Eighth Quadrennial Fellows Conference
Pittsburgh Center for Philosophy of Science
Lund University, Lund, Sweden
11 July 2016

Maxwell's Methodological Odyssey in Electromagnetism

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Introduction

Historians and philosophers of science generally tend to recast historical data in modern terms with the aim of clarifying the argument and rationally reconstructing the theory under consideration. Such an approach is productive initially for basic understanding of the theory under consideration. Indeed, we build on such works. However, we think that ultimately the goal of historical study is to understand the past on its own terms. We therefore adhere to the original text with due regard to its linguistic features, paying close attention to the terms and expressions used in the primary sources. Furthermore, we seek to determine a trajectory that faithfully traces the development of the author's ideas and his understanding of the relevant concepts. The rationale of this historiography is straightforward: we are interested in the way science has actually been practiced, in the way research has been carried out.

What was the status of electromagnetism in the 1850s? It was a divided scene: in England Faraday demonstrated experimentally novel electromagnetic phenomena and attempted to grasp them with the unifying concept he constructed, namely, lines of force. William Thomson showed with great ingenuity that electromagnetism has a mathematical structure analogous to heat conduction, and suggested that eventually all these phenomena could be understood mechanically. But it was not at all clear that the concept of lines of force was the key for comprehending phenomena in this physical domain. Indeed, Thomson sided with prominent continental physicists such as Ampère, Gauss, and Wilhelm Weber, who held to the view inspired by Newtonian mechanics: action at a distance is the concept which unifies electromagnetism. What then was the way to proceed in formulating a theory of electromagnetism?

This was probably the most urgent question which James Clerk Maxwell faced when he came on the scene at the age of 24 with the intention of contributing to this domain of physics. In particular, he had to decide which conceptual framework to choose. The concept of lines of force meant that the action of the force takes place along curved paths in contrast to the well established action at a distance in which forces act in straight lines. Maxwell acknowledged that at the time when he began his work on electromagnetism he had hardly made a single experiment in this domain; hence, the intended contribution had to be abstract. His initial goal was to turn Faraday's concept of lines of force into standard mathematical form. The expectation was that a mathematical formulation would suggest new physical connections unforeseen by Faraday and thus enrich the domain with new discoveries. The question persisted, how to proceed?

Our principal claim is that Maxwell made a conscious decision to begin with methodology. What methodology should be applied in order to reach the goal of recasting Faraday's experimental results into symbolic language? Maxwell was committed to the concept of lines of force rather than to action at a distance, and so the core of the problem was what methodology could make the concept of lines of force contend with the concept of action at a distance? To be sure, Maxwell recognized the many successes of the concept of action at a distance; clearly, discarding it would raise vehement objections. The way to proceed, going beyond what his predecessors and contemporaries had done, was to demonstrate that "lines of force" could account for "action at a distance", but that the inverse does not work. In other words, the problem is the link between methodology and physical content: what methodology would show that the concept of lines of force can account for action at a distance, that is, that the latter is a special case of the former?

We argue that Maxwell reflected deeply on this issue and came up with a brilliant solution that was neither properly appreciated at the time by his fellow physicists nor fully recognized by recent historians and philosophers of science. The solution was to invent, or rather imagine, to use Maxwell's word, a new kind of physics, and then use it in a contrived analogy in order to develop a new formalism. Maxwell began this contribution with a discussion of scientific methodology, which is quite rare in an essay on a technical subject. He warned researchers not to be lured by a "favorite hypothesis" that can lead astray, nor should they be seduced by "analytical subtleties". The way to negotiate a safe path between physical hypotheses and mathematical formulae is to contrive an analogy between a physical domain *A* and an imaginary domain *B* where the latter is completely at the physicist's disposal, and draw consequences in *B* to be applied to *A*. Maxwell's profound remarks, albeit brief, on the methodology of analogy, outline the epistemic means by which scientific knowledge, specifically in physics, is attained. In this discussion he emphasized that scientific methodology can work productively in the service of scientific research.

It is then surprising that Maxwell made three more substantial contributions to electromagnetism, each exhibiting a different methodology. This sequence of works culminated in a theory of electrodynamics, which Maxwell formulated in his *Treatise* of 1873. Within a span of some 17 years, from 1856 to 1873, Maxwell published three major papers on electromagnetism and a treatise in which he applied different methodologies, independent of one another. On the one hand, this feat of scientific ingenuity demonstrates Maxwell's versatility but, on the other, in all these publications Maxwell consistently showed that Faraday's concept of lines of force is superior to the Newtonian concept of action at a distance, which was then dominant in continental accounts of electromagnetism. While the three papers and the treatise exhibit a variety of methodologies, there is a constant theme which is retained throughout, namely, recasting Faraday's conceptual analysis of his experimental findings into formal language, that is, mathematical equations. Maxwell achieved this goal with different methodologies. We have surveyed Maxwell's sources, works by scientists such as Thomson, Tait, Stokes and Rankine, none of them had as a formative influence on Maxwell as did Faraday—his role was paramount. In this light, Maxwell's characterization of Faraday as a mathematician—despite the absence of equations in Faraday's writings—is not strange at all. Ultimately, the search for mathematical equations, the consistent theme in these contributions, was sought in order to discover physical connections that otherwise are hidden from view.

