Volumetric ultrasound imaging: modeling of waveform inversion

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ABSTRACT

Ultrasound imaging is widely used in medicine and nondestructive testing. Medical ultrasound volumetric imaging can be considered as a competitive method to X-ray CT and MRI. Recent results in breast ultrasound tomography provide spatial distribution of sound speed and attenuation coefficient based on inverse problem solution in frequency domain. Breast tissue can be characterized as relatively homogeneous medium with possible number of small diagnostically important target inhomogeneities (lesions). This paper is focused on numerical model of complex structure with objects of different sizes and properties that can be met in medicine or in various industrial or scientific cases. High resolution ultrasound imaging is sensitive to a number of effects such as reverberation, diffraction and attenuation that can degrade image quality and cause the artifacts. Those effects are investigated and eliminated by iterative waveform inversion performed in time domain for volumetric visualization of objects' structure and acoustic properties.

Keywords

Visualization, ultrasound imaging, reverberation, diffraction, attenuation, migration, tomography

1. INTRODUCTION

Ultrasound (US) imaging is based on generation of high frequency sound waves in elastic medium and registration of signals propagated through medium interior. Piezoelectric transducers acquisition geometry is designed for solution of specific problem by means of registered wave field processing to obtain a 2-D or 3-D images. Spatial resolution of final image is defined by sounding pulse dominant wavelength and usually varies in a range of fraction mm to few mm. The irregularity of investigation medium in space and physical properties causes elastic waves reflection, refraction, reverberation and attenuation on objects of size comparable or greater than wavelength of sounding pulse [Kak99]. Conventional beam-forming US imaging provides the shapes of investigating objects based on reflected signals forming a 2-D cross sections.

Volumetric US imaging is partially based on main achievements in previous geophysical imaging of the earth interior. One of the known seismic methods [Vir09] is wave form inversion (WFI). It is a qualitative reconstruction method measuring the variations of acoustic impedances originally developed in the 1980s [Tar84] for geophysical imaging of subsurface and sub-bottom [Bel90] objects. Classic WFI approach uses acquired reflection data and forward problem modeling in order to iteratively reconstruct acoustic properties of elastic medium. It can be implemented in frequency domain or time domain. The latter one was not used in practice due to high computational time consuming.

2. RECENT SOLUTIONS

Significant progress in ultrasonic volumetric imaging applied research was achieved during the last decade when various methods of US tomography (UST) methods have been developed and prototyped. Ray based computerized ultrasound tomography (CUT) was advanced by close to practical use diffraction tomography (DT) [Pin05], [Has13] and waveform tomography [San15]. Transmission US tomography results were compared with X-ray CT and MRI of breast in [Opi13]. Some authors in earlier times [Car81] and later [Dur07], [Hua16] use both reflection and transmission waves for tomographic imaging. Most of the approaches above are focused on medical diagnosis of breast pathologies relying on combination of relatively homogeneous surrounding elastic medium with small inhomogeneous pathologic objects. They use toroidal array of transducers surrounding the target object and scanning it in vertical dimension to obtain a set of slices. The joint reflection-transmission approach [Hua16] uses two parallel linear sensor arrays placed horizontally on both sides of the breast with incremental vertical scan. The most valuable results of these recent solutions are the ability to reconstruct breast structure from reflected component and sound velocity and attenuation distribution from transmitted component using different methods [Lit02]. The disadvantage of transmission tomography methods is that testing medium must have the size less than the size of scanning equipment and must be positioned between source and receiver transducers in order to register propagated waves. Such a limitation reduces the use of described methods in technical applications where sizes of objects may vary tremendously. We propose to use the volumetric US reflection inversion for visualization of complex structures containing objects of different sizes and physical properties for medical or nondestructive testing purposes. Here we meet problems with diffraction, reverberation and attenuation effects.

Diffraction is one of the main problems for US imaging. Without diffraction UST would be similar to X-ray tomography. Diffraction in breast US imaging was investigated in [Sim08] concluding better spatial resolution of diffraction tomography compared to transmission methods. Diffractions can be eliminated with migration method as shown in [Sch11] and [Gar13] developed in time and frequency domains respectively for medical application.

One of the efforts close to practical use in medicine was recently presented by [San15]. Authors use WFI principles to reconstruct sound speed in breast tissue based on Helmholtz equation solution in frequency domain. The restored images with sound speed mapping are of good quality but attenuation problem still needs to be solved.

A discussion on the pros and cons of WFI time-domain versus frequency-domain modeling [Vir09] is still open. One obvious advantage of frequency domain is computational efficiency but its disadvantage is noticeable spatial oscillations in reconstructed images. Time domain inversion despite its low efficiency provides good spatial resolution. Another its advantage is easy separation of reflection and transmission components while solving the forward problem.

