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On modelling and simulation of flow in the vocal tract with consideration of the glottis closure

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1. Introduction

In this paper the problem mathematical modelling of voice creation is addressed. The voice production mechanism is a complex process consisting of fluid-structure-acoustic interaction problem, where the coupling between fluid flow, viscoelastic tissue deformation and acoustics is crucial, see [6]. The so-called phonation onset (flutter instability) for certain airflow rate and a certain prephonatory position leads to the vocal folds to oscillation. The important aspect of the phenomena is the glottis closure (glottis is the narrowest part between the vibrating vocal folds). The problem is mathematically characterized as a problem of fluid-structure interaction with the (periodical) contact problem of the vocal folds involved. In order to include the interactions of the fluid flow with solid body deformation as well as the contact problem, a simplified model problem is considered. This model is similar to the simplified twomass model of the vocal folds of [4], see also the aeroelastic model in [3]. Here, the mathematical model is introduced and the numerical approximation of the problem is described using the residual based stabilization The simplified lumped vocal fold model with the Hertz impact forces is considered. The model is based on a suitable modification of the inlet boundary condition and the arbitrary Lagrangian-Eulerian method with a remeshing algorithm. Two strategies are suggested for treatment of the gap closure. Numerical tests are presented.

2. Flow model

First, the air flow is modelled by the system of the Navier-Stokes equations (cf. [2]) written in the ALE form (cf. [5])

$$\rho \frac{D^{\mathcal{A}}\boldsymbol{u}}{Dt} + \rho((\boldsymbol{u} - \boldsymbol{w}_D)\cdot\nabla)\boldsymbol{u} = \operatorname{div}\boldsymbol{\tau}^f, \qquad \nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{u} = (v_1, v_2)$ is the fluid velocity vector, ρ is the constant fluid density, $\boldsymbol{w}_D$ is the domain velocity, $\frac{D^{\mathcal{A}}\boldsymbol{u}}{Dt}$ denotes the ALE derivative and $\boldsymbol{\tau}^f = (\tau_{ij}^f)$ is the fluid stress tensor given by $\boldsymbol{\tau}^f = -p\mathbb{I} + \mu(\nabla\boldsymbol{u} + \nabla^T\boldsymbol{u})$. Here p is the pressure and $\mu > 0$ is the constant fluid viscosity. For the system (1) the initial and boundary conditions are prescribed. The boundary conditions are prescribed on the boundary $\partial\Omega_t^f$ of the computational domain formed by mutually disjoint parts $\partial\Omega_t^f = \Gamma_I \cup \Gamma_S \cup \Gamma_O \cup \Gamma_{Wt}$, where Γ_I denotes the inlet, Γ_O the outlet, Γ_S the axis of symmetry and Γ_{Wt} denotes either fixed or deformable wall. Except the standard boundary conditions used at the fixed or moving walss, the following combination of boundary conditions was used at the inlet and outlet

$$\text{a)} \qquad -\boldsymbol{n}\cdot\boldsymbol{\tau}^f + \frac{1}{2}\rho(\boldsymbol{u}\cdot\boldsymbol{n})^-\boldsymbol{u} = p_I\boldsymbol{n} + \frac{1}{\varepsilon}(\boldsymbol{u}-\boldsymbol{u}_I) \qquad \text{on } \Gamma_I, \tag{2}$$

$$\text{b)} \qquad -\boldsymbol{n}\cdot\boldsymbol{\tau}^f + \frac{1}{2}\rho(\boldsymbol{u}\cdot\boldsymbol{n})^-\boldsymbol{u} = 0 \qquad \text{on } \Gamma_O,$$

where $\boldsymbol{n}$ denotes the unit outward normal vector to $\partial\Omega_t^f$, $\boldsymbol{u}_I$ is the inlet velocity, p_I is a reference pressure value at the inlet, $\varepsilon > 0$ is a penalization parameter, $p_{ref} = 0$ is a reference pressure value at the outlet and α^- denotes the negative part of a real number α.

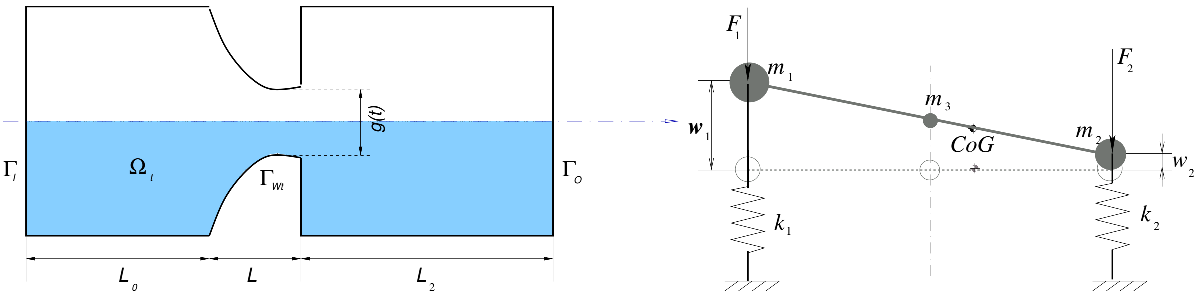

Fig. 1. The 2D computational domain Ω_t and the boundary parts (on the left). Aeroelastic two degrees of freedom model (with masses m_1, m_2, m_3) in displaced position (displacements w_1 and w_2) and resulting aerodynamic forces F_1 and F_2 (on the right).

