Time Reversal

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1. Introduction

Time reversal is a wonderfully strange concept. It sounds like science fiction at first blush, and yet plays a substantial role in the foundations of physics. For example, time reversal is often use to describe the 'arrow of time', by allowing one to say how evolving to the future is different from evolving to the past. Most fundamental laws of physics are thought to be time reversal invariant; so, when Cronin and Fitch discovered evidence that time symmetry is violated, it provided crucial new insight into the burgeoning Standard Model of particle physics (Christenson et al. 1964; Roberts 2014). Time reversal is a cornerstone of many important concepts in physics, from the Wigner (1932) derivation of Kramers degeneracy (which plays an important role in low-temperature physics and superconduction), to the boson-fermion superselection rule (Wick et al. 1952), to the Feynman-Wheeler interpretation of antimatter (Arntzenius and Greaves 2009). Through the Sakharov conditions, the understanding of time reversal is also thought to play a role in explaining the apparent large-scale asymmetry between matter and antimatter in the universe (Sakharov 1967).

This chapter introduces one little corner of the rich literature on time reversal[1], which deals with the question of what time reversal means. We'll begin with a presentation of the standard account of time reversal, with plenty of examples, followed by a

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[1]Some other fascinating issues in the time reversal literature that I will not cover here include the implications and significance of time reversal symmetry (Earman 2002; Price 1996; Farr forthcoming); time reversal symmetry violation in the weak sector (Sachs 1987; Bigi and Sanda 2009; Roberts 2014, 2015; Gołosz 2017)), the Feynman 'going backwards in time' interpretation of antimatter (Earman 1967; Arntzenius and Greaves 2009); and the significance of CPT theorem (Greaves 2010; Greaves and Thomas 2014).

popular non-standard account. I will then argue that, in spite of recent commentary to the contrary, the standard approach to the meaning of time reversal is the only one that is philosophically and physically viable. I conclude with a few open research problems about time reversal.

2. The Standard Account

Spatial rotation can be understood by studying rotated physical objects, and spatial translation by studying translated physical objects. How then are we to understand time reversal? It doesn't seem possible to physically 'reverse time'. One imagines an impassioned philosopher of physics pulling hard on their hair while exclaiming, *What would it even mean to reverse time?*

To guide intuitions, a common initial response is to imagine a film of a body in motion, like a billiard ball bouncing around a frictionless billiards table, and then to imagine that the film is reversed: this reversed film is then said to display the time-reversed motion. But as North (2008) points out, it is hard to see how the precise properties of that reversed motion follow from a simple 'film' thought experiment.

How can one make the reversed description precise? How does one know it's correct? The standard account is the following.

2.1. **Two components.** There are two components to the standard account of time reversal. The first is *order (in time) reversal.* Suppose we represent the changing states of a physical system by a curve $s : \mathbb{R} \to \mathcal{P}$ through some set of states $\mathcal{P}$, with initial state $s(0) \in \mathcal{P}$. One thing that time reversal ought to do is turn around the temporal order in which such states occur. The standard time reversal transformation does this by transforming the temporal parameter t as follows:

$$t \mapsto -t.$$

This flips time around a particular initial moment $t = 0$. That may seem a bit arbitrary at first. However, most of the time, it isn't philosophically or physically significant. One could instead write $t \mapsto -t+c$, and thereby flip time around the moment $t = c$. But this new transformation is related to the old one by a translation (forwards or backwards) by c in time, called a *time translation.* In the local physics of isolated systems, it turns out that time translation is always a symmetry, related to the conservation of energy. When that is the case, these two time reversal transformations can always be viewed as two different ways of representing the same transformation.

A more interesting question is: why is $t \mapsto f(t) = -t + c$ appropriate for time reversal, and not a transformation like $t \mapsto g(t) = e^{-t}$, which also reverses the order of events in time? This particular transformation $g(t)$ can be excluded by demanding that time reversal is an involution, meaning that $f(f(t)) = t$. The idea is just to take seriously what it means to be a 'reversal': by applying it twice, you get back to where you started. This assumption can be found in Sachs (1987) and Roberts (2012), and was explored extensively by Peterson (2015). More generally, it turns out that if one demands that time reversal is an order-reversing involution that is *linear*, so as not to 'stretch time out unevenly', then the only possible transformation is of the standard form $t \mapsto -t + c$, just as is standardly assumed[2].

The second component of time reversal, on the standard account, is the *(instantaneous) time reversal operator.* When one views a film of a classical billiard ball in reverse, both the momentum appears to have reversed direction: a ball with momentum to the right becomes one with momentum to the left, and so on.[3] The operation that implements these instantaneous changes is called the *time reversal operator* T. Some common instantaneous properties and their transformation rules under the time reversal operator are indicated in Figure 1. A central foundational question is then: how does one know which properties are preserved and which are reversed? We will discuss this question over the course of this article; for now, let us simply try to summarise what the standard account says about time reversal.

[2]More precisely: let $f : \mathbb{R} \to \mathbb{R}$ satisfy (i) *involution:* $f(f(t)) = t$; (ii) *linearity:* $f(t) = at + c$; and (iii) *order-reversal:* $t < t' \Leftrightarrow f(t) > f(t')$. Then $t = f(f(t)) = a^2t + ac + c$, hence $(a^2 - 1)t + (a + 1)c = 0$ for all $t \in \mathbb{R}$. So, $a = \pm 1$, and $c = 0$ whenever $a = 1$. But only $a = -1$ is order-reversing, and hence $f(t) = -t + c$ (see Roberts 2017, §3.3).

