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Relation of Instant Radiosity Method with Local Estimations of Monte Carlo Method

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ABSTRACT

This article discusses mathematical foundations of local estimations of the Monte Carlo method. The basic algorithm of visualization of the 3D scenes based on local estimations, which are an analog of the famous algorithm Instant Radiosity, is considered.

An algorithm for radiance object view-independent calculation based on local estimations of Monte Carlo method is shown

Additionally, questions of representation of radiance object as spherical harmonics expansion in each computational point are analyzed. The assumption of possible direct calculation of radiance object coefficients of expansion in spherical harmonics by Monte Carlo method is brought in, and problems are identified.

Keywords

Radiosity, instant radiosity, global illumination, local estimations, Monte-Carlo, spherical harmonics, view-independent global illumination.

1. INTRODUCTION

Lighting systems simulation and visualization of 3D scenes in computer graphics are based on well-known global lighting equation [Kajiya J. T. 1986].

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\int L(\mathbf{r},\hat{\mathbf{l}}')\,\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')\left|(\hat{\mathbf{N}},\hat{\mathbf{l}}')\right| d\hat{\mathbf{l}}', \quad (1)$$

where $L(\mathbf{r},\hat{\mathbf{l}})$ is the radiance at the point r in the direction $\hat{\mathbf{l}}$, $\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')$ is the bidirectional scattering distribution function (reflectance or transmittance), L_0 is the radiance of the direct radiation straight near the sources, $\hat{\mathbf{N}}$ is the normal at the point r to the surface of the scene.

The spatial angular distribution of radiance can be calculated on the global illumination ground. It will allow determining light qualitative characteristics (glare, discomfort), which will enable to calculate lighting systems for a specified quality of illumination. The spatial angular radiance distribution calculating algorithm is also the basis for the visualization of 3D scenes. Today, the radiosity is used for the lighting systems simulation. This algorithm is based on the finite element method of radiosity equation. [Goral et al. 1984] [Moon P. 1940].

$$M(\mathbf{r}) = M_0(\mathbf{r}) + \frac{\sigma}{\pi}\int_{\Sigma} M(\mathbf{r}')\Theta(\mathbf{r},\mathbf{r}')F(\mathbf{r},\mathbf{r}')d^2\mathbf{r}', \quad (2)$$

where $M(\mathbf{r})$ is the radiance at the surface point $\mathbf{r}$, $M_0(\mathbf{r})$ is radiancy at the point $\mathbf{r}$, received straight from the light source, $\Theta(\mathbf{r},\mathbf{r}')$ is the visibility function of an element $d^2\mathbf{r}'$ from point $\mathbf{r}$, $F = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}),(\mathbf{r}-\mathbf{r}'))\right|\left|(\hat{\mathbf{N}}(\mathbf{r}'),(\mathbf{r}-\mathbf{r}'))\right|}{(\mathbf{r}-\mathbf{r}')^4}$ is the elementary form-factor, $\hat{\mathbf{N}}(\mathbf{r})$ is a normal at the point $\mathbf{r}$ to the scene surface.

Should be noted that the radiosity equation has two analytic solutions. First is a well-known photometric sphere. Second is the Sobolev problem: two parallel infinite diffuse planes and isotropic point source in between [Budak V., Zheltov V. 2014].

Radiosity method on which the lighting systems modeling programs DIALux and Reluxe are based fails to take account of the reflection from non-diffuse surfaces. It markedly affects the determination accuracy of radiance spatial angular distribution.

Recently an interesting algorithm of instant radiosity was introduced [Keller A. 1997], which is a kind of local estimation algorithms of Monte Carlo method [Kalos M. 1963]. However, the phenomenological approach used for derivation makes the algorithm difficult to use in the general case. We undertook the complete proof of strict local assessment algorithms based on the global illumination equation.

In the article, we consider local and double local estimations of the Monte Carlo method for the global illumination equation. Algorithm based on the local estimations allows modeling the radiance of a scene surface point in a given direction. Also, basing on the local estimations, we proposed an algorithm of view-independent determination of the radiance angular distribution on the scene surfaces. The algorithm has obvious advantages over the radiosity method for integrating diffusely directed reflection model.