3. Structure model

The motion of the vocal fold model is governed by the displacements $w_1(t)$ and $w_2(t)$ of the two masses m_1 and m_2, respectively (see Fig. 1). The displacement vector $\boldsymbol{w} = (w_1, w_2)^T$ is obtained by the solution of the following equations (see [3] for details)

$$\mathbb{M}\ddot{\boldsymbol{w}} + \mathbb{B}\dot{\boldsymbol{w}} + \mathbb{K}\boldsymbol{w} = -\boldsymbol{F}, \tag{3}$$

where $\mathbb{M}$ is the mass matrix, $\mathbb{K}$ is the diagonal stiffness matrix with spring constants c_1, c_2 on its diagonal and $\mathbb{B}$ is the matrix of the proportional structural damping. The mass matrix is given by

$$\mathbb{M} = \begin{pmatrix} m_1 + \frac{m_3}{4} & \frac{m_3}{4} \\ \frac{m_3}{4} & m_2 + \frac{m_3}{4} \end{pmatrix}, \tag{4}$$

where m_1, m_2, m_3 are the masses shown in Fig. 1. The components of $\boldsymbol{F} = (F_1, F_2)^T$ are the aerodynamical forces (downward positive). The proportional damping matrix is chosen as $\mathbb{B} = \varepsilon_1\mathbb{M} + \varepsilon_2\mathbb{K}$.

4. Finite element and stabilized finite element methods

In order to describe the details of the application of the finite element method for solution of (stationary) boundary value flow problem, the space $\mathcal{X} \subset \boldsymbol{H}^1(\Omega_t^f)$ for velocity including the Dirichlet boundary conditions is used and the pressure space $\mathcal{Q}$ is chosen as $\mathcal{Q} = L_2(\Omega_t^f)$. The finite element approximation is then sought in the finite element spaces $\mathcal{V}_h = \mathcal{X}_h \times \mathcal{Q}_h$ constructed over an admissble triangulation τ_h of the computational domain Ω_t^f: Find an approximate solution $U_h = (\boldsymbol{u}, p) \in \mathcal{V}_h$ such that at time t holds

$$\left(\frac{D^{\mathcal{A}}\boldsymbol{u}}{Dt} + \overline{\boldsymbol{w}}\cdot\nabla\boldsymbol{u}, \boldsymbol{v}\right) + (\nu\nabla\boldsymbol{u} - p\mathbb{I}, \nabla\boldsymbol{v}) + (q, \nabla\cdot\boldsymbol{u}) + \mathcal{L}(U,V) \;\;=\;\; 0$$

for all test functions $\boldsymbol{v}$ and q. ($\overline{\boldsymbol{w}} = (\boldsymbol{u} - \boldsymbol{w}_D)$). Here, the SUPG/PSPG stabilization terms together with the div-div stabilization terms are given as

$$\mathcal{L}(U,V) = \sum_{K\in\tau_h} \delta_K \left[\left(\frac{D^{\mathcal{A}}}{Dt}\boldsymbol{u} - \nu\triangle\boldsymbol{u} + (\overline{\boldsymbol{w}}\cdot\nabla)\,\boldsymbol{u} + \nabla p, (\overline{\boldsymbol{w}}\cdot\nabla)\boldsymbol{v} + \nabla q \right)_K + \tau_K \Big(\nabla\cdot\boldsymbol{u}, \nabla\cdot\boldsymbol{v} \Big)_K \right].$$

5. Numerical analysis of Oseen problem

First, the Oseen problem is considered

$$-\nu\triangle\boldsymbol{u} + \boldsymbol{b}\cdot\nabla\boldsymbol{u} + \nabla p + \sigma\boldsymbol{u} = \boldsymbol{f}, \qquad \nabla\cdot\boldsymbol{u} = 0$$

in the computational domain $\Omega = (0,1)^2$. The problem is equipped with the Dirichlet boundary condition $\boldsymbol{u} = \boldsymbol{b}$ prescribed at $\partial\Omega$. Here we set $\sigma = 0$ is used, $\boldsymbol{b} = (\sin(\pi x), -\pi y\cos(\pi x))$ and the right hand side $\boldsymbol{f}$ is chosen in such a way, that $\boldsymbol{b}$ is solution of the Oseen problem, i.e.,

$$\boldsymbol{f}(x,y) = \nu\pi^2\sin(\pi x) + \pi\cos(\pi x)(\sin(\pi x) + \cos(\pi y)), -\nu\pi^3 y\cos(\pi x) + \pi^2 y - \pi\sin(\pi x)\sin(\pi y)).$$