The goal of this paper is to apply reflection US imaging for complex structure interior volumetric visualization.

3. PROPOSED SOLUTION

The elastic medium structure and properties are modeled and then reconstructed using the iterative reflection wave field inversion approach for a stratified elastic medium in time domain. The solution of forward and inverse problems is decomposed into several steps using both ray and wave theoretical foundations at different stages in order to investigate and eliminate (or compensate) the most critical effects that degrade image quality separately.

3.1 Forward problem

This paper presents the inversion technique by numerical modeling of the most energetic normal incident part of the field. This is provided by two assumptions: (1) emitter and receiver have zero offset in the same transducer, (2) all objects in the model are formed by planes parallel to coordinate planes as shown in Fig.1.

The proposed numerical model simulates a complex structure with objects of different shape, size and properties that can be analogous to various parts of technological systems in nondestructive testing or to abdominal body parts in medicine. The model structure (Fig.1) represents a homogeneous liquid medium of size 300x200x200 mm containing three objects inside.

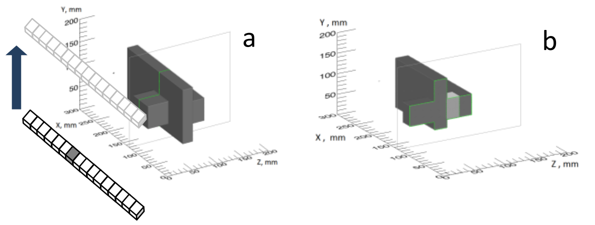

Figure 1. Numeric model and acquisition scheme: (a) model geometry and sliding transducer array forming normal incident wave field, (b) – numerical model cut by *(x=130, y, z)* plane shown on Fig.1a.

Large object of soft tissue or material has maximal sizes about 190x120x90 mm and incorporates a medium size object in the form of a channel filled out with acoustically contrast substance (solid tissue or material). Small object of size 4x4 mm simulates a lesion or debris and is located at a point with coordinates *(x=50, y=270, z=80)*.The shape of large object is complex but is formed by parallel planes in all three dimensions. This simplification allows to investigate vertically inhomogeneous model at each transducer position using one dimensional wave equation. The imaginary array of transducers is located on *(x,y)* plane and moves along the surface of liquid medium as schematically shown in Fig.1a. Thus we consider only compressional waves without shear and surface waves. In order to investigate only the normal incident part of the reflected pressure field *p(x,y,z,t)* at the medium surface *(x,y,z=0)* the transducer incrementally moves along the array in *x* direction (Fig.1a). The array scans the model moving along y direction in *(x,y)* plane with a step of 1 mm. Each element of transducer array is activated sequentially (as highlighted in gray) to send a pulse and to receive the reflected signals. Thus a virtual matrix of emitters insonifies the whole structure with normal incident plane waves in *z* direction. This acquisition scheme is similar to one used in [Hua16].

The distribution of velocities of sound waves, densities and attenuation coefficients of modeled objects are shown in Table 1.

Reverberation effect has the most influence in registered field at reflection/refraction angles close to normal.

Type of material	Velocity [m/s]	Density [g/m^3]	Attenuation coefficient [1/cm]
Liquid	1480	1.0	0.001
Soft tissue/ material	1510	1.05	1.0
Solid tissue	4080	1.9	2.0
Lesion/debris	2500	1.4	1.5
Air	331	1.29	1.26

Table 1. Acoustic properties of different objects used to form the model in Fig.1.

The problems of reverberation in conventional B-mode reflection ultrasound modality and method for its suppression were described in [Shi16]. Similar problem can be clearly demonstrated on a suggested model with parallel interfaces and zero-offset acquisition technique for reflected waves. Pressure wave propagation through the presented model along each emitter-receiver ray can be obtained using the finite-difference method for one-dimensional wave equation solution in case of impulse source. Reflectivity function $r(z)$ is obtained using acoustic impedance $\gamma(z) = \rho(z)V(z)$ distribution computed from Table 1. Reflection coefficient for air-liquid interface is $r(0) = 0.9994$. Next we substitute the spatial variable z with one-way travel time τ along the ray defined as

$$\tau(z) = \int_0^z \frac{dz}{V(z)} \qquad (1)$$

Then we model pressure field $p(\tau, t)$ by acoustic wave equation solution with appropriate boundary conditions using bent-ray approximation:

$$p_{\tau\tau} + [\ln \gamma(\tau)]' p_\tau = p_{tt} \qquad (2)$$

The solution of equation (2) provides full wave field including all reverberations.