[3]Note that, although velocity dq/dt and momentum p are proportional for structureless Newtonian particles, $dq/dt = (1/m)p$, this is not always the case. For example, in electromagnetism velocity may take the form $dq/dt = (1/m)(p + a(q))$ for some vector potential a. So, when one as being careful, the reversal of velocity and of momentum should be treated as separate concepts.

Reversed		**Preserved**	
Momentum:	$p \mapsto -p$	Position:	$q \mapsto q$
Magnetic Field:	$B \mapsto -B$	Electric Field:	$E \mapsto E$
Spin:	$\sigma \mapsto -\sigma$	Kinetic Energy:	$p^2/2m \mapsto p^2/2m$
Position wavefunction:	$\psi(x) \mapsto \psi(x)^*$	Transition probability:	$\lvert\langle\psi,\phi\rangle\rvert^2 \mapsto \lvert\langle\psi,\phi\rangle\rvert^2$

Figure 1. Some properties of the time reversal operator T.

Combining the two components, the standard account of time reversal is the following. Begin with a state space $\mathcal{P}$, and let $s(t)$ be a curve through $\mathcal{P}$ that represents the evolution of a physical system in time. The standard time reversal transformation takes $s(t)$ to a new curve $Ts(-t)$, which reverses the order of events, and also applies the time reversal operator $T : \mathcal{P} \to \mathcal{P}$ to the instantaneous state $s(t)$ at each time t. Thus, on the standard account, time reversal is a transformation of dynamical trajectories, such as those associated with solutions to a law of nature, which reverses the order of states, but also adjusts instantaneous properties like momentum and spin in an 'appropriate' way.

To keep the language clear, I will systematically use the phrase *time reversal operator* to refer to the operator $T : \mathcal{P} \to \mathcal{P}$ on instantaneous states, and the phrase *time reversal transformation* to refer to the transformation of dynamical trajectories, which also includes the reversal of order in time.

2.2. **Time reversal invariance.** Although our central discussion is about what time reversal means, a essential component of the debate involves what it means for a the laws describing a physical system to be temporally symmetric, or *time reversal invariant.*

Most physical theories can be identified with a state space $\mathcal{P}$, as well as a set of S of preferred possible trajectories, which are the 'solutions' to some law. For example, in the Newtonian description of a point particle of mass m in the presence of a force F, the states are the possible positions of the particle in space, and the trajectories are the curves with acceleration a satisfying Newton's Second Law, $F = ma$.

Consider a set of curves through a state space $\mathcal{P}$, and let S be the subset of the curves through $\mathcal{P}$ that are solutions to some law. For a bijection φ on the set of curves through $\mathcal{P}$ to be a *symmetry* or an *invariance* of the law means: if some curve is a solution to the law, then the φ-transformed curve is a solution, too. Another way to put this is: given a law with a solution set S, φ is a symmetry of the law if and only if $\varphi(S) = S$. Time reversal invariance is just a special case of this: for a law to be *time reversal invariant* means that a curve is a solution to the law only if the time-reversed curve is a solution as well.

The reader should be warned that, in some treatments of this topic, a symmetry of a law is defined to be a passive transformation that 'preserves the form' of that law. This is, unfortunately, a rather vague way to put it, and in practice it is also prone to error. When carried out correctly it usually amounts to the same thing. However, especially when one is a newcomer to the subtleties of time reversal, the reader is encouraged to stick to the more precise characterisation above. Concrete examples of it can be found below.

2.3. **Examples.** In the context of a physical theory, the standard account of what time reversal means can be given in more precise terms, and even argued for. To really dig into this, it's important to work some examples. Some encouragement for those readers who do not follow the mathematical details: don't lose hope! One can skip the technicalities of these examples and still understand the philosophical argumentation of the sections to follow. The main message of these examples is that, on the standard account of time reversal, its exact meaning can only be understood once we identify how the state space of the theory represents the world.

2.3.1. *Example: Newtonian Mechanics.* In classical mechanics applied to point particles, the state space is typically $\mathcal{P} = \mathbb{R}^{3n}$, and a state is a vector $x \in \mathbb{R}^{3n}$ describing the positions of n particles in space at a moment. The dynamical trajectories $x(t)$ of a physical system are curves in that state space that satisfy Newton's equation, $F = m \cdot (d^2/dt^2)x(t)$, for a collection of masses $m \in \mathbb{R}^n$ and a total force $F \in \mathbb{R}^3$. The time reversal operator $T : \mathcal{P} \to \mathbb{R}$ is trivial, in that it is the identity operator, since time reversal is assumed not to transform instantaneous positions. As a result, the time reversal transformation just reverses the order of trajectories:

$$x(t) \mapsto x(-t).$$

Time reversal invariance in this context means that, given some force F, if $x(t)$ is a solution to Newton's equation, then so is its time-reverse $x(-t)$. One can easily check that, if the force vector F is a function only of position in space, then Newtonian mechanics is time reversal invariant on the standard account. However, for more exotic forces, it is not generally guaranteed that classical mechanics is time reversal invariant, in spite of frequent commentary to the contrary (Roberts 2013b).

2.3.2. *Example: Hamiltonian Mechanics.* The Hamiltonian approach to classical mechanics is a theory built using differential geometry[4]. Here, things are a little more interesting. The state space is a $2n$-dimensional manifold $\mathcal{P}$ together with a symplectic form Ω, which are sometimes together called a *phase space*. The dynamical trajectories $s(t)$ are the curves that satisfy Hamilton's equations[5], for some smooth function $h : \mathcal{P} \to \mathbb{R}$ called the Hamiltonian. By Darboux's theorem, a state $\xi \in \mathcal{P}$ in a symplectic manifold admits a neighbourhood in which there is a coordinate system for which $\xi = (q_1, \ldots, q_n, p_1, \ldots, p_n)$. I will write this as $\xi = (q, p)$ for short.

