigma(k_1)\,, && (118)\\
\langle\delta s_*(\vec{k}_1)\delta s_*(\vec{k}_2)\rangle &= (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_s(k_1)\,, && (119)\\
\langle\delta\sigma_*(\vec{k}_1)\delta s_*(\vec{k}_2)\rangle &= (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)C_{s\sigma}(k_1)\,. && (120)
\end{aligned}
$$

At second-order Eqs. (78) and (79) give the leading order bispectra for the primordial curvature and isocurvature perturbations and their correlations,

$$
\begin{aligned}
\langle\zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\zeta_{\rm r}(\vec{k}_3)\rangle &= N_{,I}N_{,J}N_{,K}\langle\delta\varphi^I_*(\vec{k}_1)\delta\varphi^J_*(\vec{k}_2)\delta\varphi^K_*(\vec{k}_3)\rangle\\
&+ (2\pi)^3\delta(\Sigma_i\vec{k}_i)N_{,I}N_{,J}N_{,KL}[C^{IK}(k_1)C^{JL}(k_2)+2\text{ perms}]\,, && (121)\\
\frac{1}{3}\langle\zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\mathcal{S}_{\rm m}(\vec{k}_3)\rangle &= N_{,I}N_{,J}\Delta N_{,K}\langle\delta\varphi^I_*(\vec{k}_1)\delta\varphi^J_*(\vec{k}_2)\delta\varphi^K_*(\vec{k}_3)\rangle\\
&+ (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Big\{N_{,I}N_{,J}\Delta N_{,KL}C^{IK}(k_1)C^{JL}(k_2)\\
&+ N_{,IJ}N_{,K}\Delta N_{,L}[C^{JK}(k_1)+C^{JK}(k_2)]C^{IL}(k_3)\Big\}\,, && (122)\\
\frac{1}{9}\langle\mathcal{S}_{\rm m}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\zeta_{\rm r}(\vec{k}_3)\rangle &= \Delta N_{,I}\Delta N_{,J}N_{,K}\langle\delta\varphi^I_*(\vec{k}_1)\delta\varphi^J_*(\vec{k}_2)\delta\varphi^K_*(\vec{k}_3)\rangle\\
&+ (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Big\{\Delta N_{,I}\Delta N_{,J}N_{,KL}C^{IK}(k_1)C^{JL}(k_2)\\
&+ \Delta N_{,IJ}\Delta N_{,K}N_{,L}[C^{JK}(k_1)+C^{JK}(k_2)]C^{IL}(k_3)\Big\}\,, && (123)\\
\frac{1}{27}\langle\mathcal{S}_{\rm m}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\mathcal{S}_{\rm m}(\vec{k}_3)\rangle &= \Delta N_{,I}\Delta N_{,J}\Delta N_{,K}\langle\delta\varphi^I_*(\vec{k}_1)\delta\varphi^J_*(\vec{k}_2)\delta\varphi^K_*(\vec{k}_3)\rangle\\
&+ (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Delta N_{,I}\Delta N_{,J}\Delta N_{,KL}[C^{IK}(k_1)C^{JL}(k_2)+2\text{ perms}]\,, && (124)
\end{aligned}
$$

where $C^{II}(k)\equiv P_I(k)$.

At leading order in a slow-roll expansion the adiabatic and entropy field perturbations are independent Gaussian random fields, with [47, 48]

$$
P_s(k)\simeq P_\sigma(k)\simeq P_*(k)=\frac{H_*}{2k^3}\,,\qquad C_{s\sigma}(k)\simeq 0\,, \qquad (125)
$$

and the primordial bispectra simplify considerably in the slow-roll limit, where we drop the terms in brackets in Eqs. (78) and (79), to give

$$\langle\zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\zeta_{\rm r}(\vec{k}_3)\rangle \simeq (2\pi)^3\delta(\Sigma_i\vec{k}_i)N_{,s}^2N_{ss}\left[P_*(k_1)P_*(k_2)+2\text{ perms}\right] , \tag{126}$$

