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BASIC PROPERTIES OF A BISTABLE MECHANICAL OSCILLATOR

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Abstract: The contribution deals with analysis of dynamical properties of 1 DOF bistable mechanical oscillators. These systems appear in many technical applications which advantageously use their both static and dynamic properties. First, a mechanical mode based on von Mises truss is introduced and corresponding mathematical model is formulated. Then, the model is used to show the basic properties of the system and some parametric studies are performed.

Keywords: bistable system, vibration, bifurcation, stability

1 Introduction

In many devices, such as switches, clasps, closures and others, mechanisms experiencing stable equilibrium in two distinct positions are desired. Bistable are employed in micro-mechanisms for micro-valves, micro-switches or relays, bistable display systems and multi-stable mechanisms. As an example, to achieve a bistable configuration of a mechanical system, buckling of a beam can be mentioned. A short survey on mechanisms with bistable configuration can be found in [1]. They focus on mechanisms assembled from compliant segments and solve problems connected with the proper design. A very specific application of bistable mechanism can be found in an easy-chair design [2]. Such a chair is designed for people suffering with arthritis and it helps to rise from seated position and to diminish the painful experience due to the inflammation of their joints. Further, bistable mechanisms are often used in piezoelectric vibration-driven systems to increase the average locomotion speed of the system [6]. These mechanisms are employed based on the inspiration by limbless animals like snakes and earthworms, in crawling robots.

There is a huge number of literature devoting to bistable systems. Their similarity consists in the fact, that in many cases the systems can be described by Duffing equation, which contains the fundamental dynamical properties of bistable systems. For example, the nonlinear equation of motion of a mathematical pendulum can be approximated by the series expansion leading to cubic nonlinear force term (Duffing equation). Similarly, a geometrically nonlinear system consisting two linear springs and a mass can be approximated by the Duffing equation capturing the basic dynamical properties [3].

In this paper, we take into account a mechanical model displayed in Fig. 1. It consists of a mass m sliding on a straight line (plotted dashed). The mass is connected by two identical springs to a frame. The distance between connecting points of the springs to the frame equals to $2h$. The line segment AB is perpendicular to the direction of motion of the mass point m. The free length of each spring with stiffness k is denoted as l_0. The inclination of the system is determined by the angle α. It is assumed, that the mass point is in a Earth gravity field. Then the potential energy of the system is formed by the inclination of the mass motion (given by dashed line). If the inclination angle $\alpha = 0$, the potential energy is symmetrical with respect to vertical axis. If the inclination angle $\alpha \neq 0$, the potential energy becomes non-symmetrical. Then, there are three equilibria, where two are stable ones (x_{S1}, x_{S2}) and one unstable x_{N1}, see Fig. 1. In the next, the attention is paid on formulation of the mathematical model of the presented system and on the mathematical model gained by approximation of the original model by Duffing model. It is shown that if even the models model could be treated as identical ones, they do not show the identical dynamical nor statical response. Let us derive the mathematical model of the system

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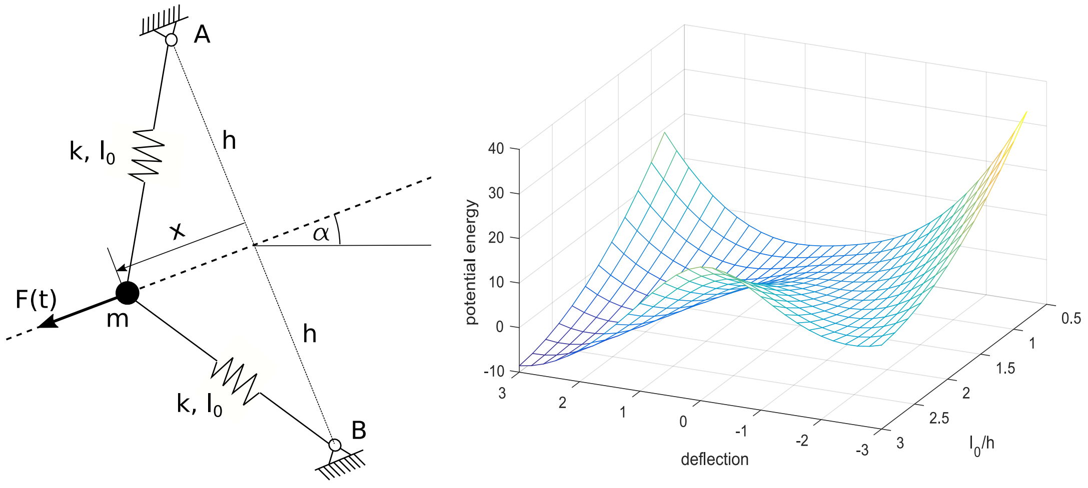