Often, though not always, the q components in Darboux coordinates are interpreted as position values, while the p components are interpreted as momentum. In that case, the time reversal operator is a function $T : \mathcal{P} \to \mathcal{P}$ that preserves position and reverses momentum, $T(q, p) = (q, -p)$. The time reversal transformation then has the following effect on dynamical trajectories:

$$q(t), p(t) \mapsto q(-t), -p(-t).$$

[4]A classic reference for this perspective on classical mechanics is Arnold (1989), or the more recent Marsden and Ratiu (2010). I adopt their notation in this chapter. Philosophers of physics influenced by the Chicago School sometimes use Penrose's abstract index notation instead, for which see Geroch (1974).

[5]In geometric expression, this says that the trajectories are the integral curves of the vector field X for which $\iota_X \Omega = d\Omega$. In (Darboux) coordinate form, Hamilton's equations are the system of $2n$ equations $(d/dt)q_i(t) = (\partial/\partial p)h(q, p)$ and $(d/dt)p_i(t) = -(\partial/\partial q)h(q, p)$, for each $i = 1, \ldots, n$.

That is: the same positions are occupied in the time-reversed description, but in the reverse order, and with the instantaneous momentum of each state turned around.

However, it is equally possible to use the q's to represent momentum and the p's to represent position, or some other quantity entirely. So, in Hamiltonian mechanics, time reversal really cannot be given explicit meaning until one has interpreted what how a state represents reality, and in particular what the q's and p's represent. Nevertheless, there is a general feature that all time reversal transformations in Hamiltonian mechanics are assumed to share (e.g. in Abraham and Marsden 1978, Definition 4.3.12), which is that they are *antisymplectic.* This means that the push-forward T^* of T reverses the sign of the symplectic form, $T^*\Omega = -\Omega$. Reversing p in Darboux coordinates is one example of an antisymplectic transformation: writing the symplectic form in terms of the wedge-product as $\Omega = dq \wedge dp$, we find that $(q, p) \mapsto (q, -p)$ induces a transformation $\Omega \mapsto -\Omega$.

Viewing time reversal in Hamiltonian mechanics as some particular antisymplectic bijection $T : \mathcal{P} \to \mathcal{P}$, there is an easy way to interpret all the other antisymplectic transformations: they are a combination of time reversal plus some other (non-time-reversing) symmetry. To see this, let $A : \mathcal{P} \to \mathcal{P}$ be any other antisymplectic transformation. Then $S = A \circ T^{-1}$ is symplectic, and $A = S \circ T$. In practice, such a transformation $S \circ T$ might represent parity and time reversal (PT), or spatial translation and time reversal, or any other symplectic transformation followed by the reversal of time.

Time reversal invariance in this context means that if $\xi(t)$ is a solution to Hamilton's equation with Hamiltonian h, then so is the time-reversed curve $T\xi(-t)$, where $T : \mathcal{P} \to \mathcal{P}$ is the antisymplectic time reversal operator. This is easily checked to be equivalent to the statement that the time reversal operator leaves the Hamiltonian invariant, in that $h \circ T(\xi) = h(\xi)$ for all $\xi \in \mathcal{P}$.

2.3.3. *Example: Quantum Mechanics.* The quantum mechanical expression of time reversal is very similar to that of classical Hamiltonian mechanics. Let the space of (pure) states be a Hilbert space $\mathcal{P} = \mathcal{H}$. The dynamical trajectories $\psi(t)$ are the curves that satisfy the law of unitary evolution $\psi(t) = e^{-itH}\psi$ for some self-adjoint H and initial state $\psi \in \mathcal{H}$; in differential form this is just the Schrödinger equation $i(d/dt)\psi(t) = H\psi(t)$. As in Hamiltonian mechanics, the explicit definition of time reversal in quantum mechanics depends on what we take the states ψ to represent. Suppose we take them to represent 'position wavefunctions' in the following sense. Let $\mathcal{H} = \mathcal{L}_2(\mathbb{R}^n)$ be the Hilbert space of square-integrable functions $\psi : \mathbb{R}^n \to \mathbb{C}$. Define the operators Q, P as in the Schrödinger representation, by $Q\psi(x) := x\psi(x)$ and $P\psi(x) = i(d/dx)\psi(x)$ (on the appropriate domains). If we interpret Q and P as the 'position observable' and 'momentum observable', respectively, then the time reversal operator is defined by $T\psi(x) = \psi(x)^*$, and the time reversal transformation is given by:

$$\psi(t) \mapsto \psi(-t)^*.$$

One can check that transformation has the effect of preserving position and reversing momentum: $TQT^{-1} = Q$ and $TPT^{-1} = -P$.

Of course, the formal operators Q and P do not have to be interpreted as position and momentum, just like in classical Hamiltonian mechanics. For example, in the momentum representation, one would interpret this operator Q as momentum. However, as in classical Hamiltonian mechanics, it is generally assumed that all time reversal operators share a property, which is that of being *antiunitary*. An antiunitary operator A is one that satisfies $A^*A = AA^* = I$, but which is antilinear: $A(a\psi + b\phi) = a^*\psi + b^*\phi$, for all vectors ψ, ϕ and for all $a, b \in \mathbb{C}$. This implies that $\langle A\psi, A\phi\rangle = \langle\psi, \phi\rangle^*$, for all $\psi, \phi \in \mathcal{H}$. Once we are convinced that time reversal preserves Q and reverses P, then it follows that T must be antiunitary[6]. And, in an irreducible representation of the canonical commutation relations in Weyl form, this T is in fact the unique antiunitary operator (up to a multiplicative constant) that preserves Q and reverses P (Roberts 2017, Proposition 2). However, some identification of what the observables represent is needed in order to make this kind of argument.