The computations were performed for different values of the viscosity coefficient nu. First convergence of the Galerkin finite element approximations $\boldsymbol{u}_h^G$ to the exact solution $\boldsymbol{u} = \boldsymbol{b}$ is investigated, $p(x,y) = \sin(\pi x)\cos(\pi y)$ for $\nu = 0.05$ (here, relatively high viscosity was chosen in order to obtain stable Galerkin approximations even on coarser meshes). For approximation of flow problem the Taylor Hood finite elements were used. The errors in H^1 norm are shown in Table 1. These results are compared to the results of stabilized formulation of the same problem, which shows that the used residual based stabilization does not polute the solution, see Table 2. The convergence orders in both cases agreee well with the theoretical estimate. For the stabilized method such a convergence rates are well preserved for the values $\nu = 10^{-3}, \ldots, 10^{-6}$ with a slow down observed only for coarse grid configuration. Let us emphasize, that as the convection $\boldsymbol{b}$ equals the exact solution $\boldsymbol{u}$, the Dirichlet problem for Navier-Stokes equations can be formulated with the same analytical solution. Similar convergence analysis was performed with analogous results.

6. Gap closure treatment, remeshing and remaping

In order to treat the gap closure, the large deformation of the mesh needs to be addressed. This can lead to the high distortion of the mesh mainly in the region of the glottis. Here, the two possible solution are discussed. The first one based on the restriction of the applied model at the part of the domain which corresponds to the closed area. This is realized using the solution of an artificial problem at this part of the mesh in combination with a modified definition of mesh deformation (ALE mapping).

The other possibility is the use of the remeshing algorithm together with conservative remapping of the flow variables. First, the remeshing algorithm is avoided here with the consideration of several

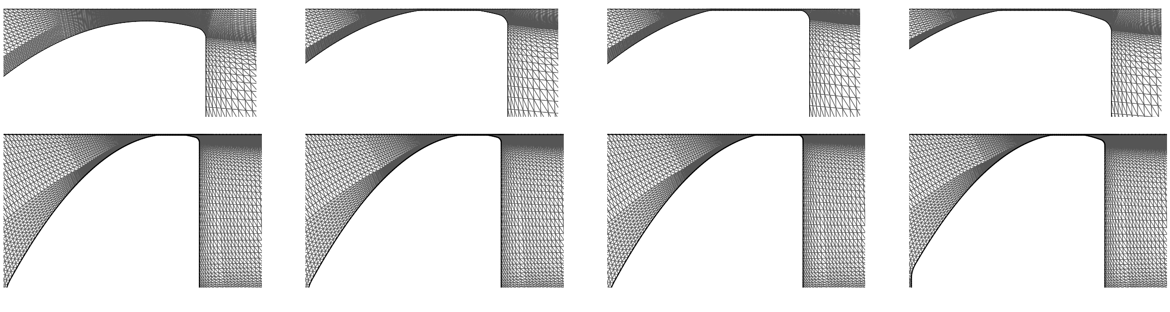

Fig. 2. Remeshing of the mesh for the (almost) closed glottal part

Table 1. Convergence of Galerkin FE method to the solution of the Oseen problem

h_{max}	$H^1(u)$	$H^1(v)$	$H^1(p)$	q_{H1u}	q_{H1v}	q_{H1p}
0.333174	0.148971	0.278814	0.824015			
0.166358	0.0294769	0.0389521	0.306831	2.33	2.83	1.42
0.0881204	0.00751673	0.00970303	0.155735	2.15	2.19	1.07
0.0449673	0.00178417	0.00225841	0.0781441	2.14	2.17	1.03
0.0230627	0.000444434	0.00055994	0.038375	2.08	2.09	1.07
0.0118955	0.000107972	0.000139096	0.0192135	2.14	2.1	1.04

Table 2. Convergence of Stabilized FE method to the solution of the Oseen problem

h_{max}	$H^1(u)$	$H^1(v)$	$H^1(p)$	q_{H1u}	q_{H1v}	q_{H1p}
0.333174	0.148971	0.278814	0.824015			
0.158053	0.0385103	0.0479786	0.332202	1.81	2.36	1.22
0.0877183	0.00912475	0.0113478	0.162359	2.45	2.45	1.22
0.0451748	0.00239609	0.00279304	0.0821471	2.02	2.11	1.03
0.0235271	0.000593443	0.000695859	0.0402822	2.14	2.13	1.09
0.0119951	0.000148245	0.000174687	0.0201917	2.06	2.05	1.03

meshes suitable for different cases of glottis closure or opening. Here, four configurations are considered characterized by their displacement in terms of w_1 and w_2. In dependence on the current displacement $w_1(t)$ and $w_2(t)$ of the vocal fold either the current underlying mesh is deformed or a remeshing step is used. For the remeshing step the conservative remapping of the flow quantities is applied. The algorithms with the conservative remapping of momentum(velocity) components followed by the projection on the divergence-free space was tested. Fig. 2 shows the mesh in the deformed positions for the case of the closed gap.

In order to treat such a gap closing several modifications of two algorithms were tested. The first algorithm is based on artificial fluid formulation, where part of the flow in the computational domain is modelled with the aid of the artificial porous media flow, see [1]. The second algorithm considers the relaxad contact formulation with a combination with the well tuned inlet boundary conditions.

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