The four principal stations along the seventeen-year trajectory of Maxwell's published writings devoted to electrodynamics, are the following: (1) the paper of 1858, "On Faraday's lines of force"; (2) the paper of 1861–1862, "On physical lines of force"; (3) the paper of 1865, "A dynamical theory of the electromagnetic field"; culminating in (4) Maxwell's magnum opus of 1873, *A treatise on electricity and magnetism*. Tracing this trajectory yields insights into the various methodologies which Maxwell pursued in the course of constructing his epoch making electrodynamical theory. We call this trajectory Maxwell's methodological odyssey in electromagnetism.

Odyssey expresses transition, a passage from one station to another, while enriching one's experience. What makes it possible to speak of an odyssey is a consistent thread that runs through all the four stations and thus makes them elements of a trajectory. This thread is an expression of commitment to two related concepts, namely, lines of force and a medium—the latter being the bearer of the former. The commitment is to the physical reality of lines of force and to the medium as the agent, the bearer of the phenomena of this physical domain. Maxwell's commitment is then to Faraday's conceptual framework; he never abandoned or modified it.

The advantage for historians and philosophers of treating the entire trajectory from 1856 to 1873 as a unified whole is that it reveals Maxwell's methodological ingenuity. Moreover, this framework of discussion is designed to highlight the shifts in methodology and to emphasize the relation between form and content. In each paper Maxwell chose a methodology that would work best for that stage in embedding Faraday's concept of lines of force in a powerful theory. Put differently, the framework of the theory is just as important as the empirical facts in this physical domain.

While Maxwell was committed to the unifying concept of lines of force, with respect to methodology he had no specific commitment. Evidently, Faraday's conceptual framework can thrive in many methodological contexts. We distinguish between method and methodology, although Maxwell did not have distinct terms for them. A method is a plan of action or an ordered systematic arrangement of phenomena; it is essentially organizational, that is, a tool. A methodology is a procedure for attaining knowledge which is employed, more or less systematically, in a field of study as a mode of investigation and inquiry. Methodology is the application of heuristic rules of research as well as the epistemic grounding of scientific knowledge. While a set of methods can function in some methodology, it should be recognized that methods and methodology are not equivalent. Our philosophical focus lies at the juncture where knowledge is generated, that is, where new claims about nature are made and defended on the basis of some methodology. The trajectory of Maxwell's methodologies in electromagnetism begins with what he initially called "mathematical analogy" in 1856. In 1861–1862 Maxwell shifted methodology and pursued the consequences of a hypothesis of a mechanical nature, the so called molecular vortices hypothesis, taking the lines of force to be physically real. In 1865 Maxwell discarded his methodology of 1861–1862 and applied the methodology of formal analogy in which the equations represent descriptions of the phenomena. Finally, in the *Treatise* of 1873 Maxwell referred to all his previous methodologies and appealed to a new one, even though they seem to contradict each other.

Let us look closely at each station:

Station 1

The first station calls attention right in its title to the motivating concept, Faraday's lines of force. In many respects, this first contribution to the study of electromagnetism is the most revolutionary, although each of the four contributions is innovative in its own way. In this first paper Maxwell transformed the traditional methodology of analogy, perfected by Thomson, into a powerful new technique which draws a parallel between a set of physical phenomena and an imaginary arrangement, entirely contrived by the physicist. The crucial move was to imagine the lines of force as tubes, fill them with incompressible fluid complete with sources and sinks. Maxwell then developed an imaginary hydrodynamics to turn the geometrical scheme of Faraday into a "physical" one, which now included both the direction of the force and its intensity. This novel imaginary physics served as an analogue which, in turn, facilitated the construction of a new formalism. Faraday's geometrical scheme had the power to unify electromagnetic phenomena, but only geometrically; Maxwell rendered it physical with the introduction of the intensity of the force in addition to its direction. This was done by appealing—surprisingly—to imagination; indeed, that was the strength of the new methodology. We call this methodology "contrived analogy", for Maxwell constructed an analogy which worked, as he explicitly put it, in one direction only: from his imaginary physical scheme to the physics of electromagnetism. The use of analogy in scientific discourse is very old, but making an analogy between observable phenomena and imaginary physics was something new and unexpected.