[6]Proof: given that $[Q, P] = i$, and that T preserves Q and reverses P, it follows that $TiT^{-1} = T[Q, P]T^{-1} = [TQT^{-1}, TPT^{-1}] = -[Q, P] = -i$, which implies that T cannot be unitary. Adopting the assumptions of Wigner's theorem, it follows that T is antiunitary.

Once we do fix some antiunitary operator T as time reversal, it turns out here too that the antiunitary operators are precisely those that can be written as unitary (non-time-reversing) operator combined with T. Namely, given an antiunitary A, we define $U = AT^{-1}$, whence it follows that U is unitary and $A = UT$.

A complication introduced in quantum theory is the existence of internal degrees of freedom associated with observables like spin. For example, consider the standard representation of the Pauli matrices on a 2-dimensional Hilbert space $\mathcal{H}$:

$$I = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} & i \\ -i & \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}.$$

In this context, the time reversal operator is defined by,

$$T\psi := \sigma_y \psi^*,$$

where the vectors ψ are written in the eigenstate basis associated with the σ_3 operator. This operator T is easily verified to have the property of reversing the spin observables, in that $T\sigma_i T^{-1} = -\sigma_i$ for each $i = 1, 2, 3$. Conversely, in an irreducible representation of the canonical anticommutation relations, this T is in fact the unique antiunitary operator (up to a multiplicative constant) that reverses all the spin observables in this way (Roberts 2017, Proposition 3).

In quantum mechanics, time reversal invariance means that if $\psi(t)$ describes a unitary solution to the Schrödinger equation for some Hamiltonian H, in that if $\psi(t) = e^{-itH}\psi$ for some $\psi \in \mathcal{H}$, then the time-reversed trajectory $T\psi(-t)$ is a unitary solution too, in that $T\psi(-t) = e^{-itH}\phi$ for some $\phi \in \mathcal{H}$. This turns out to be equivalent to the statement that the antiunitary time reversal operator T commutes with the Hamiltonian, $TH = HT$.

Why think of time reversal in this way? In one sense, the answer is easy: we want an operator that preserves position, and reverses momentum and spin. But why do we want that? There are in fact systematic ways to answer this. For example, if one demands a time reversal transformation that is non-trivial, in that it allows at least one non-zero Hamiltonian that is time reversal invariant, then this is already enough to guarantee that the time reversal operator is antiunitary (Roberts 2017, Proposition

1). Symmetry arguments from the homogeneity and isotropy of space can then be used to determine the transformation rules for position, momentum and spin. The result of this analysis is a derivation of the standard time reversal transformation for quantum mechanics. I will not go into further details here; they can found in the paper cited above.

2.3.4. *Example: Relativistic field theory.* Most relativistic field theories can be described with a globally hyperbolic Lorentzian manifold[7] (M, g_{ab}), together with a collection of classical or quantum fields satisfying some field law. Classical electromagnetism is one such example: there is a vector field J^a representing the 4-current, and an antisymmetric rank-2 tensor field F_{ab} representing the Maxwell-Faraday field, which together satisfy Maxwell's equations.

Such theories often have a Hamiltonian formulation, in which case time reversal can be understood using the same techniques described above for classical and quantum mechanics. However, Malament (2004) has proposed another way to understand time reversal in this context, which is both natural and powerful. In short, suppose we identify the direction of time with a systematic choice of light cone lobes to represent the 'future'. Then we can treat time reversal as the reversal of those lobes, as in Figure 2. It is then often possible to identify the physical fields that plausibly exhibit some dependence on the choice of a future direction, and use this to work out how they transform when these lobes are reversed.

In more precise terms: a *temporal orientation* for (M, g_{ab}) is an equivalence class of smooth timelike vector fields with the property that every pair of its elements ξ^a, χ^b 'points in the same direction', in that $g_{ab}\xi^a\chi^b > 0$. Every globally hyperbolic spacetime admits exactly two temporal orientations, corresponding to the two lobes of each light cone. In practice, it is convenient to describe an orientation by choosing a representative smooth vector field τ^a from the equivalence class, so long as we remember

[7]For an introduction to the foundations of relativity theory, the *locus classicus* is Malament (2012). In this section I adopt the notation and terminology from this book, including Penrose 'abstract index' notation for tensors in this section, since it is so standard in this context — with apologies that it is a different tensor notion than the one used above.

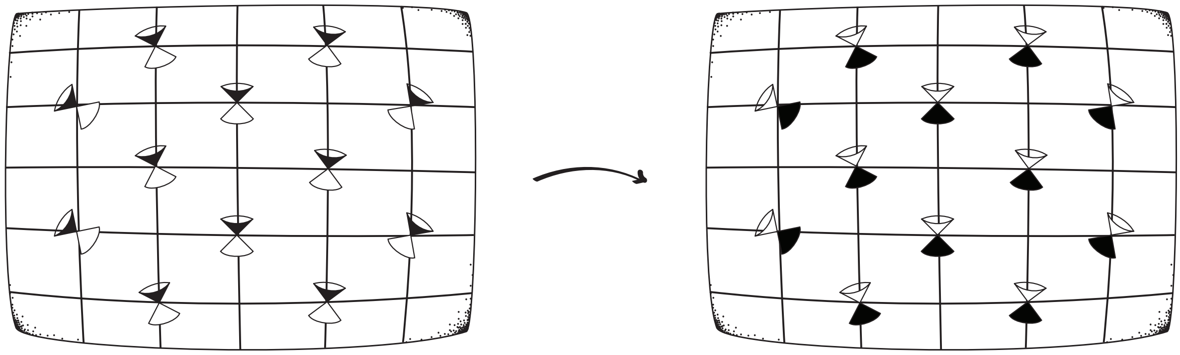

FIGURE 2. Time reversal viewed as reversal of temporal orientation: the black lobe represents the 'future direction' in each case.

