In Search of Goals: Increasing Ice Hockey's Attractiveness by a Sides-Swap[1]

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Abstract

The popularity and business impact of major sports have been growing globally over time. This paper focuses on ice hockey, specifically the National Hockey League in North America. It reports a striking irregularity in ice hockey's scoring dynamics relative to comparable sports such as soccer and rugby, namely a scoring surge in the middle section of the game. We explore an explanation for this irregularity related to the convention on the spatial location of the teams' benches (which are fixed throughout the game) and on-ice sides (which are switched every period). Because a large number of the players' substitutions occur while the play is in progress, this convention determines the distance forwards and defenders need to travel to make a substitution, and thus indirectly substitution strategies and scoring. We consider two simple operational changes that could increase the number of goals in the NHL by approximately 5% and 10% respectively, corresponding to roughly 350 and 700 additional goals each season. This would partly offset the current downward scoring trend and thus enhance the game's attractiveness. The estimated impact of the proposed reforms, one of which is largely costless, is robust across several specifications – using per-minute and per-second scoring data and controlling for various factors, such as bookmakers' odds.

Keywords: Sport, ice hockey, discrete choice models, scoring dynamics

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1 Introduction

It is well established that the design of rules, processes and regulations affects the outcomes of various contests – including those of a political, economic or sporting nature.[3] This paper focuses on the latter area, and offers a demonstration of the power of design on improving the quality of "production" in ice hockey, one of the most popular sports in North America and Europe. Importantly, it estimates the effects of two simple operational changes regarding the teams' sides/bench positions (one of them ostensibly costless) that are likely to increase the game's aesthetic attractiveness through greater goal scoring. As such, they would be beneficial to all stakeholder groups as well as its business model (see e.g. NHL.com, 2015).

Ice hockey's academic interest in the business-disciplines literature dates back to Jones (1969), and even further back in terms of applications in operations research (see, for instance, Lindsey, 1963). Subsequent research has investigated a number of specific problems, such as Fleurent and Ferland (1993), who optimize the scheduling problem following league expansion (for a more recent 'scheduling' contribution see Nurmi et al., 2014). Pettigrew (2014) evaluated playoff-qualification likelihood bias arising from an unbalanced conference realignment – a manifestation of problems arising from an imperfect tournament design (see McGarry and Shutz, 1997). Meanwhile, literature further afield has centred on the unique scientific aspects of hockey, including various contributions in economics (Lopez, 2015), sociology (Paul, 2003) and psychology (Gee and Leith, 2007).

More generally, the financials clearly demonstrate ice hockey's economic significance in terms of its sheer size and scope. Total season attendances in the National Hockey League (NHL) in Canada and the United States exceed 21 million spectators. The NHL's revenue is $4 billion per annum (2014-15) and the combined annual revenue of its 30 constituent teams is a comparable $3.8 billion. This does not include revenues to partners benefiting from related trade, such as stadiums, merchandise, retailers and media.[4]

Our paper first reports a striking difference in ice hockey's scoring dynamics (a popular analytical tool; see Merritt and Clauset, 2014) compared to soccer and rugby. It does so using a unique data set

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[3] For examples, see Jackman (1987), Asano and Matsushima (2014) and Wright (2014), respectively.

[4] For attendance statistics see, for example, www.hockeydb.com/nhl-attendance; for business data see, for instance, www.statista.com/topics/960/national-hockey-league.

containing approximately 25,000 NHL games played over the period from 1995 to 2015. While in soccer and rugby the (instantaneous) probability of goal/try scoring increases over the course of the match, in ice hockey there is a substantial discontinuity in the second period, in which the probability of a goal being scored is noticeably higher than in the first and third periods (see Figures 3-4 in Section 2).

We offer an explanation for this interesting and surprising phenomenon, drawing attention to a seemingly unrelated logistic feature of ice hockey that has not been examined in the academic literature. It relates to the convention of the spatial location of the teams' *bench and sides positions*. Specifically, at the start of the game, each team's bench is located closer to its defensive zone than its offensive zone. However, while the teams keep the same bench throughout the game, they switch sides on the ice rink each period.[5] This implies that in the second period, the benches are closer to the teams' respective offensive zones, unlike in the other two periods, in which they are closer to their defensive zones (see the left panel of Figure 1).

Why should this matter for goal scoring? The key relevant feature in ice hockey is the frequent substitution of the five "skaters" on the ice (i.e. all players except the goaltender), called the "line change", and the way it occurs. A player's average appearance on ice, i.e. the so-call "shift-length", is generally between 35 and 55 seconds (with defenders serving slightly longer shifts than forwards).[6] Importantly, most line changes actually take place while the game continues, the so-called "changing on the fly".

