Roughness in lubricated rolling contact: the dry contact limit

A V Olver* and **D Dini**
Department of Mechanical Engineering, Imperial College, London, UK

The manuscript was received on 11 July 2007 and was accepted after revision for publication on 30 July 2007.

DOI: 10.1243/13506501JET318

Abstract: A difficulty with the standard fast Fourier transform (FFT) perturbation model of roughness in lubricated rolling contacts is that it does not necessarily converge towards the elastic case as the film thickness is reduced; rather it leads to a situation in which all the roughness is completely flattened. This is rarely the case for real engineering surfaces.

Here, it is shown that this difficulty can be avoided by carrying out a Fourier transform of the elastostatically flattened roughness and using the resulting (complex) amplitude as the low-film thickness limit of each Fourier component in the elastohydrodynamic lubrication (EHL) analysis.

Results give a plausible convergence to the elastostatic solution, which is nevertheless consistent with the expected near-full-film EHL behaviour and which becomes identical to the earlier model for roughness that, statically, can be fully flattened. As expected, hydrodynamic action persists at the finest scale, even for very thin films.

Keywords: elastohydrodynamic lubrication, rolling contact, roughness, Fourier transform

1 INTRODUCTION

A major preoccupation of tribology over the last twenty years has been the modelling of the effect of rough surfaces on elastohydrodynamic lubrication (EHL). During the last few years there has been an increasing interest in the development of *approximate* models of the EHL of rough surfaces [**1–6**]. These stemmed from the numerical modelling of rolling contacts in which one surface was wavy (having harmonic, or sinusoidal, roughness). For pure rolling, provided the amplitude of the roughness was not too great, it was found that, within the contact, the roughness was compressed by an amount which depended on the ratio of its wavelength to the length of the 'inlet sweep', the region in the inlet to the contact where the pressure develops. Correlation curves were developed so that the degree of compression of harmonic roughness in any direction could be estimated for both point and line contacts [**1**, **2**]. Long waves would be completely flattened while shorter waves would be only partially so – and the shortest waves would pass through unaffected.

This led to the idea that general roughness could be handled by first finding its Fourier transform (e.g. by using the fast Fourier transform (FFT)), thus decomposing it into harmonic components. Next, the compression of each is found using the correlation curve. This is equivalent to applying a filter in the wavelength domain – the 'Hooke filter'. Finally, the compressed profile is constructed using the inverse transform [**3**]. Implicit in this is that the waves behave independently, a condition arguably satisfied when the roughness is small compared with the mean thickness of the fluid film. In practice, it was found that the procedure could be used with reasonable accuracy, even for quite large features [**3**].

In the case of rolling–sliding contacts, the behaviour is more complex since each wave creates a flowrate perturbation in the inlet which, in turn, leads to a decaying 'complementary wave' that passes through the contact at the mean rolling speed, u. Since this is not the speed of either surface, the complementary wave has a different wavelength from the parent roughness [**4**, **5**] and its amplitude and phase are difficult to determine by any simple method [**4**, **6**]. Despite these complications, general roughness can still be

**Corresponding author: Department of Mechanical Engineering, Imperial College, Exhibition Road, London SW7 2AZ, UK. email: a.v.olver@imperial.ac.uk*

handled by the Fourier transform approach and this has led to great interest in developing the method for determining pressures, stresses and fatigue lives for components such as rolling bearing and gears [**7**, **8**].

However, one aspect of the FFT approach to the problem of general roughness presents a serious problem when applied to real engineering surfaces. The correlation curve of Hooke *et al.* shows complete flattening when the inlet sweep is short compared with the wavelength. This occurs not only when the wave is long, but also when the speed and/or viscosity are low so that hydrodynamic action ceases (and hence the inlet sweep is short). The implication is that roughness will be completely flattened when the fluid film is lost at low speed or for the rougher surfaces. However, this is not true; most hard engineering surfaces used in EHL contacts have roughness, which is *not* completely flattened under elastostatic conditions [**9**].

It may be objected that the whole FFT approach must be abandoned in this case since statically incompressible roughness will result in the contact fragmenting into smaller parts (asperities!) and so the concept in which roughness perturbs an essentially parallel film is invalid. However, it seemed that an approach in which the asymptotes were correct would be an improvement, even if the range of validity was still limited. Here, a simple linear transformation of the Hooke's correlation is suggested, in which the low film thickness limit is modified to take account of the correct elastostatic behaviour. This means that consistent predictions of roughness compression and of pressure can easily be made for any ratio of roughness height to mean film thickness (lambda, λ).