$$\frac{1}{3}\langle\zeta_{\rm r}(\vec{k}_1)\zeta_{\rm r}(\vec{k}_2)\mathcal{S}_{\rm m}(\vec{k}_3)\rangle \simeq (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Big\{N_{,s}N_{,ss}\Delta N_{,s}\left[P_*(k_1)+P_*(k_2)\right]P_*(k_3) + N_{,s}^2\Delta N_{,ss}P_*(k_1)P_*(k_2)\Big\} , \tag{127}$$

$$\frac{1}{9}\langle\mathcal{S}_{\rm m}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\zeta_{\rm r}(\vec{k}_3)\rangle \simeq (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Big\{N_{,s}\Delta N_{,s}\Delta N_{,ss}\left[P_*(k_1)+P_*(k_2)\right]P_*(k_3) + N_{,ss}\Delta N_{,s}^2P_*(k_1)P_*(k_2)\Big\} , \tag{128}$$

$$\frac{1}{27}\langle\mathcal{S}_{\rm m}(\vec{k}_1)\mathcal{S}_{\rm m}(\vec{k}_2)\mathcal{S}_{\rm m}(\vec{k}_3)\rangle \simeq (2\pi)^3\delta(\Sigma_i\vec{k}_i)\Delta N_{,s}^2\Delta N_{,ss}\left[P_*(k_1)P_*(k_2)+2\text{ perms}\right] . \tag{129}$$

# References


[1] A. D. Linde and V. F. Mukhanov, "Nongaussian isocurvature perturbations from inflation," Phys. Rev. D **56**, 535 (1997) [arXiv:astro-ph/9610219].

[2] L. Boubekeur and D. H. Lyth, Phys. Rev. D **73**, 021301 (2006) [arXiv:astro-ph/0504046].

[3] M. Sasaki and E. D. Stewart, "A General analytic formula for the spectral index of the density perturbations produced during inflation," Prog. Theor. Phys. **95**, 71 (1996) [arXiv:astro-ph/9507001]; A. A. Starobinsky, "Multicomponent de Sitter (inflationary) stages and the generation of perturbations," JETP Lett. **42**, 152 (1985) [Pisma Zh. Eksp. Teor. Fiz. **42**, 124 (1985)].

[4] D. H. Lyth and Y. Rodriguez, "The inflationary prediction for primordial non-gaussianity," Phys. Rev. Lett. **95**, 121302 (2005) [arXiv:astro-ph/0504045].

[5] K. Enqvist and M. S. Sloth, "Adiabatic CMB perturbations in pre big bang string cosmology," Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214]; D. H. Lyth and D. Wands, "Generating the curvature perturbation without an inflaton," Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002]; T. Moroi and T. Takahashi, "Effects of cosmological moduli fields on cosmic microwave background," Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].

[6] D. H. Lyth, C. Ungarelli and D. Wands, "The primordial density perturbation in the curvaton scenario," Phys. Rev. D **67**, 023503 (2003) [arXiv:astro-ph/0208055].

[7] C. Gordon and A. Lewis, "Observational constraints on the curvaton model of inflation," Phys. Rev. D **67**, 123513 (2003) [arXiv:astro-ph/0212248].

[8] M. Beltran, "Isocurvature, non-gaussianity and the curvaton model," Phys. Rev. D **78**, 023530 (2008) [arXiv:0804.1097 [astro-ph]].

[9] D. Langlois and F. Vernizzi, "Mixed inflaton and curvaton perturbations," Phys. Rev. D **70**, 063522 (2004) [arXiv:astro-ph/0403258].

[10] F. Ferrer, S. Rasanen and J. Valiviita, "Correlated isocurvature perturbations from mixed inflaton-curvaton decay," JCAP **0410**, 010 (2004) [arXiv:astro-ph/0407300].