Diffractions blur inhomogeneities reducing spatial resolution on reflected US images [Has13] thus, for example, forming speckles.

Since ray approach does not presume diffraction effect [Kak99] we compute diffractions from all of inhomogeneous points in 3-D space based on fundamental principle of Huygens' secondary source. The response in (x, y, t) space from such a source forms a diffraction hyperboloid according to:

$$t(x,y) = \sqrt{t_0^2 + \frac{(x-x_0)^2+(y-y_0)^2}{v^2(x_0,y_0,t_0)}} \quad (3)$$

where x_0, y_0, t_0 – are coordinates of diffraction point, and v is sound velocity. In 3-D we receive hyperboloids at each point of diffraction. The edges and corners of the model are blurred with diffractions as can be seen on superposed reverberation and diffraction field shown in Fig. 2.

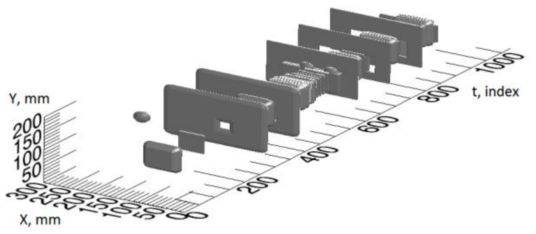

Figure 2. Reflected wave field including reverberations and diffractions.

Computational equivalent of δ-function used for modeling in previous steps is an ideal approach but to model a physical process we need to take in account a transducer transfer function (TF). A typical piezo element TF can be modeled as described in [Myl07]. In order to use reflected waves amplitudes in inversion we need to register signals in the far field. We can estimate the near field distance as $\frac{D^2}{4\lambda}$, where D is transducer diameter and λ is generated wavelength. For liquid material used in the model of this paper and for 1 MHz pulse and D=10 mm this distance is about 17 mm which is suitable for acquisition geometry we describe. Sounding pulse similar to one in [Myl07] has central frequency about 1 MHz. The superposed field with multiple reflections and diffractions is convolved with pulse described above.

Attenuation was modeled as exponential coefficient applied to amplitudes of waves while they travel up and down the model interior including multiples according to equation (2):

$$a(t) = s(t)\, e^{-bt} \quad (4)$$

where b – is attenuation coefficient from Table 1, in which b values for medical application correspond to water, bone, soft tissue and lesion. Dispersion is not considered as it is small in soft tissue [Kak99].

Despite the influence of attenuation reverberations cause artifacts in the form of dominating false reflectors.

Only the first and some reverberated reflectors and diffractors can be observed on final images due to attenuated energy. Small object usually seen as a speckle in regular US B-forming scan forms a hyperboloid in 3-D (Fig.2).

To reconstruct the model we need to solve inverse problem.

3.2 Inverse problem

We make inversion of final reflected wave field in few steps in order to restore the model structure.

At first step, we need to deconvolve the registered field $a(x,y,t)$ by transducer transfer function. Since each transducer transfer function $H(f)$ can be measured precisely the convolutional integral equation can be solved in frequency domain to receive a deconvolved impulse response $g(t)$ and can be embedded into equipment to be applied in-vivo:

$$\mathrm{g}(t) = \boldsymbol{F^{-1}}[G(f)] = \boldsymbol{F^{-1}}\left[\frac{A(f)H^*(f)}{|H(f)|^2+\alpha^2}\right] \quad (5)$$

where $A(f), G(f)$- are Fourier transforms of registered and deconvolved signals, $\boldsymbol{F^{-1}}$ denotes inverse Fourier transform , $\alpha = \alpha(f)$ regularization parameter practically depending on noise level and its frequency composition [Bel90]. At US frequencies mainly electronic noise affects the signals so α can be taken as a standard deviation of equipment self-noise. The main role of parameter α in present model is to compensate spectrum distortions by attenuation. Deconvoled pulse contains some noise produced by non- ideal inverse filtering.

The next step is diffraction artifacts elimination in 3-D space. According to [Kak99] the first Born approximation is valid only when the scattered field is smaller than the incident field. Since absolute values of reflection coefficients are smaller than 1 total reflected and reverberated energy at receiver is less than it was generated by source. Diffraction tomography assumes that scattering arises from incident field only.