2. GLOBAL ILLUMINATION EQUATION

From the integral equation for the solid angle, one can go to the well-known integral equation of Fredholm second kind on surfaces

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\int_{(\Sigma)} L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})F(\mathbf{r}',\mathbf{r})d^2r', \quad (3)$$

where $F(\mathbf{r}',\mathbf{r}) = \dfrac{\left|(\hat{\mathbf{N}}(\mathbf{r}),\mathbf{r}-\mathbf{r}')(\hat{\mathbf{N}}(\mathbf{r}'),\mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^4}\Theta(\mathbf{r}',\mathbf{r})$,

$\hat{\mathbf{l}}' = \dfrac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$.

One can construct an algorithm based on (3) for its solution by Monte Carlo method. However, wandering along the surfaces Σ of the scene visualization is not a trivial task. Conventional scheme of wandering of the Monte Carlo method is constructed in space, which requires the integral to integration over the volume.

Integral over the volume

$$d^3r' = |\mathbf{r}-\mathbf{r}'|^2\, dr'd\hat{\mathbf{l}}',$$

$$d\hat{\mathbf{l}}' = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}'),\mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^2}d^2r'. \quad (4)$$

For integration over dr' we will use equivalent transformation with usage δ-function properties

$$\int_{(\Sigma)} L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})F(\mathbf{r}',\mathbf{r})d^2r' \equiv$$

$$\equiv \int_0^{\infty}\int L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})\frac{F(\mathbf{r}',\mathbf{r})}{\left|(\hat{\mathbf{N}}(\mathbf{r}'),\hat{\mathbf{l}}')\right|}|\mathbf{r}-\mathbf{r}'|^2 \times$$

$$\times\frac{\left|(\hat{\mathbf{N}}(\mathbf{r}'),\hat{\mathbf{l}}')\right|d^2r'}{|\mathbf{r}-\mathbf{r}'|^2}\delta(\xi_0-|\mathbf{r}-\mathbf{r}'|)dr', \quad (5)$$

where ξ_0 is a solution of the surface Σ equation $\Pi(\mathbf{r})=0$: $\Pi(\mathbf{r}-\xi_0\hat{\mathbf{l}}')=0$.

The surface equation can be included directly in (5) because the ratios

$$\xi_0 - |\mathbf{r}-\mathbf{r}'| = 0 \text{ and } \Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}'|\hat{\mathbf{l}}') = 0 \quad (6)$$

are equivalent.

At that, it is important to consider the δ -function properties.

$$\int_a^b \delta(f(x))dx = \frac{1}{\left|\frac{df(x)}{dx}\right|_{x=x_0}}\int_a^b \delta(x-x_0)dx, f(x_0) = 0. \quad (7)$$

Accordingly, we will get for global illumination equation

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\int L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})G(\mathbf{r}',\mathbf{r})d^3r' \quad (8)$$

where the new geometric factor

$$G(\mathbf{r}',\mathbf{r}) = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}),\mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^3}\Theta(\mathbf{r}',\mathbf{r})\times$$

$$\times\left|\frac{d\Pi(\mathbf{r}-\xi\hat{\mathbf{l}}')}{d\xi}\right|_{\xi=|\mathbf{r}-\mathbf{r}_0|}\delta\left(\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}'|\hat{\mathbf{l}}')\right), \quad (9)$$

where $\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}_0|\hat{\mathbf{l}}')=0$, $\hat{\mathbf{l}}' = \dfrac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|}$.

Should be noted that equation (8) is derived for the radiance of a point on the surface Σ. However, light qualitative characteristics (glare, discomfort) are indissolubly related to observer: the radiance should be determined by some point in space. Formally, for the 3D visualization the radiance on the camera in space is also determined. Let us consider the equation about an arbitrary point in space.

The radiance angular distribution $L_\Sigma(\mathbf{r},\hat{\mathbf{l}})$ on a closed surface Σ is defined by the equation Π(r) = 0. It is required to determine the distribution of radiance in an arbitrary point **r** in volume *V* limited by the surface Σ. The volume is filled with a completely transparent medium.