[11] D. Polarski and A. A. Starobinsky, "Spectra of perturbations produced by double inflation with an intermediate matter dominated stage," Nucl. Phys. B **385**, 623 (1992).

[12] C. Gordon, D. Wands, B. A. Bassett and R. Maartens, "Adiabatic and entropy perturbations from inflation," Phys. Rev. D **63**, 023506 (2001) [arXiv:astro-ph/0009131].

[13] S. Groot Nibbelink and B. J. W. van Tent, "Scalar perturbations during multiple field slow-roll inflation," Class. Quant. Grav. **19**, 613 (2002) [arXiv:hep-ph/0107272].

[14] G. I. Rigopoulos, E. P. S. Shellard and B. W. van Tent, "Non-linear perturbations in multiple-field inflation," Phys. Rev. D **73**, 083521 (2006) [arXiv:astro-ph/0504508].

[15] D. Langlois and F. Vernizzi, "Nonlinear perturbations of cosmological scalar fields," JCAP **0702**, 017 (2007) [arXiv:astro-ph/0610064].

[16] D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, "Primordial fluctuations and non-Gaussianities in multi-field DBI inflation,", Phys. Rev. Lett. **101**, 061301 (2008) [arXiv:0804.3139 [hep-th]]; D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, "Primordial perturbations and non-Gaussianities in DBI and general multi-field inflation," Phys. Rev. D **78**, 063523 (2008) [arXiv:0806.0336 [hep-th]].

[17] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, "Non-Gaussianity from isocurvature perturbations," arXiv:0808.0009 [astro-ph].

[18] M. Kawasaki, K. Nakayama and F. Takahashi, "Non-Gaussianity from Baryon Asymmetry," arXiv:0809.2242 [hep-ph].

[19] D. Langlois and F. Vernizzi, "Evolution of nonlinear cosmological perturbations," Phys. Rev. Lett. **95**, 091303 (2005) [arXiv:astro-ph/0503416].

[20] D. Langlois and F. Vernizzi, "Conserved nonlinear quantities in cosmology," Phys. Rev. D **72**, 103501 (2005) [arXiv:astro-ph/0509078].

[21] D. Langlois and F. Vernizzi, "Nonlinear perturbations for dissipative and interacting relativistic fluids," JCAP **0602**, 014 (2006) [arXiv:astro-ph/0601271].

[22] D. H. Lyth, K. A. Malik and M. Sasaki, "A general proof of the conservation of the curvature perturbation," JCAP **0505**, 004 (2005) [arXiv:astro-ph/0411220].

[23] M. Sasaki, J. Valiviita and D. Wands, "Non-gaussianity of the primordial perturbation in the curvaton model," Phys. Rev. D **74**, 103003 (2006) [arXiv:astro-ph/0607627].

[24] D. Polarski and A. A. Starobinsky, "Isocurvature perturbations in multiple inflationary models," Phys. Rev. D **50**, 6123 (1994) [arXiv:astro-ph/9404061].

[25] F. Vernizzi, "On the conservation of second-order cosmological perturbations in a scalar field dominated Universe," Phys. Rev. D **71**, 061301 (2005) [arXiv:astro-ph/0411463].

[26] D. Seery and J. E. Lidsey, "Primordial non-gaussianities from multiple-field inflation," JCAP **0509**, 011 (2005) [arXiv:astro-ph/0506056].

[27] K. A. Malik, D. Wands and C. Ungarelli, "Large-scale curvature and entropy perturbations for multiple interacting fluids," Phys. Rev. D **67**, 063516 (2003) [arXiv:astro-ph/0211602].

[28] K. A. Malik and D. H. Lyth, "A numerical study of non-gaussianity in the curvaton scenario," JCAP **0609**, 008 (2006) [arXiv:astro-ph/0604387].