that any other choice would have served just as well. Then we can understand a time-reversing transformation to be one that reverses the temporal orientation, $\tau^a \mapsto -\tau^a$. The effect of this is that, if an event p 'happens before' an event q in a spacetime structure (M, g_{ab}, τ^a), in that p is connected to q by a future-directed timelike curve, then 'q happens before p' in the time-reversed structure $(M, g_{ab}, -\tau^a)$

This lead North to propose that time reversal means: "*Only* flip the temporal orientation vector field" (North 2008, p.212). However, it's important to add that one must occasionally do other things as well, in order to distinguish time reversal from (say) parity-time reversal. To exclude the latter, we must also require that the time reversal transformation reverse total orientation, $\varepsilon_{abcd} \mapsto -\varepsilon_{abcd}$. When combined with time reversal, this ensures that no 3-volume element ε_{abc} associated with a spatial surface reverses orientation; that is, it ensures that only time (and not space) is reversed[8].

The remaining step is to figure out how this definition of time reversal transforms a matter field. Suppose we associate each spacetime structure of the form $(M, g_{ab}, \tau^a, \varepsilon_{abcd})$ with a matter field, represented by the vector field ξ^a, which we define to be $\xi^a := k\tau^a$ for some $k \in \mathbb{R}$. Then the time reversal transformation $(\varepsilon_{abcd}, \tau^a \mapsto -\tau^a) \mapsto (-\varepsilon_{abcd}, -\tau^a)$ induces a transformation $\xi^a \mapsto -\xi^a$ of the matter

[8]The 3-volume element associated with a future-directed timelike vector ξ^a is given by $\varepsilon_{abc} := \xi^n \varepsilon_{anbc}$. Moreover, reversing temporal orientation induces a change in sign in the future-directed vector $\xi_a \mapsto -\xi_a$. We thus find that by reversing the sign of both ε_{abcd} as well as the temporal orientation, we induce a transformation $\varepsilon_{bcd} \mapsto -(-\varepsilon_{bcd}) = \varepsilon_{bcd}$, and so the 3-volume is preserved.

field, in that if $(M, g_{ab}, \varepsilon_{abcd}, \tau^a)$ is associated with ξ^a, then the transformed structure $(M, g_{ab}, -\varepsilon_{abcd}, -\tau^a)$ is associated with $-\xi^a$. This transformation occurs because the matter field is 'linked' to the direction of time by its very definition. Malament (2004) points out that the fields of electromagnetism share this property. The student (and everyone) will do best to study Malament's lucid article directly; I will not go into more details here. Instead I will just state the conclusion: on a natural understanding of the Maxwell-Faraday tensor F_{ab}, and the associated electric field E^a and magnetic field B^a in some reference frame, the effect of the time-reversal transformation defined above is to preserve E^a, while reversing F_{ab} and B^a:

$$F_{ab} \mapsto -F_{ab} \qquad B^a \mapsto -B^a \qquad E^a \mapsto E^a.$$

This characterisation of time reversal notably requires a temporally and totally oriented spacetime[9]. Without such a global notion of future and past, there is of course no global define time reversal, either. Peterson (2015) has suggested this is a concern about the approach. However, it is possible to give a similar analysis of time reversal for non-temporally-oriented spacetimes, too. For example, given a non-temporally-orientable spacetime, one can define time reversal with respect to a point by considering a (possibly small) region around that point that is temporally orientable. Or, one can construct the universal covering space, which can be used to represent all the same local physical facts, and which is in general temporally orientable (Hawking and Ellis 1973, §6.1). Thus, although 'flipping the temporal orientation' is a guiding idea here, temporal orientability is really no barrier to defining time reversal.

3. The Albert-Callender 'pancake account'

The above discussion presents the standard account of time reversal, and some of the arguments that have been given for it. In this section, I will present (and

[9]Gödel spacetime is not temporally orientable. Gödel (1949) argued from this to the ideality of time; see Savitt (1994); Earman (1995, Appendix to §6); Dorato (2002); Belot (2005); Manchak (2016).

then critique) a creative non-standard account due to Albert and Callender, which has recently become popular among some philosophers[10]

3.1. **Critique of the Standard Account.** Some of the properties of the instantaneous time reversal operators T discussed above (and in the table of Figure 1) are intuitively reasonable: the time reverse of a billiard ball appears to warrant reversing both velocity and momentum. Some are also automatic, such as the fact that the velocity dx/dt of a trajectory $x(t)$ reverses sign under the transformation $t \mapsto -t$.

However, at least to the newcomer, other properties of the standard account less intuitive: the spin and magnetic field reverse, and position wavefunctions are conjugated. At least on a superficial level, these transformations do not arise from reversing the sign of the 'little t' index describing order in time. So, why do these changes occur? We have seen some positive reasons in the examples above: they can be derived by thinking about how these objects depend on the temporal orientation of spacetime, or by demanding a time reversal transformation that is non-trivial. But let's also consider a more disruptive alternative: to reject the standard account as untenable.

David Albert responded to the textbook definition of time reversal in this way:

> "the books identify precisely that transformation as the transformation of 'time-reversal.' ... The thing is that this identification is wrong. Magnetic fields are not the sorts of things that any proper time-reversal transformation can possibly turn around. Magnetic fields are not — either logically or conceptually — the *rates of change* of anything. ... [Time reversal] can involve nothing whatsoever other than reversing the velocities of the particles" (Albert 2000, p.20)

[10]An early expression of a similar view can also be found in Horwich (1989).