By the solution of the radiative transfer equation for a transparent medium, radiance along the ray does not change. Therefore, the radiance of the point **r** in the direction $\hat{\mathbf{l}}$ is equal to the surface at the point of intersection of the surface with a ray from a point **r** at the direction $\hat{\mathbf{l}}$:

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_\Sigma(\mathbf{r}-\xi\hat{\mathbf{l}},\hat{\mathbf{l}}), \qquad (10)$$

where ξ – root of the equation

$$\Pi(\mathbf{r}-\xi\hat{\mathbf{l}}) = 0 \qquad (11)$$

The last correlations can be made user-friendlier analytically when properties of δ-function are used:

$$L(\mathbf{r},\hat{\mathbf{l}}) = C_{01}\int_{(V')} L_\Sigma(\mathbf{r}',\hat{\mathbf{l}})\delta(\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}'|\hat{\mathbf{l}}))\times$$

$$\times\delta(\hat{\mathbf{l}}-\hat{\mathbf{l}}_0)\frac{d^3r'}{(\mathbf{r}-\mathbf{r}')^2}, \qquad (12)$$

where $C_{01} = \left|\left|\frac{d\Pi(\mathbf{r}-\xi\hat{\mathbf{l}})}{d\xi}\right|\right|_{\xi=|\mathbf{r}-\mathbf{r}_0|}$ is related to the properties of the integral of δ-function with a composite argument, $\hat{\mathbf{l}}_0 = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$.

Combining the above expression for scene surface radiance (8) and radiance for a point in space (12), we can write the final expression

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_{\Sigma 0}(\mathbf{r}_\Sigma,\hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi}C_{01}\int L_\Sigma(\mathbf{r}_1,\hat{\mathbf{l}}')\sigma(\mathbf{r}_2;\hat{\mathbf{l}}',\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r}_2)\times$$

$$\times\delta(\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}_2|\hat{\mathbf{l}}))d^3r_1d^3r_2\ , \qquad (13)$$

where point $\mathbf{r}_\Sigma$ corresponds to the point of intersection of the camera sight line with surface Σ.

Thus, the last equation describes the radiance in any point of space.

3. LOCAL ESTIMATIONS

Local estimates were formulated in atomic physics [Kalos M.H. 1963] and continued its development in the optics of the atmosphere and ocean when solving the radiation transport equation [Marchuk G.I. 1980]. Note that global illumination equation is an implication of the radiative transfer equation in a vacuum. Let's consider local estimations for global illumination equation.

Local Estimation

The solution (8) can be shown as Neumann series

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\int L_0(\mathbf{r}_1,\hat{\mathbf{l}}_1)\sigma(\mathbf{r};\hat{\mathbf{l}}_1,\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r})d^3r_1$$

$$+\frac{1}{\pi}\int\frac{1}{\pi}\int L_0(\mathbf{r}_1,\hat{\mathbf{l}}_1)\sigma(\mathbf{r}_2;\hat{\mathbf{l}}_1,\hat{\mathbf{l}}_2)G(\mathbf{r}_1,\mathbf{r}_2)d^3r_1\sigma(\mathbf{r};\hat{\mathbf{l}}_2,\hat{\mathbf{l}})\times$$

$$\times G(\mathbf{r}_2,\mathbf{r})d^3r_1 + \qquad (14)$$

All terms of the series - definite integrals, which will be calculated by the Monte Carlo

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{\pi}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r},\hat{\mathbf{l}})} +$$

$$+\frac{1}{\pi^2}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i},\mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i})}\times$$

$$\times\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{2i},\hat{\mathbf{l}})G(\mathbf{r}_{2i},\mathbf{r})}{p_2(\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i}\to\mathbf{r},\hat{\mathbf{l}})} + \qquad (15)$$

Combining the sums into one

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\pi}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r},\hat{\mathbf{l}})}\right.$$

$$+\frac{1}{\pi^2}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i},\mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i})}\times$$

$$\left.\times\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{2i},\hat{\mathbf{l}})G(\mathbf{r}_{2i},\mathbf{r})}{p_2(\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i}\to\mathbf{r},\hat{\mathbf{l}})} + \right) \qquad (16)$$

The last expression can be interpreted as a Markov chain wandering ray with the contribution by the kernel

$$k(x_i\to x) = \frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_i,\hat{\mathbf{l}})G(\mathbf{r}_i,\mathbf{r})}{p_2(x_i\to x)}. \qquad (17)$$

Similar expressions were presented in [Budak et al. 2015.] As a result of the construction of the Markov chain, we can evaluate the radiance at a given point in a given direction on the scene surface. Such estimation may be called local estimation of Monte Carlo. Similarly to the estimation, made for the transport equation in atmospheric optics [Marchuk G.I. 1980].