The sides/bench convention thus promotes attacking play in the second period by making it more challenging for the two defenders to make a substitution; this is why it is called the "long change". Furthermore, the second period setting makes it easier for the three forwards to get behind the defenders and score from a quick counter-attack.[7] Conversely, the defenders' "short change" in the

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[5] In terms of the convention, see, for example, rule 636d of the NHL Rule Book, available at www.usahockeyrulebook.com/page/show/1084715-rule-636-start-of-game-and-time-of-match-time-outs. In regards to the literature, even Thomas (2007), whose Figure 1 (p. 2) clearly shows the same scoring pattern as in our Figure 3, does not discuss it. The short blog posts by Plank (2010) and Pettigrew (2013) seem to be the first to discuss this phenomenon.

[6] The median shift-length was 43.8 seconds in the 2015-16 regular season, see the data at www.sportingcharts.com/nhl/stats/average-ice-time-per-shift/2015/.

[7] For actual NHL examples of sides/bench related counter-attacks, see www.youtube.com/watch?v=Ubp3xhl_UAA or www.youtube.com/watch?v=LU-XWPzo8io. For an informative explanation of changing on the fly in the NHL, see www.youtube.com/watch?v=QdaiacSrya0.

first and third periods implies the opposite – a more defensive style of play with a more gradual attack into a well-formed defense. The logistic conventions, described above, therefore directly imply increased scoring in the second period, in line with the findings from Figures 3-4.

This "second-period scoring surge" in ice hockey is very robust, it appears across every single year in our sample and for any prior scoring dynamics (i.e. for any goal history up to any point of the second period, as apparent in Figure 5). We discuss later its possible alternative explanations, such as differences in the degree of competitive balance across sports and loss aversion – finding them unlikely to be driving the phenomenon. So, while our results do not constitute direct empirical evidence of a causal link from sides/bench positions to scoring, there appear to be no obvious alternative sources of possible variations between the periods that could plausibly explain the effect.[8]

The key reason for our examination of this scoring surge is its potential to increase scoring in the NHL, given that the number of goals has been falling over time (as apparent in Figure 2). The League is under enormous ongoing pressure from its competitors for market share – see Henrickson (2012) on how spatial competition geographically often leads to team relocation. This competitive pressure is arguably a major reason behind the NHL's frequent changes to the rules of play, most of which are designed to make the game a more aesthetically pleasing product via more attacking (higher scoring). Leeds and von Allmen (2014) discuss ways in which the NHL changed a range of rules for precisely this reason – to arrest a steady fall in the average number of NHL goals (for the regular season and playoffs combined) from above 7.5 in the first half of the 1980s to below 5 in 2003-04. Upon the resumption from the "lockout" (canceled) season of 2004-05, the newly implemented changes proved successful in that they increased this average to 5.91 in 2005-06. However, there has once again been a steady decline in scoring since then, with the average falling to 5.19 by the 2014-15 season.

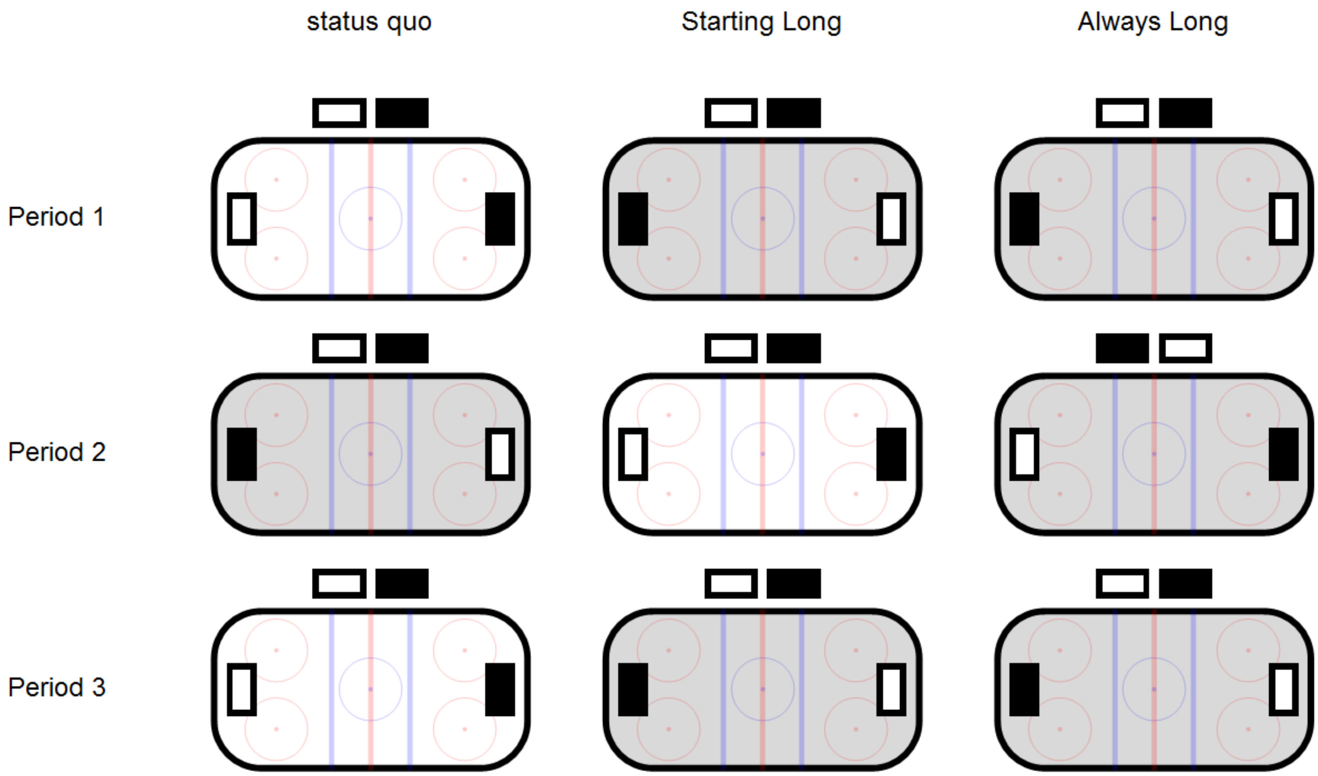
Given the declines in scoring and mounting competitive pressures, it is unsurprising to see intensified efforts by NHL officials to look for ways to induce more offensive play and more goal scoring. For example, the NHL stated – in relation to the rule changes introduced for the 2014-15 season – that: "…*the majority of the changes have been written into the rulebook with the underlying goal of increasing offense*". Similarly, the League informs that its general managers are still: "…*discussing*