Diffraction effect is modeled based on the principle that each inhomogeneous point of medium is a Huygens' secondary source with angle-dependent point aperture for all types of waves including reverberations. So we can use migration procedure of pressure wave front to suppress all diffractions in $g(x, y, t)$ space. One of the robust migration methods is based on Kirchhoff integral [Sch11]. For practical use the term proportional to $\frac{1}{R^2}$ is omitted [Yil87]. In order to get output field $g_{out}(t)$ for any output spatial point (x, y, z) from input field $g_{in}(x_{in}, y_{in}, z, t)$ we can use it in the following form:

$$g_{out}(x, y, z, t) =$$

$$= \iint dx\, dy\, \frac{\cos(\theta)}{vR} \frac{\partial}{\partial t} g_{in}(x_{in}, y_{in}, z, t - R/v) \quad (6)$$

where $R = \sqrt{(x - x_{in})^2 + (y - y_{in})^2 + z}$, $\cos(\theta) = 0$ for presented model with parallel interfaces and normal incident waves. Migration procedure is applied to each point of registered wave field to obtain the migrated field $\hat{p}(x, y, t) = g_{out}(x, y, z = 0, t)$ at the receiving surface.

The reverberations from acoustically contrast objects dominate the rest of informative reflections and need to be suppressed. The inverse finite-difference scheme for equation (2) deploys a backpropagation algorithm providing all reverberation suppression and reflectivity function $\hat{r}(\tau)$ restoration from deconvolved and migrated impulse response $\hat{p}(t)$.

The most critical step in inversion process is attenuation correction. The exponential correction reverse to (4) is applied with adjusted power to compensate the attenuated reflectivity. The exponent power parameter can be estimated as a product of double depth of ultrasound penetration, presumed averaged attenuation coefficient and tuning coefficient which was obtained based on the limit for the maximal reflection coefficient value $|\hat{r}(\tau)| \leq 1$. Tuning coefficient was found equal to 0.75 as an optimal value for the case of presented model (Fig. 3). Such function can be designed in real systems with sophisticated algorithms for adjustment by operator while imaging process.

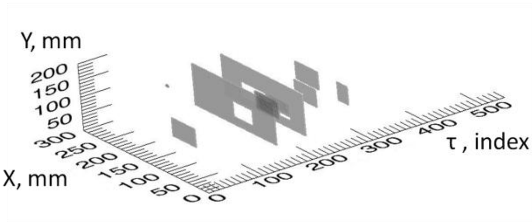

Figure 3. Reflection wave field after inversion and attenuation correction.

Acoustic impedance distribution can be obtained from restored reflectivity using [Old83] :

$$\hat{\gamma}(\tau) = \exp\left(\int_0^{\tau} 2\hat{r}(\vartheta) d\vartheta\right) \quad (7)$$

The restored impedance is a function of one-way travel time τ. The restored and initial arrival times have good correlation but absolute values for acoustically contrast internal object differs a lot mainly due to strong influence of attenuation especially in solid tissue or hard material of internal medium size object.

In suggested inversion approach the impedance is restored as a function of one-way travel time and to get it in space we need to split the product of density and velocity. Velocity function provides the main contribution to this product, so we indirectly receive a velocity distribution. Since the exact velocity distribution is known for the presented model the impedance is scaled accordingly to be presented as a spatial 3-D function to compare it with original model (Fig.1).

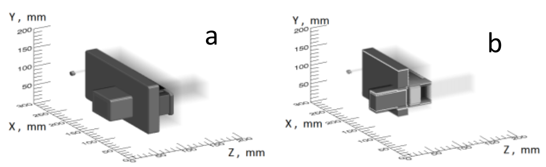

Figure 4. Restored model: acoustic impedance volumetric distribution as a spatial function (a), the same as (a) cut by *(x=130, y, z)* plane.

Computational noise at all steps of inverse problem resulted in some distortions in restored impedance model as can be seen in Fig.4. The edges of the large object were smoothed while migration step. Despite the distorted impedance values the objects of all sizes can be revealed in shapes close to initial ones.

4. CONCLUSIONS

An effort was made to volumetrically visualize the objects of different sizes and shapes with quantitative estimate of their acoustic impedances. This result was based on time domain inversion of US reflected field registered on the surface of vertically inhomogeneous elastic medium insonified by normal incident plane waves. Deconvolution and migration were applied to improve the spatial resolution of the final image in the form of acoustic impedance function of two spatial coordinates and one-way travel time. Sound velocity distribution can be estimated from impedances that may lead to present final structure volumetrically. Attenuation is a difficult effect to compensate by suggested inversion approach.

5. REFERENCES

[Bel90] Belykh, I. N., Inversion of seismoacoustic sounding data. In Russian geology and geophysics, Vol. 3, pp 105-111., (in Russian with abstract in English), 1990.

[Car81] Carson P. L., Meyer C. R., Scherzinger A. L., and Oughton T. V., "Breast imaging in coronal planes with simultaneous pulse echo and transmission ultrasound," In Science, Vol. 214, p. 1141-1143, 1981.