Thus, local estimation allows calculating surface radiance at a given scene point in a given direction.

Let's consider the algorithm for radiance calculation with the local estimates of Monte Carlo. Accept that we have some 3D scene. We fix points and the directions on the surface at which we want to determine the radiance.

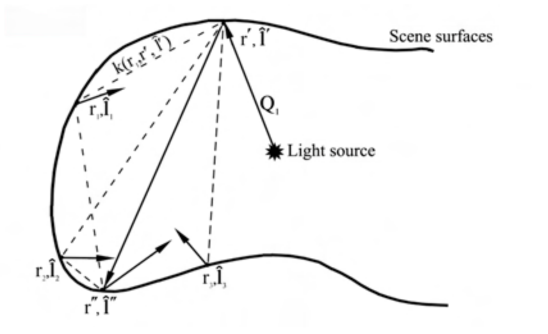

Figure 2: Algorithm scheme of radiance evaluation by local estimation

Let us cast the ray from the source. The most efficient way to select a ray is an importance sampling, but any other known samples for Monte Carlo methods can be used. This ray will receive the weight corresponding to the radiance. Determine the point of intersection of the ray with a scene element. Then we can evaluate the equation (8) kernel (17) for each of the test points and calculate directly reflected radiance in the test point taking into account its reflection coefficient in the test direction. Then, casting a new ray, considering reflectance coefficient. Its weight decreases. The process continues iteratively until the ray's weight is below the threshold or until it leaves the scene. Then again we take a new ray from the source. When statistics is accumulated, averaged and normalized, we will get the radiance directly at predetermined points in a given direction. Also, should be noted that the algorithm can be implemented in "Russian roulette" principle. Method is outlined in Figure 2.

Double Local Estimation

Let's take a look at the local estimation algorithm construction to determine the radiance of a given point in space. In equation (13) appears an additional δ-function $\delta(\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}_2|\hat{\mathbf{l}}))$ which depends on the direction $\hat{\mathbf{l}}$. It makes direct modeling impossible. It is clearly seen in the graphic interpretation in Figure 3.

Suppose we have a given point **r** in space and a direction $\hat{\mathbf{l}}$ in which we want to determine the radiance. We begin to build a Markov chain. As it seen in Figure 3, it is impossible to get from the chain point in test direction at the test point. To solve the problem we fix an additional node - the point on the surface - and do calculations through it. This approach is called a double local estimation [Marchuk GI 1980].

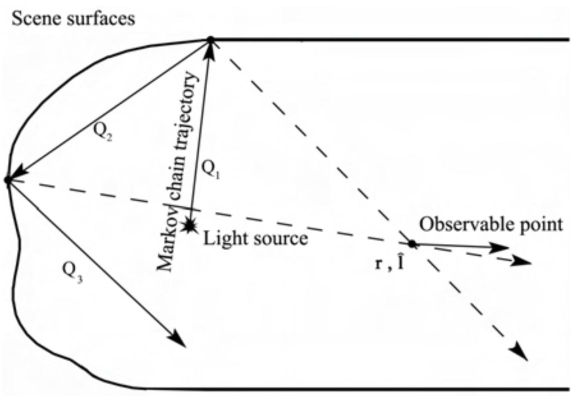

Figure 3: Geometric description of the impossibility of radiance modeling direct in the spatial point

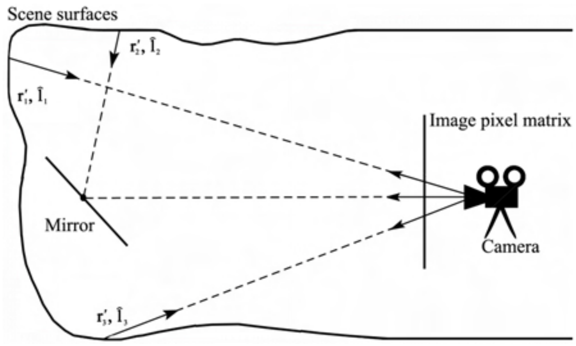

Figure 4: Scheme of scenes three-dimensional visualization construction

The further algorithm will be no different from the local estimation.