2 ANALYSIS

Line contact with transverse roughness is considered, so that the problem relates entirely to a two-dimensional contact where one surface has a defined initial profile. This is the situation described in references [**2**] and [**3**] by Hooke and Li. The following linear transformation of their correlation is proposed

$$g(\Lambda_i) = \frac{A_i^{\mathrm{L}}}{A_i^0}(\Lambda_i) = \frac{A_i^{\mathrm{E}}}{A_i^0} + \left(1 - \frac{A_i^{\mathrm{E}}}{A_i^0}\right) f(\Lambda_i); \quad i = 0 \text{ to } (N-1) \tag{1}$$

Here, the real function $f(\Lambda_i)$ is Hooke's correlation for the amplitude of the harmonic roughness (Fig. 1). Since $f(0) = 1$ and $f(\infty) = 0$, equation (1) simply linearly scales the function, f, so that the $\Lambda_i = \infty$ (low film thickness) limit is replaced by the (complex) elastostatic value; that is, $g(\infty) = A_i^{\mathrm{E}}/A_i^0$. Whether this linearity is justified is, of course conjectural, but there seems no physical reason to expect any more complicated behaviour. The amplitudes here are all treated as the *complex* values from the Fourier transform, so the phase information is preserved.

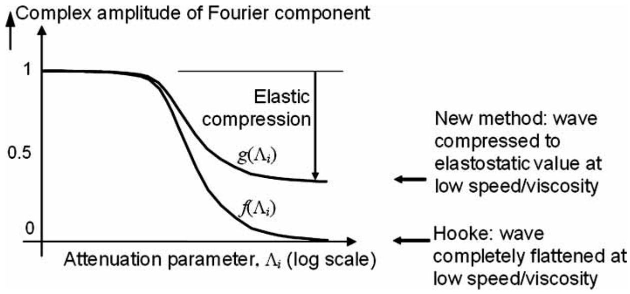

Fig. 1 Schematic of the correlation due to Hooke *et al.* for the amplitude reduction of roughness in rolling contacts. The method proposed here interprets Hooke's vertical axis as the corresponding complex Fourier amplitude and proposes a modified low-film thickness limit identical with the amplitude derived from the FFT of elastostatic solution

The amplitudes A_i^{E} are found from the Fourier transform of the roughness profile when compressed statically, derived from a suitable elastostatic solution [**9**], so that the analysis automatically approaches the same elastostatic condition as Λ_i is increased (and the film thickness reduced). For longer waves, or for *any* wave resulting from a sufficiently low initial roughness, the wave is flattened in the unlubricated condition, so $A_i^{\mathrm{E}} = 0$ and the algorithm is unchanged from that of Hooke.

Since the partial compression of a single wave results in a sinusoidal displacement, and since the resulting pressure is also sinusoidal [**10**], the pressure amplitude for each wave was found from

$$p_i = (A_i^0 - A_i^{\mathrm{L}})\frac{\pi E^*}{\lambda_i} \tag{2}$$

The actual pressure distribution is the summation of that from all the individual waves and was found by performing an inverse FFT. Since the pressure is found from the relative displacements of the surfaces, the mean value of the pressure found in this way, is indeterminate. This was, therefore, set using the two conditions that the pressure must be zero outside the contact and must equal the Hertzian value for the smooth elastostatic case. Thus, the calculated pressures were added to the Hertz pressure and shifted uniformly in magnitude (vertically) to achieve zero pressure externally.

All the calculations were performed using commercial spreadsheet software having a built-in FFT functionality. The test surface was sampled at $N = 1024$ points with a spacing of $0.25\,\mu$m using a stylus instrument.