Figure 1: Considered bistable mechanical model with geometric nonlinearity (left), corresponding potential energy plot (right)

presented in 1. The potential energy has following form

$$V = k\left(l_0 - \sqrt{h^2 + x^2}\right)^2 - mg\,x\sin\alpha, \tag{1}$$

where x denotes generalized coordinate representing position of the mass, g stands for gravitational acceleration. Kinetic energy then becomes

$$E_k = \frac{1}{2}m\dot{x}^2. \tag{2}$$

Further, we suppose linear damping of the system. The linearity means, it is proportional to the velocity of the mass. The Rayleigh dissipation function has then following form

$$R = \frac{1}{2}b\dot{x}^2 \tag{3}$$

with damping coefficient b. In general, we assume frequency dependent excitation acting on the mass point

$$F(t) = F_0\sin\omega t. \tag{4}$$

Using equations (1) - (4), as well as introducing $\Omega^2 = k/m$, $2D\Omega = b/m$ and $f_0 = F_0/m$, the following equation of motion of the slider is derived

$$\ddot{x} + 2D\Omega\dot{x} + 2\Omega^2 x\left(1 - \frac{l_0}{\sqrt{h^2 + x^2}}\right) = f_0\sin\omega t + g\sin\alpha. \tag{5}$$

In the literature, mathematical models like (5) are usually transformed in the form of Duffing equation, e.g. see [4]. Following the same procedure, i.e. expanding the fourth term of the equation (5) into a series assuming small oscillations and neglecting terms of order $O(x^4)$ and higher, one has [3]

$$\ddot{x} + 2D\Omega\dot{x} + \Omega^2\left(2\left(1 - \frac{l_0}{h}\right)x + \frac{l_0}{h^3}x^3\right) = f_0\sin\omega t + g\sin\alpha. \tag{6}$$

2 Analysis

Both statical and dynamical properties are analysed comparing the two derived models: original model (5) and Duffing model (6). Even if Duffing models are usually used to understand fundamental properties of dynamical systems, to perform a quantitative analysis, Duffing models are not ever sufficient depending on system parameter as shown further. The properties of both models can differ significantly.

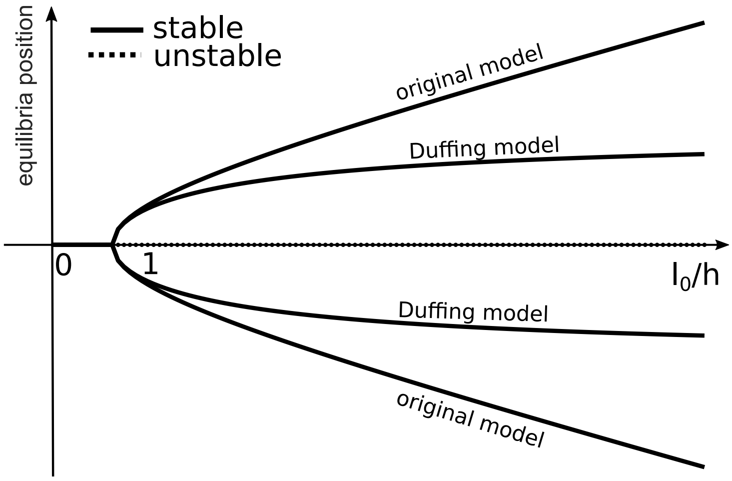

Figure 2: Stable and unstable equilibria of the model

2.1 Fixed points – static analysis

Neglecting time dependent excitation, fixed points can be found. Here, fixed points represent static equilibrium solutions, which can be characterized by zero velocity $\dot{x}^* = 0$ and zero static force $F_s(x^*) = 0$. Following this condition, static equilibria can be derived

a) original model: $x_{S1,2} = \pm\sqrt{l_0^2 - h^2}$ and $x_U = 0$,

a) Duffing model: $x_{S1,2} = \pm\sqrt{2h^2(1 - l_0/h)}$ and $x_U = 0$.