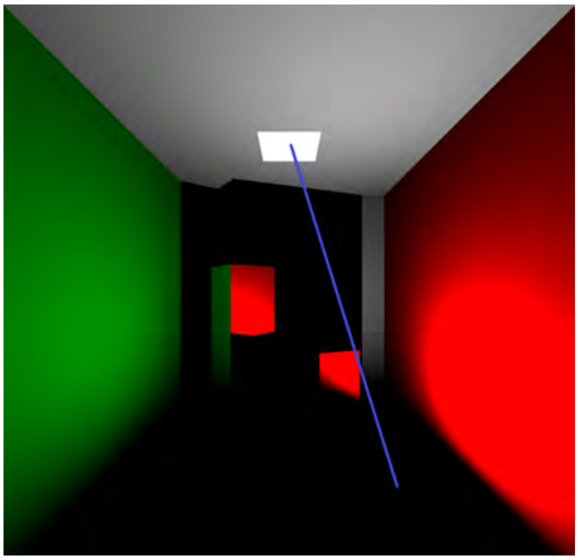

Figure 5: Scene rendering by one ray on one node of Markov chain

On Figure 5 presented the rendering of the 3D scene on one node of the Markov chain. In fact, even on one node, we get all the images at once, taking into account multiple reflections. Certainly, it is not accurate. The figure shows an image of the scene Figure 6 considering a second node of the Markov chain for the same ray. After drawing a large number of rays, we can get the final image shown in Figure 7.

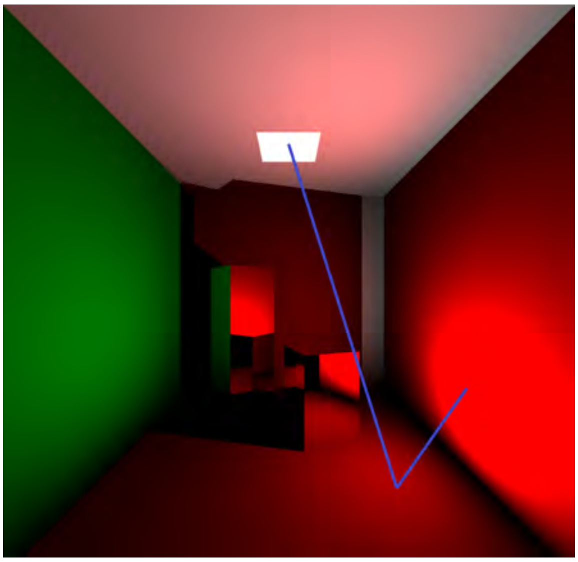

Figure 6: Scene rendering by one ray on two nodes of Markov chain

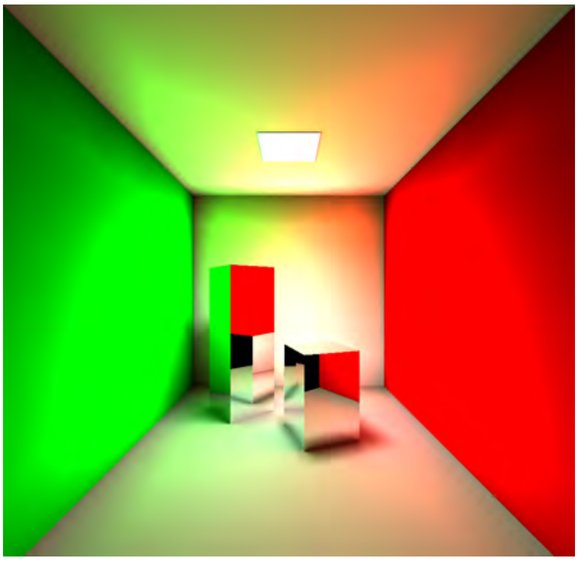

Figure 7: Scene rendering by 1000 rays on average five nodes of Markov chain

Local Estimations and Instant Radiosity

Local estimations of Monte Carlo were proposed for the first time in phenomenological approach in the work of [Keller A. 1997] and were called Instant Radiosity.