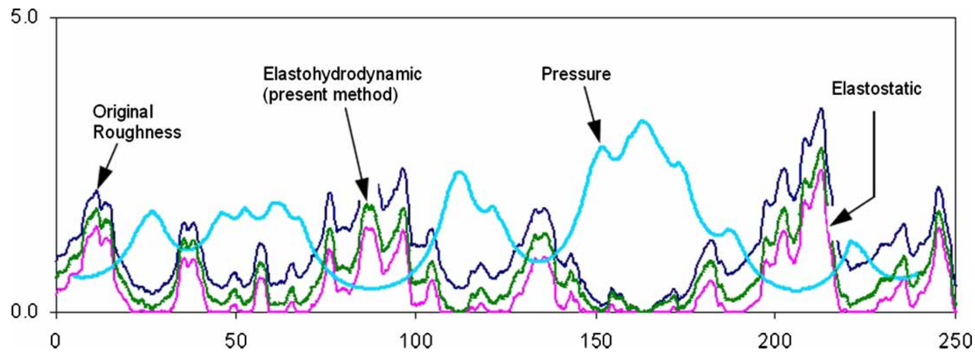

Fig. 2 Original and compressed (elastohydrodynamic) roughness profile for $u = 3.05\,\mathrm{m/s}$. The fully compressed elastostatic profile is shown for comparison. $R_e = 4.91\,\mathrm{mm}$, $p_0 = 1.69\,\mathrm{GPa}$, $\eta_0 = 0.005\,465\,\mathrm{Pa\,s}$, $\alpha = 12.25\,\mathrm{GPa}^{-1}$, $E^* = 110\,\mathrm{GPa}$. Horizontal scale in μm; vertical scale in μm (profiles) and GPa (pressure)

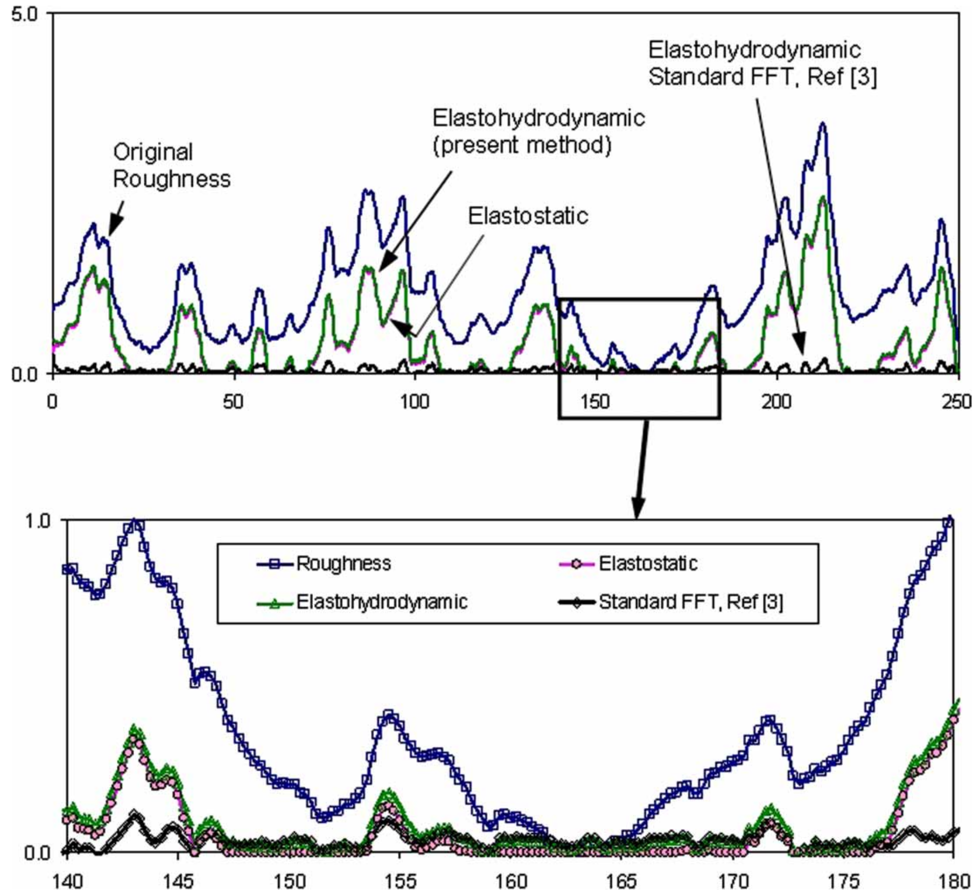

Fig. 3 Compressed elastohydrodynamic roughness for an entrainment speed of 10 mm/s, compared with that corresponding to the the initial and dry-contact (elastostatic) condition (same curves repeated from Fig. 2). Also shown is the EHL roughness calculated by the standard FFT perturbation method from reference [**3**]. The initial roughness is compressed to nearly the elastostatic condition whereas the standard method predicts a more complete flattening of the profile. In both cases, the very finest features are relatively unaffected. (see detail in lower graph) Other conditions as for Fig. 2. Both scales in μm

3 RESULTS

Figure 2 shows some typical results for a set of conditions relating to a disc experiment described elsewhere [**8**] except that here, pure rolling was assumed. The profiles are shown inverted, i.e. with the crests uppermost and the valleys below, so that the vertical axis represents the distance from the highest crest. The computed curve falls between the uncompressed and elastostatic value and the pressures exceed Hertz near the crests but are much lower in the valleys.