The modification turns an analogy between two different physical domains into an analogy between an imaginary system—the flow of an incompressible fluid—and nature—electromagnetic phenomena. Clearly, this contrived analogy was designed to be used as a vehicle for mathematical reasoning. Maxwell's ingenious methodological idea goes far beyond Faraday's concept of lines of force. Maxwell stressed the generality of the method, for it can account for any kind of force. In particular, it does not exclude the attracting force of action at a distance. This is significant for, in principle, the methodology does not exclude theories based on action at a distance—the fundamental concept that was dominant at the time. Maxwell showed confidence in Faraday's concept of lines of force but, at this juncture in his electromagnetic studies, he was apparently reluctant to dismiss the Newtonian approach altogether. Maxwell's stated goal was to recast Faraday's verbal description of the phenomena into mathematical symbolism and thus to obtain a new perspective on the phenomena.

Station 2

However, four years later, in his second contribution to electromagnetism, Maxwell applied an entirely different methodology from that in the first contribution. In fact, the analogical approach was dropped altogether and Maxwell pursued a more traditional methodology, that is, hypothesis based physics. At this second station Maxwell stated that he had changed his approach. The contrived analogy of the first paper was merely instrumental in showing that the mathematical formulation of electromagnetic phenomena could be based on the concept of lines of force. But, in the second paper, on physical lines of force, Maxwell's goal was to account for the phenomena, that is, he now presupposed a mechanical hypothesis to explain the phenomena. The change in the underlying methodology is dramatic: Maxwell discarded the instrumental approach and sought a hypothesis at the micro-level to account for the phenomena at the macro-level. The question now was, What tensions in, or motions of, a medium are capable of *producing* mechanically the observed electromagnetic phenomena? Maxwell's mechanical point of view involved illustrations based on the molecular vortex hypothesis. Clearly, the change is categorical, from analogical argumentation to causal reasoning.

Moreover, Maxwell drew for the benefit of the reader a figure depicting this scheme; hence the appeal to mechanical illustrations. While in the first paper he took a formal-analogical approach, in the second, as the title indicates, he took a physical approach, that is, a theory closely related to a mechanical illustration. Most contemporary commentators take this illustration to be a model but, in fact, Maxwell called it "hypothesis" and treated it as such. In that sense the fundamental methodology of station 2 is traditional.

What is, however, revolutionary are the discoveries Maxwell reported in this paper. The two major discoveries presented in the second paper are, first, the displacement current and, second, the suspicion that light is an electromagnetic phenomenon. The second discovery is formulated cautiously because evidently Maxwell was not entirely convinced of this discovery, given that the evidence was circumstantial. Undoubtedly, these two discoveries substantially changed this domain of physics, and they were linked to Faraday's unifying concept of lines of force; indeed, in this paper, Maxwell took the lines of force to be real. While in his first paper Maxwell showed how the concept can accommodate the commonly accepted approach that depend on action at a distance, in the second paper Maxwell openly expressed his dissatisfaction with it.

Station 3

This was not the case anymore in the third contribution. Three years after the publication of the second paper, Maxwell assumed yet another novel perspective on electromagnetism and applied a new methodology which took account of his previous findings, without any reference to their methodologies. This suggests a break, that is, the third paper is not simply a continuation of Maxwell's previous work. Station 3 departs from the methodologies of the previous contributions in many ways, but it still adheres to the underlying unifying concept of lines of force. In fact, it makes this concept the foundation of the theory and rejects altogether the Newtonian scheme. Undoubtedly, the third contribution expresses confidence both in the two discoveries reported in the second paper and in Faraday's unifying concept.

The third paper offers a dynamic theory of the electromagnetic field in which the displacement current and light as an electromagnetic phenomenon are considered assumptions. Maxwell stated that the argumentative structure of his new methodology was reversed: results of paper two were now presupposed in the theory. Maxwell called this new theory of the electromagnetic field *dynamical* because it assumes a field surrounding electrified or magnetic bodies, in which matter in motion subject to forces produces the observed electromagnetic phenomena. He specified that the motion of the assumed "aetherial substance" which constitutes the field, produces electromagnetic phenomena, that is, it is the cause of these phenomena. In this third study, the medium does the work and therefore the mechanical scheme of the molecular vortices of Station 2 is superfluous. However, the overriding task now was not to offer an explanatory mechanism by which the phenomena are produced; rather, Maxwell's goal was to construct a satisfactory formal theory compatible with the phenomena.

Maxwell identified two kinds of energy that can exist in the medium: "actual" and "potential". The fundamental property of the medium, namely, its elasticity, becomes then essential, since in virtue of this elasticity both kinds of energy can be stored in the medium. In this novel approach, which assumes an energetic medium, a specific mechanical scheme is unnecessary, for the medium takes on the physics of electromagnetism. Maxwell no longer appealed to a specific mechanism, and remarked that dynamical illustrations, such as a flywheel, are to be considered merely as assisting the reader to understand the phenomenon.