Should be noted that the algorithm described in [Keller A. 1997] is different from the one proposed in this article and based on the local estimations. The author divides the process into two stages - forming the virtual light sources and a calculation of their contribution. From our point of view, the construction of Markov chains and calculation of its nodes contribution are closely interrelated. Should be noted that analysis of instant radiosity method in [Pharr M., Humphreys G. 2010] is based on a similar processes separation approach.

In the approach wet put forth in our work, these processes are not divided. Because of that, we can see the whole image at any time. In other words, we can get a complete image at once even by one ray of Markov chain.

The geometric factor kernel (17) of global illumination contains a well-known feature $\frac{1}{(\mathbf{r}-\mathbf{r}')^3}$ that leads to local estimation infinite dispersion [Kollig. T., Keller A. 2004]. There are two algorithms for its elimination: with proposed equation kernel restriction and further solutions refinement by integration within this volume; or with integration by a small area around the observation point in which averaging of results will take place [Kalos MH 1963].

4. VIEW-INDEPENDENT LOCAL ESTIMATION

Radiosity method is not widespread in computer graphics. Nevertheless, it found its application in lighting design systems.

Currently, the radiosity method is used in two main software products for lighting systems design, DIAlux and Relux. The main advantage of the method is that it allows calculating of global illumination without camera position - a view-independent calculation. However, it uses a diffuse reflection model, which cannot describe real materials. Moreover, the calculated illumination distribution is not a characteristic perceived by the eye.

Based on the described local estimations of Monte Carlo method, we can build a new algorithm for calculating global illumination without camera position and with any reflection model.

Algorithm

As in the case of radiosity, the calculations will be made on the mesh. This mesh can be both simple static, formed before the calculations, based on simple criteria, or dynamically generated directly during the computation. Should be noted that according to the circumstantial evidence we assume that DIAlux uses static mesh. At the same time, our mesh will differ significantly from one used in radiosity method. In the radiosity method, the radiance is averaged by the mesh element. In our case, we can directly calculate the radiance at a given point in a fixed direction and can perform calculations on the mesh nodes.

For simplicity, we will use a static mesh. Suppose we have some initial scene. We divide it into smaller mesh nodes and define a uniform step zenith θ and azimuthal angle φ direction by the normally oriented hemisphere. These are directions at fixed points in which we calculate the radiance $L(\mathbf{r},\hat{\mathbf{l}})$.

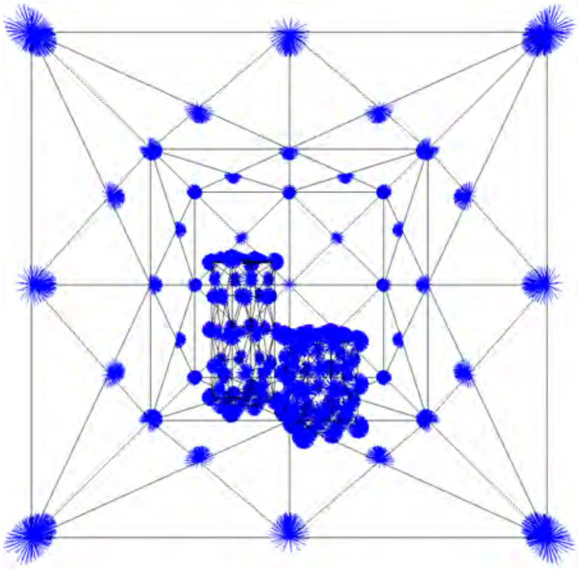

Figure 9: Definition of calculation points and directions of the view-independent calculation

The further algorithm will be no different from the local estimation discussed earlier. After calculation using the local estimation, we will get the radiance values at the mesh nodes in the set of directions.

To draw an analogy with radiosity method, further, when doing final image collection or analyzing illumination in lighting calculations, we will be interested in the radiance of arbitrary points on the surface. However, while the radiance value in radiosity method does not depend on the position of the viewpoint (camera), in our case the radiance will depend on it.