Figure 3 shows similar results for a very low entrainment speed. (The original roughness and elastostatic profiles are repeated from Fig. 2.) The lubricated profile is now very close to the elastostatic one except (see detail) that the very finest features are still substantially unaffected. Also shown for comparison is the lubricated profile calculated using the standard method described by Hooke and Li [**3**], which shows a much greater degree of flattening even than for the elastostatic case – a clearly unsatisfactory result.

Corresponding pressures are shown in Fig. 4. The pressure is now zero in the major valleys but the pressures near the crests are more uniform than the corresponding elastostatic curve, again reflecting the fact that the finest features are not compressed.

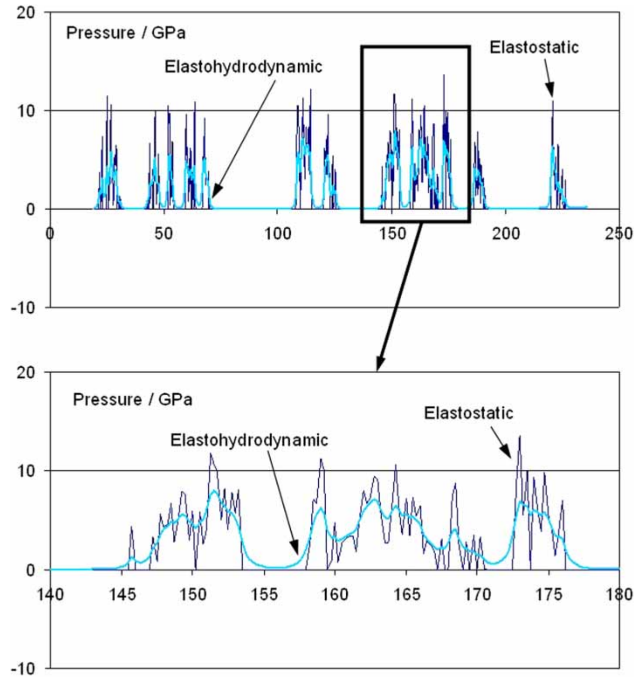

Fig. 4 Comparison of asperity pressures for the elastostatic and a lubricated (10 mm/s) condition, corresponding to the profiles given in Fig. 3. The pressure in the major valleys is zero but the distributions of pressure near the crests are lower and wider than under unlubricated conditions. Vertical scale in GPa, horizontal in μm

4 DISCUSSION

The Fourier transform of the elastostatically compressed roughness has been used to create a description of EHL which is both consistent with the well-established perturbation method and which approaches the correct low-film thickness limit. This enables consistent predictions of pressure and film shape over a wide range of conditions spanning the full film, mixed, and boundary regimes.

However, the present work contributes little to the actual understanding of the complex problem of mixed lubrication or even on the manner in which very thin fluid films may break down to yield boundary (solid–solid) lubrication in the neighbourhood of the crests of surface roughness. Nevertheless, the simplicity and stability of the algorithm and its intrinsic compliance with the known limits to the problem (full film and elastostatic) might in themselves be the envy of proponents of more rigorous approaches.

Extension of the method to two dimensions using the two-dimensional FFT method, widely used for image processing applications, would appear to be straightforward since longitudinal as well as transverse correlations (filters) describing the correct EHL behaviour have been published [**3**]. Although rolling–sliding problems are more complex because of the complementary wave, these too can in principle be treated in a similar way.

Validation of the method, particularly in the mixed regime is more challenging. Not only are experiments in this regime difficult because of wear and damage at asperity crests and the non-stationarity of real surfaces, but rigorous computational modelling is also difficult at very low lambda values because of the possibility of cavitation, near-surface fluid behaviour (slip, viscous sublayers) and of boundary film formation itself. However, equation (1) lends itself to modification and development as more information becomes available, either by changing the elastostatic limit (e.g. to account for surface films) or by introducing non-uniformity into the scaling.

5 CONCLUSION

A simple method is proposed to ensure that estimates of surface shape and pressure in rough lubricated rolling contacts conform to the known elastostatic behaviour at the low film thickness limit. This allows consistent predictions to be made over a wide range of conditions with measured roughness profiles.