Maxwell's goal was to find a set of equations that would describe the phenomena. He stated that the equations of the theory had been deduced from purely experimental evidence: no mechanism or hypothesis intervened; rather, it was entirely based on Faraday's verbal account of the phenomena plus Maxwell's own discoveries of the displacement current and that light is an electromagnetic phenomenon.

Station 4

As indicated, Station 4 is the culmination of Maxwell's methodological odyssey in electromagnetism. In 1873, eight years after his previous contribution to electromagnetism, Maxwell published his monumental work, *A treatise on electricity and magnetism*. It should be noted that Maxwell did not appeal to any new phenomena of electromagnetism; so, from an empirical perspective, there was no motive to revise the theory or to propose a new one. However, from a theoretical perspective, there were many novelties to be adopted and applied. Maxwell's *Treatise* is a mélange of methodologies and scientific ideas intended for the student of physics as well for the expert physicist at the cutting edge of this scientific domain. Yet, however eclectic the *Treatise* may appear, it has a unifying theme which reflects a systematic methodology. The unifying concepts are energy and lines of force, and the methodology is a sophisticated extension of the methodology of Station 3. Essentially, in Station 4 Maxwell modified the methodology of Thomson and Tait that they had developed in their *Treatise on natural philosophy* of 1867. In their *Treatise* Thomson and Tait applied the methodology of "abstract dynamics" to the domain of mechanics. For his part, Maxwell tailored Thomson and Tait's principles of abstract dynamics to the domain of electromagnetism. In this way he turned electromagnetism into electrodynamics. Maxwell considered the medium the seat of energy and endowed it with dynamical properties by transmitting forces through the medium via lines of force. The emphasis on energy, rather than on force, goes back to the insights of Rankine in the 1850s, for he noted that energy is the same whether it is mechanical, electric, magnetic, or chemical. Maxwell's treatment of electromagnetic phenomena from the point of view of energy was among his many novelties. To make this approach viable, Maxwell needed to introduce ingenious modifications to a set of mathematical tools that were available at the time but had never been applied directly to the phenomena of electromagnetism, such as the Lagrangian and the Hamiltonian.

As a mélange of methodologies and scientific ideas Station 4 is rich in discussions at both the micro- and macro-level. Theories are offered at different levels of generality that were intended to encompass all the known phenomena of electromagnetism. It is apparent that Maxwell insisted on maintaining the intuitive physical grasp of the phenomena over and above the powerful formalism he modified to suit the treatment of the phenomena of electromagnetism. In sum, he sought to translate formal expressions into mental imagery; indeed, he rendered this requirement a criterion for viable theories.

Summary

There are several recurring features in these four distinct stations. We have already noted the commitment to lines of force and a medium which is what makes it meaningful to speak of an odyssey at all. But there is another recurring feature which is no less striking. Maxwell did not design any methodology *de novo*; rather, he modified existing ones. We hasten to add that this does not diminish his achievement in any way. Indeed, the whole is greater than the sum of its parts; that is, Maxwell's various methodologies are greater than his specific debts to his predecessors. Maxwell's creative responses to the works of Faraday as well as those of Thomson, Helmholtz, Stokes, Tait, Rankine, Lagrange and Hamilton, among others, show him to be an extraordinarily ingenious thinker capable of capitalizing on past achievements in electromagnetism, both theoretical and experimental, in unforeseen ways. The integration of past successes with his innovative ideas allowed Maxwell to construct a novel theory of electrodynamics that has stood the test of time.

From a philosophical perspective, we observe that the trend of the trajectory is towards greater abstraction while, at the same time, Maxwell insisted on mental images accessible without mathematical symbolism. As Maxwell put it,

> Scientific truth should be presented in different forms, and should be regarded as equally scientific, whether it appears in the robust form and the vivid colouring of a physical illustration, or in the tenuity and paleness of a symbolical expression.

This was not the view of many of Maxwell's contemporaries, especially the German physicists. For them, parallel representation was not an issue. For Maxwell it was paramount.

Our historiography of addressing Maxwell's contribution to electromagnetism in its entirety and, within this body of scientific work, tracing the transition from station to station yields an overview with a unique vantage point on Maxwell's motivations in taking this journey. This historiography of methodologies results in insights into the practice of a great scientist. The odyssey we identify as historians and philosophers of science in Maxwell's electromagnetic studies is neither a metaphor nor a poetical account of his originality; rather, for us, it is a historiographical tool for uncovering the working of a leading scientist in the nineteenth century whose accomplishments we still admire today and can serve as a source of inspiration.