To determine the radiance at any point of the element, we need first to find the radiance at the vertexes of a triangular mesh element in the direction of the observer. It can be done by approximating the radiance calculated in directions distributed over a hemisphere. Then we can calculate the radiance at a given point within the triangular element through barycentric coordinates.

Obviously, the accuracy will directly depend on the size of scene partition element and the number of directions in which the radiance will be calculated.

The problem of choosing the number of directions for calculations is beyond the scope of this study. However, our preliminary studies show that, for example, to the Phong model in real scenes, the higher the degree of the cosine, the greater the number of directions necessary to describe that.

Figures 10 and 11 shows an example of view independent calculation of the Sobolev problem scene with rectangle downlight. The scene surfaces have Phong reflectance with the power of cosine equals 16.

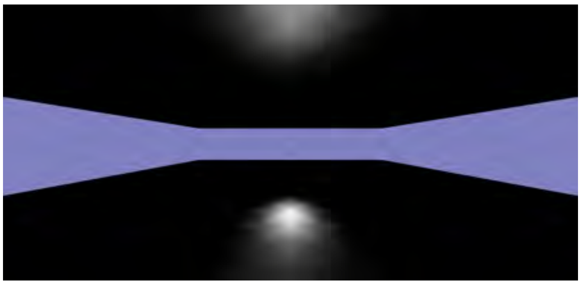

Figure 10: The Sobolev problem scene view-independent rendering

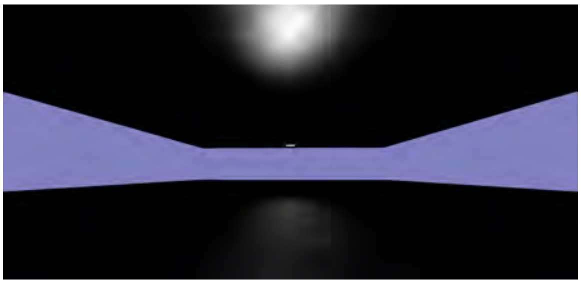

Figure 11: The Sobolev problem scene view-independent rendering

Using spherical harmonics approximation

As described above, when analyzing the obtained results we have a problem of radiance approximating at a point in the direction. We suggest a well-known expansion by spherical functions for this. To do so, we can expand the radiance at each point by spherical harmonics

$$L(\mathbf{r},\hat{\mathbf{l}}) = \sum_{n=0}^{N}\sum_{m=-n}^{n} C_n^m(\mathbf{r})\,\mathrm{Y}_n^m(\hat{\mathbf{l}}) =$$

$$= \sum_{n=0}^{N}\sum_{m=0}^{n}\left(A_n^m(\mathbf{r})\cos\varphi + B_n^m(\mathbf{r})\sin\varphi\right)\mathrm{P}_n^m(\hat{\mathbf{l}}\cdot\hat{\mathbf{z}}), \quad (18)$$

where

$$C_n^m(\mathbf{r}) = \int \mathrm{Y}_n^m(\hat{\mathbf{l}})L(\mathbf{r},\hat{\mathbf{l}})d\hat{\mathbf{l}}. \quad (19)$$

As a result, for each vertex of the mesh after the calculation, we will store expansion coefficients $A_n^m(\mathbf{r})$ and $B_n^m(\mathbf{r})$instead of radiance by the set of generated directions $L(\mathbf{r},\hat{\mathbf{l}})$. Therefore, we can also determine the radiance in the desired direction $\hat{\mathbf{l}}$ based on these coefficients. According to our preliminary study, this can significantly reduce the amount of stored information.

Also, should be noted that for the greater efficiency of the algorithm of directions at the zenith angle θ in the formation of directions mesh at the point it is better to choose in zeros of the Legendre polynomials. It will further on when determining expansion coefficients by integrating allow using the Gaussian quadrature, which gives the exact value by integration.

An important fact is that the loss of "energy" does not depend on spherical harmonics series terms number. Each next following term clarifies the solution.

Radiance object expansion by spherical harmonics also essential from the lighting technology science perspective, because we can see that some members of the series have a rigorous physical interpretation, for example, the coefficient $A_0^0(\mathbf{r})$ will be the same as scalar irradiance - the radiance integral over the solid angle since $Y_0^0(\theta,\varphi)= 1$. Coefficient $A_1^0(\mathbf{r})$ - as light vector module because $Y_1^0(\theta,\varphi) = \cos\theta$.