However, caution must be exercised in applying it, particularly to the mixed regime in which even more rigorous modelling has yet to be validated by accurate experiments.

The results suggest that as the film thickness falls, the valley pressures drop but that the finest roughness features are less affected so that some hydrodynamic ('cushioning') action persists even when the film thickness is very low.

REFERENCES

1 **Hooke, C. J.** and **Venner, C. H.** Surface roughness attenuation in line and point contacts. *Proc. Instn Mech. Engrs, Part J: J. Engineering Tribology*, 2000, **214**(J5), 439–444.
2 **Hooke, C. J.** and **Li, K. Y.** Rapid calculation of the pressures and clearances in rough, elastohydrodynamically lubricated contacts under pure rolling. Part 1: low amplitude, sinusoidal roughness. *Proc. IMechE, Part C: J. Mechanical Engineering Science*, 2006, **220**(C6), 901–913.
3 **Hooke, C. J.** and **Li, K. Y.** Rapid calculation of the pressures and clearances in rough, elastohydrodynamically lubricated contacts under pure rolling. Part 2: general roughness. *Proc. IMechE, Part C: J. Mechanical Engineering Science*, 2006, **220**(C6), 915–925.
4 **Greenwood, J. A.** Transverse roughness in elastohydrodynamic lubrication. *Proc. Instn Mech. Engrs, Part J: J. Engineering Tribology*, 1999, **213**(J5), 383–396.
5 **Hooke, C. J.** The effects of roughness in EHL contacts. In Proceedings of the 31st Leeds–Lyon Symposium on Tribology, Lyon, September 2004, 2005, pp. 31–46 (Elsevier, Amsterdam).
6 **Hooke, C. J.** Roughness in rolling-sliding elastohydrodynamic lubricated contacts. *Proc. IMechE, Part J: J. Engineering Tribology*, 2006, **220**(J3), 259–271.
7 **Chapkov, A. D., Colin, F.,** and **Lubrecht, A. A.** Influence of harmonic surface roughness on the fatigue life of elastohydrodynamic lubricated contacts. *Proc. IMechE, Part J: J. Engineering Tribology*, 2006, **220**, 287–294.
8 **Olver, A. V., Lainé, E., Hili, J.,** and **Dini, D.** Application of FFT harmonic surface roughness model to the development of micropitting damage in elastohydrodynamic contacts. Paper submitted to *Trans. Am. Soc. Mech. Eng., J. Tribology*, June 2007.
9 **Webster, M. N.** and **Sayles, R. S.** A numerical model for the elastic, frictionless contact of real rough surfaces. *Trans. Am. Soc. Mech. Eng., J. Tribology*, 1986, **108**, 314–320
10 **Johnson, K. L.** *Contact mechanics*, 1985, p. 398 (Cambridge University Press, Cambridge).

APPENDIX

Notation

A_i^0	complex amplitude of the ith Fourier component for the measured (uncompressed) surface roughness, $i = 0$ to $(N-1)$
A_i^{E}	complex amplitude of the ith Fourier component for the compressed surface roughness in elastostatic contact
A_i^{L}	complex amplitude of the ith Fourier component for the compressed surface roughness in lubricated (micro-EHL) contact
b	Hertz semi-width
E^*	effective elastic modulus $\dfrac{1}{E^*} = \dfrac{1-\nu_1^2}{E_1} + \dfrac{1-\nu_2^2}{E_2}$
$f(\Lambda_i)$	amplitude ratio for pure rolling [**2**, **3**]
$g(\Lambda_i)$	amplitude ratio for pure rolling (present paper)
G	Greenwood speed parameter: $G = \dfrac{\alpha\eta_0 u}{R_e}$
i	sample number $i = 0$ to $(N-1)$
L	total sample length
N	number of samples
p_i	complex amplitude of the ith pressure wave
P	Greenwood load parameter αp_0
p_0	maximum Hertzian pressure
q	coefficient relating p_0 and $b : q = \sqrt{\dfrac{bE^*}{4p_0R_e}}$
R_e	radius of relative curvature in the entrainment direction
u	entrainment velocity
α	pressure–viscosity coefficient
η_0	dynamic viscosity at ambient pressure
λ_i	wavelength of ith Fourier component $\lambda_i = \dfrac{L}{i}$
Λ_i	attenuation correlation parameter in pure rolling $\Lambda_i = \dfrac{\lambda_i}{b} q P^{3/2} G^{-2}$
ϕ_i	phase of the ith FFT component