[29] F. Vernizzi, "Generating cosmological perturbations with mass variations," Nucl. Phys. Proc. Suppl. **148**, 120 (2005) [arXiv:astro-ph/0503175].

[30] J. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models," JHEP **0305**, 013 (2003) [arXiv:astro-ph/0210603].

[31] V. Acquaviva, N. Bartolo, S. Matarrese and A. Riotto, "Second-order cosmological perturbations from inflation," Nucl. Phys. B **667**, 119 (2003) [arXiv:astro-ph/0209156].

[32] K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, “Non-Gaussianity, Spectral Index and Tensor Modes in Mixed Inflaton and Curvaton Models,” Phys. Rev. D **78**, 023513 (2008) [arXiv:0802.4138 [astro-ph]].

[33] N. Bartolo, S. Matarrese and A. Riotto, “Evolution of second-order cosmological perturbations and non-Gaussianity,” JCAP **0401**, 003 (2004) [arXiv:astro-ph/0309692].

[34] E. Komatsu and D. N. Spergel, “Acoustic signatures in the primary microwave background bispectrum,” Phys. Rev. D **63**, 063002 (2001) [arXiv:astro-ph/0005036].

[35] S. Gupta, K. A. Malik and D. Wands, Phys. Rev. D **69**, 063513 (2004) [arXiv:astro-ph/0311562].

[36] S. Weinberg, “Must cosmological perturbations remain non-adiabatic after multi-field inflation?,” Phys. Rev. D **70**, 083522 (2004) [arXiv:astro-ph/0405397].

[37] E. Komatsu *et al.* [WMAP Collaboration], “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Cosmological Interpretation,” arXiv:0803.0547 [astro-ph].

[38] J. Garcia-Bellido and D. Wands, “General relativity as an attractor in scalar - tensor stochastic inflation,” Phys. Rev. D **52**, 5636 (1995) [arXiv:gr-qc/9503049].

[39] V. F. Mukhanov and P. J. Steinhardt, “Density perturbations in multifield inflationary models,” Phys. Lett. B **422**, 52 (1998) [arXiv:astro-ph/9710038].

[40] F. Vernizzi and D. Wands, “Non-Gaussianities in two-field inflation,” JCAP **0605**, 019 (2006) [arXiv:astro-ph/0603799].

[41] L. Alabidi and D. H. Lyth, “Inflation models and observation,” JCAP **0605**, 016 (2006) [arXiv:astro-ph/0510441].

[42] G. I. Rigopoulos, E. P. S. Shellard and B. J. W. van Tent, “Large non-Gaussianity in multiple-field inflation,” Phys. Rev. D **73**, 083522 (2006) [arXiv:astro-ph/0506704].

[43] F. Bernardeau and J. P. Uzan, “Non-Gaussianity in multi-field inflation,” Phys. Rev. D **66**, 103506 (2002) [arXiv:hep-ph/0207295].

[44] D. Langlois, “Correlated adiabatic and isocurvature perturbations from double inflation,” Phys. Rev. D **59**, 123512 (1999) [arXiv:astro-ph/9906080].

[45] D. Wands, K. A. Malik, D. H. Lyth and A. R. Liddle, “A new approach to the evolution of cosmological perturbations on large scales,” Phys. Rev. D **62**, 043527 (2000) [arXiv:astro-ph/0003278].

[46] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, “Non-Gaussianity from inflation: Theory and observations,” Phys. Rept. **402**, 103 (2004) [arXiv:astro-ph/0406398].

[47] C. T. Byrnes and D. Wands, “Curvature and isocurvature perturbations from two-field inflation in a slow-roll expansion,” Phys. Rev. D **74**, 043529 (2006) [arXiv:astro-ph/0605679].

[48] Z. Lalak, D. Langlois, S. Pokorski and K. Turzynski, “Curvature and isocurvature perturbations in two-field inflation,” JCAP **0707**, 014 (2007) [arXiv:0704.0212 [hep-th]].