Craig Callender independently came to the same conclusion[11], writing that "David Albert... argues — rightly in my opinion — that the traditional definition of [time-reversal invariance], which I have just given, is in fact gibberish. It does not make sense to time-reverse a truly instantaneous state of a system" (Callender 2000, p.254).

Their proposal, instead, is that time reversal is nothing more than the reversal of the order of states in time. This is to adopt the first component of the standard account of time reversal, that the order or trajectories is reversed, but not the second, by rejecting the use of a time reversal operator T on instantaneous states. Equivalently, Albert and Callender adopt a time reversal operator T that is the identity operator, $T = I$, which only transforms instantaneous states trivially to themselves.

The Albert-Callender objection is natural given a certain perspective on the passage of time. According to it, all moments are stacked up like a stack of inert pancakes, so that time reversal consists solely in a reversal of the order of pancakes in the stack (Figure 3). As Malament (2004) puts it, this view presumes that physical phenomena described at each moment "just lie there" at any given instant, without any connection to the direction of time, unless they happen to be defined (like velocity) in a way that depends on the time parameter t. The ordinary description of a magnetic field is not like this, nor is any other quantity appearing in the left column of Figure 1. As a result, Albert and Callender conclude, the instantaneous description of these quantities should be left unchanged by time reversal.

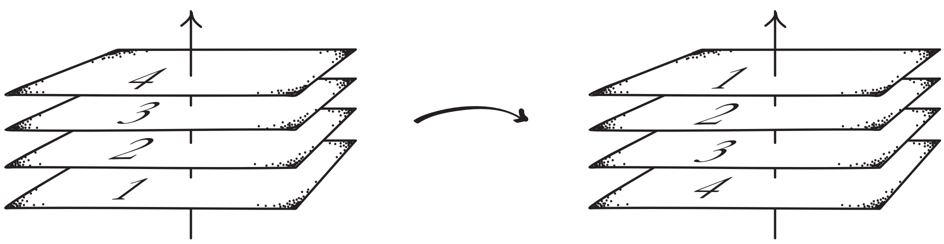

FIGURE 3. The Albert-Callender 'pancake' approach to time reversal: order reversal of inert spatial slices with no temporal properties.

[11]This work drew on Callender's 1997 dissertation. See also Horwich (1989, §3) for a related view.

3.2. **Consequence for time symmetry.** The 'pancake objection', if accepted, leads to a radical revision how most people currently understand time reversal. For example, all of the interesting applications described in the Introduction to this chapter would evaporate, or at least no longer be disassociated with the 'true' meaning of time reversal according to Callender and Albert. In particular, the widely-held belief that most isolated systems are time reversal invariant would no longer be true. Albert and Callender themselves emphasise this fact; for example, Albert writes:

> [Electromagnetism] is *not* invariant under time-reversal. Period. And neither (it turns out) is quantum mechanics, and neither is relativistic quantum field theory, and neither is general relativity, and neither is supergravity, and neither is supersymmetric quantum string theory, and neither (for that matter) are any of the candidates for a fundamental theory that anybody has taken seriously since Newton. (Albert 2000, p.14)

Indeed, even classical Hamiltonian mechanics fails to be invariant under time reversal on this perspective, in all but the most trivial cases. It's instructive to review this example in some mathematical detail: suppose we have a classical phase space $\mathcal{P} = \mathbb{R}^{2n}$ with a Hamiltonian function $h : \mathcal{P} \to \mathbb{R}$. Time reversal invariance means, as usual, that if a curve is a solution to Hamilton's equations with this Hamiltonian, then so is its time-reverse, now viewed as the mere order-reversal $t \mapsto -t$ of states. So, in local coordinates (q, p), suppose a system is time reversal invariant in this non-standard sense: given a solution $(q(t), p(t))$ to Hamilton's equations, its 'pancake time reverse' $(q(-t), p(-t))$ is a solution as well, for all times $t \in \mathbb{R}$:

$$\begin{aligned} \tfrac{d}{dt}q(t) &= \tfrac{\partial}{\partial p}h(q,p) & \qquad \tfrac{d}{dt}q(-t) &= \tfrac{\partial}{\partial p}h(q,p) \\ \tfrac{d}{dt}p(t) &= -\tfrac{\partial}{\partial q}h(q,p) & \qquad \tfrac{d}{dt}p(-t) &= -\tfrac{\partial}{\partial q}h(q,p) \end{aligned}$$

Since these equations hold for all $t \in \mathbb{R}$, they continue to hold if we apply the substitution $t \mapsto -t$ to the two equations on the right. But, combining this with the equations on the left, we find that both $\partial h/\partial q$ and $\partial h/\partial p$ are equal to their negatives, and hence,

$$\tfrac{\partial}{\partial q}h(q,p) = \tfrac{\partial}{\partial p}h(q,p) = 0,$$

which implies that $h(q,p) = c$ is constant across all points $(q,p) \in \mathcal{P}$. This means that, on the 'pancake' account of Albert and Callender, the only time reversal invariant system is the trivial one in which energy is constant across all states[12]. This is in particular a system in which nothing ever changes. So, a consequence of the pancake perspective on time reversal is that time reversal invariance only holds of the most trivial physical descriptions, for which no change in time is ever possible.

3.3. **Philosophical Implications.** In spite of these radical consequences for physics, many have recently entertained the Albert-Callender perspective and explored its philosophical consequences. For example, Farr and Reutlinger (2013) suggest it could be used in support of the Russellian 'deflationary' account of causation, although they ultimately endorse the standard view of time reversal instead. Allori (2015) adopts the Albert-Callender perspective wholesale, and uses it to argue that electromagnetism contains an interpretive paradox. Similarly, Castellani and Ismael (2016) adopt it in their response to the claim that time reversal is a counterexample to Curie's Principle[13] (Roberts 2013a).