[Dur07] Duric N., Littrup L., Poulo P., Babkin A., Pevzner R., Holsapple E., Rama O., Glide C.. Detection of breast cancer with ultrasound tomography: First results with the Computed Ultrasound Risk Evaluation (CURE) prototype. In Med. Phys., Vol. 34, p. 773-785, 2007.

[Gar13] Garcia, D., Le Tarnec, L., Muth, S., Montagnon, E., Porée, J., Cloutier, G. Stolt's f-k. Migration for Plane Wave Ultrasound Imaging. In IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, Vol. 60, No. 9, p.1853-1865, Sep. 2013.

[Has13] Hasegawa, H., Kanai, H. Comparison of Spatial Resolution in Parallel Beamforming and Diffraction Tomography. In Proceedings of Symposium on Ultrasonic Electronics, Vol. 34, p. 315-316. Nov. 2013.

[Hua16] Huang L., Shin J., Chen T., Lin Y., Gao K., Intrator M., Hanson K. Breast ultrasound tomography with two parallel transducer arrays. In Medical Imaging 2016: Physics of Medical Imaging, Proc. of SPIE, 97830, Vol. 9783, Mar. 2016.

[Kak99] Kak, A.C., Slaney, M. Principles of computerized tomographic imaging. New York: IEEE Press, 1999.

[Lit02] Littrup, P., Duric, N., Leach Jr., R. R., Azevedo, S. G., Candy, J. V., Moore, T., Chambers, D. H., Mast, J. E., and Holsapple, E. T., "Characterizing tissue with acoustic parameters derived from ultrasound data". In SPIE: Ultrasonic Imaging and Signal Proceesing, Insana, M. and Walker, W. F., eds., Proc. SPIE 4687, pp 354–361, 2002.

[Mue16] Mueller, P., Schuermann, M., Guck, J. The Theory of Diffraction Tomography. In press. 2016. Available from: https://arxiv.org/pdf/1507.00466.pdf.

[Myl07] Mylvaganam, J. Characterization of medical piezoelectric ultrasound transducers using pulse echo methods. M.S. thesis, Norwegian University of science and technology, 2007.

[Old83] Oldenburg, D.W., Scheuer, N., Levy, S. Recovery of the acoustic impedance from reflection seismograms. In Geophysics, Vol. 48, p. 1318-1337, 1983.

[Opi13] Opieliński, K., Pruchnicki, P., Gudra, T., Podgórski, P., Kraśnicki, T., Kurcz, J., Sąsiadek, M. Ultrasound Transmission Tomography Imaging of Structure of Breast Elastography Phantom Compared to US, CT and MRI. In Archives of Acoustics, Vol. 38, No. 3, p. 321–334, 2013.

[Pin05] Pintavirooj C., Jaruwongrungsee K., Withayachumnankul W., Hamamoto K., Daochai S. Ultrasonic Diffraction Tomography: The Experimental Result. In proc. WSCG, Poster proceedings, 2005.

[San15] Sandhu G.Y., Li C., Roy O., Schmidt S., Duric N. Frequency domain ultrasound waveform tomography: breast imaging using a ring transducer. In Phys. Med. Biol., Vol. 60, p.5381–5398, 2015.

[Sch11] Schmidt, S., Duric, N., Li, C., Roy, O., Huang Zh.-F. Modification of Kirchhoff migration with variable sound speed and attenuation for acoustic imaging of media and application to tomographic imaging of the breast. In Medical Physics, Vol. 38, No. 2, p.998-1007. Feb. 2011.

[Shi16] Shin, J., Chen, Y., Malhi, H., Yen, J.T. Ultrasonic Reverberation Clutter Suppression Using Multiphase Apodization With Cross Correlation. In IEEE Transactions On Ultrasonics, Ferroelectrics, And Frequency Control, Vol. 63, No. 11, p. 1947-1956., Nov. 2016.

[Sim08] Simonetti, F., Huang L., Duric N. and Littrup P. Diffraction and coherence in breast ultrasound tomography: A study with a toroidal array. Technical report 08-4616, Los Alamos National Laboratory, 2008.

[Tar84] Tarantola, A., Inversion of seismic reflection data in the acoustic approximation: In Geophysics, vol.49, p. 1259–1266. 1984.

[Vir09] Virieux J. and Operto S. An overview of full-waveform inversion in exploration geophysics In Geophysics, Vol. 74, No. 6, Nov.-Dec. 2009.

[Yil87] Yilmaz, O. Seismic data processing. In Investigations in geophysics: Vol. 2, 1987.

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