



## Modelling of the turbine blade by new finite element

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The paper deals with 1D finite element modelling of a turbine blade. The proposed finite element has only 16 degrees of freedom (DOF) and enables to achieve a complex model of turbo-machine with a relatively small DOF number, more details will be described in the article [1]. It is a great advantage but the pre-processing for the blade cross section parameters of the individual blades is a little bit more complicated. This approach means to solve a warping function in chosen cross sections of the blade and simultaneously calculate the geometrical parameters of those cross sections. The second step represents an approximation of the obtained parameters along the axis coordinate  $\zeta$  by means of spline functions. The next step is gradual computation of local finite element matrices and assemblage of the global blade matrices.

The velocity of infinitesimal mass point is taken into account as a sum of cross section shear stress center  $S$  velocity corresponding to the translation motion and the secondary velocity from spherical motion with center in point  $S$ . Moreover, the axial motion of arbitrary point is affected by warping of the cross section caused by torsion deformation.

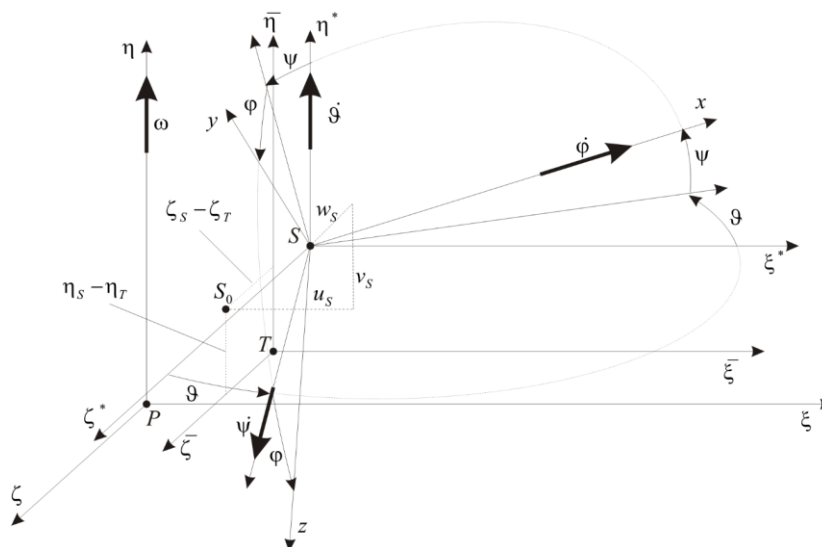


Fig. 1. Used coordinate systems

Having determined the displacements and velocities of the arbitrary cross section point according to Fig. 1 we can come to the relations for kinetic and potential energy of the blade finite element. Using e.g. Lagrange's equations it is possible to determine matrices of blade finite element. After assembly of all elements the whole blade equation of motion can be written in form

$$\mathbf{M} \ddot{\mathbf{q}}(t) + \omega \mathbf{G} \dot{\mathbf{q}}(t) + (\mathbf{K} - \omega^2 \mathbf{M}_D - \omega^2 \mathbf{M}_M) \mathbf{q}(t) = \mathbf{f}_Z(t) + \mathbf{f}_D, \quad (1)$$

where  $\mathbf{M}$  is mass matrix,  $\mathbf{G}$  is matrix of gyroscopic forces,  $\mathbf{K}$  is stiffness matrix,  $\omega^2 \mathbf{M}_M$  is membrane forces matrix,  $\omega^2 \mathbf{M}_D$  is circulation matrix,  $\mathbf{f}_z(t)$  is vector of external forces and  $\mathbf{f}_D$  is constant vector of centrifugal forces. The presented methodology was validated on one blade modelled by 3D elements and by the presented finite 1D blade elements. Comparison of the first two mode shapes obtained by the use of both approaches is shown in Fig. 2.

$$\Omega_1 = 779 \text{ rad / s} \quad \Omega_1 = 871 \text{ rad / s} \quad \Omega_2 = 1451 \text{ rad / s} \quad \Omega_2 = 1486 \text{ rad / s}$$

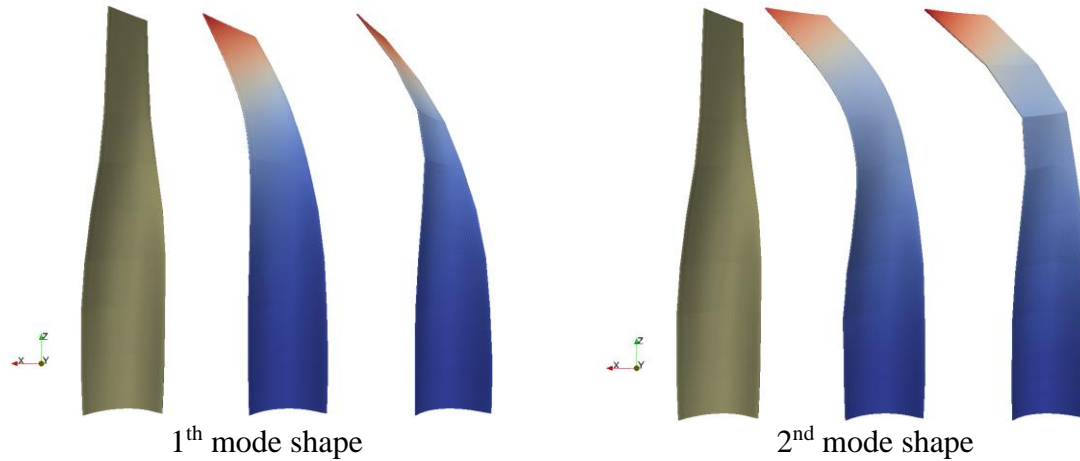


Fig. 2. Comparison of the mode shapes calculated using 3D and 1D approach

The model using presented finite element is described only by 154 DOF and 19 finite 1D blade elements. From the other side the model consisting of 3D elements has about  $4.10^5$  DOF. The modal analysis was performed for  $\omega = 0 \text{ rad / s}$ . As we can see the presented methodology means the great save of time and the capacity of computer for blade system analysis.

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### References

- [1] Dupal, J., Zajicek, M., Lukes, V., 1D finite element for modelling of turbine blade vibration in the field of centrifugal forces, Applied and Computational Mechanics (2018) 1-17. (under review)