In this last case, to save Curie's principle, Castellani and Ismael argue that it is reasonable to reject the standard account of time reversal and adopt the Albert-Callender perspective instead:

> "In physical terms, time reversal should leave the states intrinsically untouched and just change their order. If we cleave to that understanding of time reversal, none of the counterexamples Roberts offers constitutes a failure of [Curie's Principle]" (Castellani and Ismael 2016, p.1011).

[12]This argument is a 'one liner' from the geometric perspective on Hamiltonian mechanics: let $(\mathcal{P}, \Omega)$ be a symplectic manifold, and suppose that X and $-X$ both describe the Hamiltonian vector field generated by $h : \mathcal{P} \to \mathbb{R}$. This means that, $dh = \iota_X\Omega = \iota_{(-X)}\Omega = -\iota_X\Omega = -dh$. The exterior derivative dh thus vanishes, which implies that h is a constant function.

[13]Curie's principle is the claim that "when certain causes produce certain effects, the elements of symmetry of the causes must be found in the produced effects" (Curie 1894, p.394); English translation in Brading and Castellani (2003, §17).

I am not convinced. In the next sections, I will identify a few objections to the Albert-Callender 'pancake' perspective on time reversal, and thus to the philosophical agendas that make use of it above.

4. Against the Pancake Account

This 'pancake' account of time reversal is a creative perspective on the nature of time, and certainly worth exploring. However, I will argue in this section that it is not an equally legitimate competitor to the standard account: it suffers from at least five concerns, all of which are avoided by the standard account of time reversal.

4.1. **Concern 1: Questionable motivation.** The main motivation for the Albert-Callender view is that the only instantaneous properties that depend on the direction of time are "rates of change", such as the linear change in position with respect to time (velocity), or rotational change in position with respect to time (angular velocity). What is the justification for this claim? In physics, as well as in ordinary language, there appear to be many instantaneous properties that do depend on temporal direction, and which are not rates of change of anything in time.

FIGURE 4. Brave or cowardly? This instantaneous property of the soldier appears to depend on the direction of time.

For example, consider a soldier racing towards a deadly dragon (Figure 4). At some moment, the soldier might be said to have the instantaneous property of 'bravery'. But what happens when this description is transformed to its time reverse, in which the soldier is back-pedalling away from the dragon? Now the soldier in the same moment might be justifiably described as 'cowardly'. In other words, the instantaneous properties of 'bravery' and 'cowardice' display a dependence on the direction of time, in spite of not being necessarily being characterised by a rate of change.

The same goes for physics: some degrees of freedom, such as an electron's spin or a magnetic field in some reference frame, are not necessarily the rate of change of anything, and yet may be linked to the direction of time. To deny that this is the case requires a positive argument. The burden of providing such an argument is a challenge for the supporter of the pancake account.

4.2. **Concern 2: Momentum in the wrong direction.** Further conceptual difficulties arise on the pancake account. For example, consider the description of a state $\xi = (q, p)$ in classical Hamiltonian mechanics, with q interpreted as position and p as momentum. By only reversing the order of states in time, and not the instantaneous states $\xi = (q, p)$, the pancake account entails a reversal of velocity dq/dt, but not of momentum. In other words, velocity and momentum after time reversal point in the opposite directions!

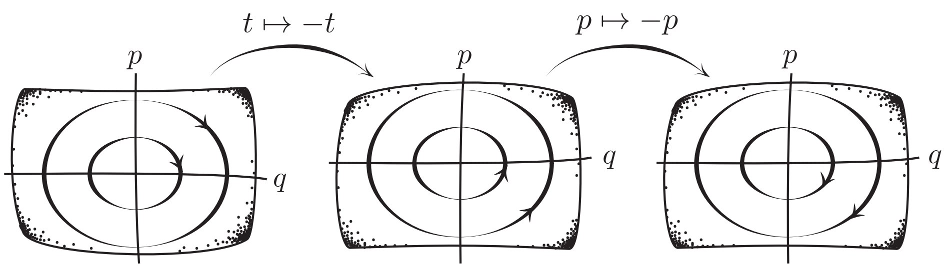

FIGURE 5. Time reversal of a harmonic oscillator's phase space: the pancake account only includes the first (order-reversing) transformation, leading to momentum and velocity in opposite directions; the standard account avoids this by including the second (instantaneous state) transformation as well.

This problem is avoided in the standard account, where we reverse momentum too, as shown in Figure 5. Recognising this issue, North (2011, p.317) suggests that Albert might avoid this problem by taking momentum to be a "non-intrinsic, non-fundamental quantity" satisfying $p = mv$. This is effectively to deny the Hamiltonian description of classical mechanics, where (q, p) can represent a great many things independently of the dynamics. And, it is simply not always the case that momentum always has the form $p = mv$ (see footnote 3 above).

4.3. **Concern 3: No non-trivial examples.** In the 20th century, it was discovered that certain decay events (for example, a decay from a kaon into an antikaon) occur with a different transition probability than the reverse decay event (from an antikaon into a kaon). This is a rare and unusual phenomenon: almost all known decay events have a transition probability that is the same in both directions. On the standard account, having equal transition probabilities in both directions is equivalent to time reversal invariance; Roberts (2014) called this 'Kabir's principle'. This provides a way to explain the strangeness of decay events in things like neutral kaons: although most interactions are time symmetric, neutral kaon decay is asymmetric in time.

On the pancake account, no comparable explanation of the difference between these interactions is available. As discussed in Section 3.2, the pancake account treats all non-trivial processes as equally asymmetric in time. There are no non-trivial examples of time symmetric systems. As a result, this transformation is of little physical interest if we wish to identify the distinction between temporal symmetries and asymmetries in realistic physical systems. Thus, a further burden on the pancake account is to explain why it is useful.