Both models show unstable equilibrium at the origin but the stable solutions differ and the difference grows as the geometric parameters differ. If $l_0/h \approx 1$, then the stable solutions almost coincide. As soon as the condition is broken, the stable equilibria differ. For illustration, the position of equilibria is plotted in Fig. 2. Based on the plot in Fig. 2, the solution x_U is stable for $l_0/h < 1$ (free length of the spring is less than the distance h and the zeros equilibrium is stable and no other equilibria exist). If $l_0/h > 1$, there appear two stable equilibria and the zero equilibrium becomes unstable. Comparing both models, the zero equilibrium agrees in both stable and unstable modes but the non-zero equilibria differ. The difference is higher as the ratio l_0/h is higher. The Duffing approximation seems to be more stiff as the ratio grows. It has roots in the expansion of the original spring force around the equilibrium considering small deflections.

In the next, for the other analysis, the original model is used.

2.2 Response to harmonic excitation - limit cycles and chaotic response

The dynamic response of the model (4) is analysed using pseudo-bifurcations diagrams, where the horizontal axis corresponds to excitation frequency ω and on the vertical axis the extremes (maxima and minima) of the mass point deflection are plotted, i.e. a pseudo-Poincaré map is used.

Let us consider following values of parameters: $l_0/h = 1.1$, $D = 0.05$, $m = 1$, $k = 10$ and the excitation amplitude $f_0 = 0.2$. Following the bifurcation diagram in Fig. 3 the response shows different vibration regimes. They can be classified as follows

- *limit cycles* can be detected by number of deflections extremes; there are 1-period and 2-period limit cycles in the response,
- *period doubling* can be detected as a transition from 1-period limit cycle to 2-period limit cysle which starts a transition to chaotic response,
- *well hopping* is induced by chaotic vibration regimes,

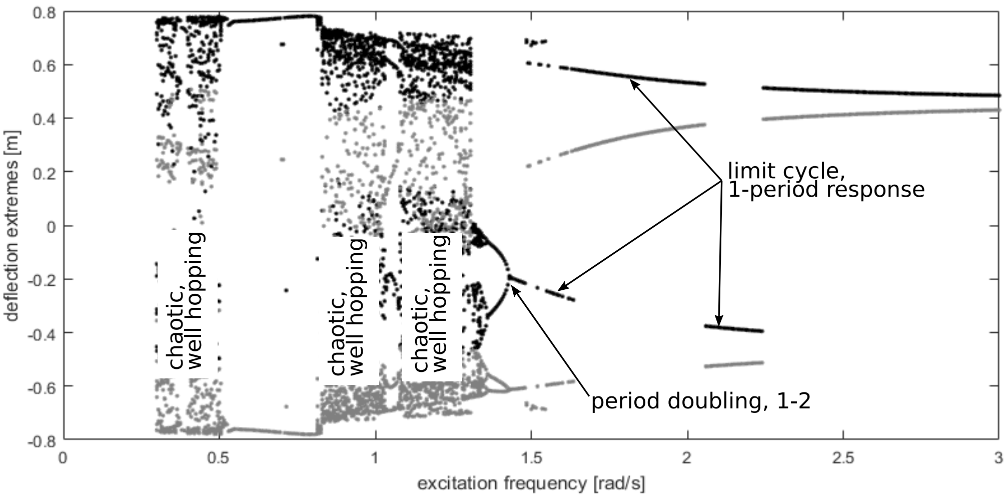

Figure 3: Bifurcation diagram of original model, $l_0/h = 1.1$, $f_0 = 0.2$

- *chaotic response* forms three distinct areas with respect to excitation frequency which serves as a bifurcation parameter. The chaotic response is interesting from the energy point of view of the system response. In these areas, the oscillator amplifies the motion of the base, therefore these regime are suitable for e.g. energy harvesting systems.

3 Conclusion

The paper studies basic properties of bistable mechanical systems which are based on von Mises truss principle. The system potential is unsymmetric based on the angle of inclination. Based on performed analyses the system exhibits different dynamical regimes which can be advantageously used e.g. in energy harvesting because the system transforms and amplifies the vibration of a base in a wide operation range.

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