Way to increase Spherical Harmonics Transformation (SHT) algorithm performance

Reducing the amount of information stored in the brightness distribution at the points of the scene is can be achieved by introduction of an additional algorithm (SHT), which certainly makes a negative contribution to the performance of the whole algorithm. Taking into account the actual for today resolutions, it becomes obvious that the cyclical structure of SHT procedure imposes the requirement for the high performance of this algorithm.

It is clear from the definition that integration by volume is required to expand a function in a series of spherical harmonics. However, one may notice that the integral over the azimuthal angle φ (the sum over **n** in (19)) is the Fourier series. It is known that there is an effective numerical method, called the Fast Fourier Transform (FFT) for the Fourier transform procedure. Thus, from the algorithmic point of view, the double integral is reduced to the single on ϑ, and inner integral can be calculated using the FFT [Martin J. Mohlenkamp 1999].

One property of associated Legendre polynomials $Y_n^m(\cos\varphi)$ is that it is either even or odd across φ=π/2 as *n-m* is even or odd. The use of equalities also reduces computation time by a factor of two [Martin J. Mohlenkamp 1999].

Other important factors affecting performance are the calculation of $Y_n^m(\cos\varphi)$ and the step of sampling, which will affect the speed of calculation of the integral. Taking into account specific of our task iterative calculation of the associated Legendre polynomials seems expensive. However, we guess that any radiance object should fit well into the only single matrix of angles sampling, which enables us to calculate polynomials in advance. We guess that sampling rate 2n should be sufficient.

Spherical Harmonics Local Estimation

One of the problems of radiosity method is a rather large consumption of memory for storing the information on the mesh. In our method of View-Independent local estimation, this amount also increases by some directions for each node. As described above when using radiance decomposition by spherical harmonics we can reduce the amount of stored information. Thus, if we look for a solution directly in the spherical functions, we can immediately calculate the expansion coefficients for the given points. In this case, the expression (20) can be directly estimated by Monte-Carlo method already on one node, and then (15) can be written as

$$
\begin{aligned}
C_n^m(\mathbf{r}) = {} & \frac{1}{N}\sum_i L_0(\mathbf{r},\hat{\mathbf{l}})\,Y_n^m(\hat{\mathbf{l}}) + \\
& + \frac{1}{\pi}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r},\hat{\mathbf{l}})}Y_n^m(\hat{\mathbf{l}}) + \\
& + \frac{1}{\pi^2}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i},\mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i})} + \\
& + \frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{2i},\hat{\mathbf{l}})G(\mathbf{r}_{2i},\mathbf{r})}{p_2(\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i}\to\mathbf{r},\hat{\mathbf{l}})}Y_n^m(\hat{\mathbf{l}}) + \qquad (20)
\end{aligned}
$$

Thus, in the spherical harmonics local estimation algorithm we do not fix the directions at the vertexes of the mesh, but on each node of the Markov chain, we determine a random direction of reflection for each vertex and calculate the expansion coefficients with one radiance value. The statistics are collected directly in the expansion coefficients $A_n^m(\mathbf{r})$ and $B_n^m(\mathbf{r})$.

5. CONCLUSION AND FUTURE WORK

Local estimation of the Monte Carlo method allows calculating the radiance of a given point on the surface or in space in a particular direction. Obtained expressions show us the connection between Instant Radiosity method and local estimation, harmonically complementing the already known method.

Local estimations allow obtaining the spatially angular distribution of radiance, and view-independent simulation algorithm of allocation allows to get to a new level of illumination quality analysis. Avoiding reflections diffuse model used in lighting simulation nowadays is a necessary stage in the transition from designing of lighting systems with defined quantitative characteristics to the simulation of lighting systems with specified quality characteristics.

The work still has numerous unsolved issues. In the future, our attention will be paid to:

- distinctive features in the core of the global illumination equation;
- finding a solution directly in spherical functions decomposition coefficients;

performance issues and as the ultimate goal - the quality of lighting.

6. ACKNOWLEDGMENTS

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