4.4. **Concern 4: Radical underdetermination.** A further issue is that the pancake account of time reversal, at least as it has so-far been presented, is radically underdetermined. The claim that time reversal transforms a trajectory to one "with the temporal order inverted" (Callender 2000, p.253) is compatible with infinitely many transformations. One commonly identifies $t \mapsto -t$ as an order-reversing transformation. But $t \mapsto \sqrt[3]{a - t^3}$ is an order-reversing transformation as well, and indeed it is even an involution, meaning that applying it twice yields the identity transformation. Callender suggests that his time reversal transformation moves objects such that "if their old co-ordinates were t, their new ones are $-t$, assuming the axis of reflection is the co-ordinate origin" (Callender 2000, p.253). But what justifies this? An emphasis on mere order-reversal is certainly not enough. Thus, a challenge for the pancake supporter is to determine which of the many order-reversing transformations really counts as time reversal.

In order to guarantee an order-reversing transformation of the form $t \mapsto -t$, one might think in more detail about which properties of matter and spacetime are linked to the direction of time, as we did when we argued for this property in Section 2.1. There we asserted things like 'time reversal is an involution' and 'time reversal does not stretch time out unevenly'. But once one has started down this road, it is hard to see why other reasonable principles guiding the definition of time reversal should be excluded. Can we also now ask that time reversal be a transformation that allows for some non-trivial systems to be time symmetric? Can we ask that time reversal respect the dependence of a physical field on light cone structure? If the answer is yes, then the defences of Malament (2004) and Roberts (2017) lead straight to the standard definition of time reversal. There appears to be little to prevent this, once reasonable steps are taken to avoid the underdetermination of the pancake account.

4.5. **Concern 5: Relativistic fields.** It is particularly hard to make sense of the pancake account in the context of fields on a relativistic spacetime. For example, the toy matter field of Section 2.3.4, defined by $\xi^a = k\tau^a$, appears to be somehow forbidden on the pancake account: on that view, a vector field that is not a rate of change of anything must 'just lie there' under time reversal. But the vector field ξ^a manifestly does not, at least on Malament's account of time reversal.

Arntzenius (2004) has pointed out a further difficulty, that saying electric and magnetic fields 'just lie there' at a given instant suggests they do not transform under Lorentz boosts, in spite of the well-known empirical fact that they do. In fact, the situation is even worse: on Malament's reconstruction, electric and magnetic fields can only be meaningfully defined with respect to a temporal orientation, and so there is no way to reverse one without inducing a transformation of the other. The pancake account of time reversal appears to not even be wrong in this context: it is incoherent.

In spite of this difficulty with the pancake account, some authors still demand "taking it seriously" in the context of electromagnetism (c.f. Allori 2015, p.4). Arntzenius and Greaves (2009, §3.4) have made a valiant attempt to do so, by proposing that we interpret at least Albert as holding the background assumption that "spacetime is in fact Newtonian, velocities are spatial 3-vectors, and so are the electric and magnetic fields", a view that they attribute to him on the basis of personal correspondence (Arntzenius and Greaves 2009, p.568).

However, it is not clear that even this is enough: in order to make the view coherent, we must define electric and magnetic fields in such a way that they (a) correctly describe electromagnetic phenomena, and (b) do not depend on the direction of time for their definitions. I do not see how this is possible on the pancake account. If the charge moves in one direction along a straight conducting wire, then it is associated with a magnetic field that curls around in a particular orientation. If the charge moves in the other direction, the magnetic field has the opposite orientation. So, to in order to correctly describe this magnetic field, even on a Newtonian spatial slice, we need to know the direction of motion of a charged particle. And the direction of motion, even on the Albert-Callender account, depends on the direction of time, however we formulate it mathematically.

5. Conclusion and Open Research Questions

This chapter has tried to give an overview of the standard account of time reversal, and to present a critique of a popular competing view. Much remains to be learned about the foundations of time reversal, even on this narrow topic of its meaning. We thus conclude by mentioning a few open research questions, of varying degrees of difficulty, which are suitable for researchers or for student research essays.

(1) *Time vs Motion reversal.* Ballentine (1998, p.377) wrote, following Wigner (1931, p.325), that "the term 'time reversal' is misleading, and the operation... would be more accurately described as motion reversal". Evaluate this claim.

(2) *Curie's principle.* Evaluate the Castellani and Ismael (2016) response to the critique of Curie's Principle given by Roberts (2013a, 2015) and Norton (2016).

(3) *Time reversal in Open Systems.* Most derivations of the meaning of time reversal assume a context of local physics for isolated systems; see e.g. the discussion of Section §2.1. What can be said about the physics of large-scale or open systems? On the latter, see Maroney (2010, §2.2) for a helpful start.

(4) *Deriving the Lagrangian time reversal operator.* Formulate an assumption about what time reversal ought to mean in the context of Lagrangian mechanics on a tangent bundle TM (c.f. De León and Rodrigues 1989, §7). Then derive the basic mathematical properties of the time reversal operator, in the spirit of Malament (2004) and Roberts (2017). Check whether its image under the Legendre transformation is antisymplectic on the cotangent bundle T^*M with its canonical symplectic form.

(5) *Spacetime approach to quantum time reversal.* Consider any rigorous formulation of quantum field theory on curved spacetime (c.f. Wald 1994; Haag 1996; Araki 1999). Derive the meaning of time reversal in this context by examining what transformation is induced when temporal orientation is reversed. *Suggestion:* As a starting point, consider the result of Varadarajan (2007, Lemma 9.9).

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