[8] We discuss later how differences in the variance of the defenders' shift-length across periods could provide direct evidence that the sides/bench convention affects on-ice behavior and scoring. In the final section we also consider the effect of power plays if penalty calls differ across the periods.

potential ways to increase scoring".[9] A recent change in this direction, agreed to in March 2016, was to compact the goaltender equipment.

In order to contribute to this debate and policy evaluation, we estimate the marginal scoring effect of two straightforward operational changes: "Starting Long" and "Always Long". They are depicted graphically in Figure 1, together with the status quo.

Figure 1: Diagrammatic representation of the currently used convention on sides/bench positions (left panel) and the two considered alternatives.



Notes. Black and white rectangles denote the location of the respective teams' benches and sides (defensive zones). The shaded rinks indicate the periods featuring the long change and thus more attacking (higher scoring) play.

Under both considered settings, there would be a change either in the teams' initial sides on the ice or in their bench positions. In either case, the defenders' long change would apply in period 1, i.e. the teams' benches would be located closer to their offensive (rather than defensive) zone. Under Starting Long, the teams would (as is the case currently) change their sides on the rink each period and keep the same benches throughout the game.

[9] For the former quote, see www.nhl.com/news/analysis-rules-changes-could-create-more-scoring/c-730220, for the latter, see www.nhl.com/ice/m_news.htm?id=787279.

This proposal would therefore increase the number of goals scored in the first and third periods, compared to the status quo, while decreasing it in the second period. The Always Long reform would circumvent this period 2 scoring decline, but it would require an additional change compared to the status quo. Specifically, the teams would either not change sides every period, or they would switch the benches every period.[10] As a consequence, the long change would apply in all three periods.

Both considered reforms would lead to more goal scoring in total. To provide some quantitative insights, in Section 3 we report their estimated effect using three distinct empirical specifications and accounting for various factors that impact scoring dynamics. Our results are highly robust across the specifications, both in per-minute and per-second models.[11]

The estimates suggest that if the teams begin the match with their benches closer to their offensive zone in periods 1 and 3 (as opposed to period 2, which is currently the case), the amount of goals scored in periods 1 and 3 would increase by around 16.5% and 14.5%, respectively. This amounts to a rise of 0.55 goals per match over these two periods combined, i.e. under the Always Long reform. The operationally easier Starting Long reform would see a reduction in scoring in period 2, which we estimate to be around 14%, and such a reform would therefore lead to an increase of 0.27 goals in total scoring.

In either case, the effect is substantial. It corresponds to approximately 327 additional goals each NHL regular season, and 24 in the playoffs, under the Starting Long reform. The figures for the Always Long reform would be approximately double that. Given that the former reform does not encompass any major costs or drawbacks (two relatively minor ones are discussed in the Conclusions), the case for testing it is indeed strong.

2 Empirical Facts On Within-game Scoring Dynamics

It is useful to start with long-term scoring trends in ice hockey. Figure 2 plots the average number of goals per game in the NHL by season since 1967-68, and reveals that a downward scoring trend has been observed since the early 1980s.

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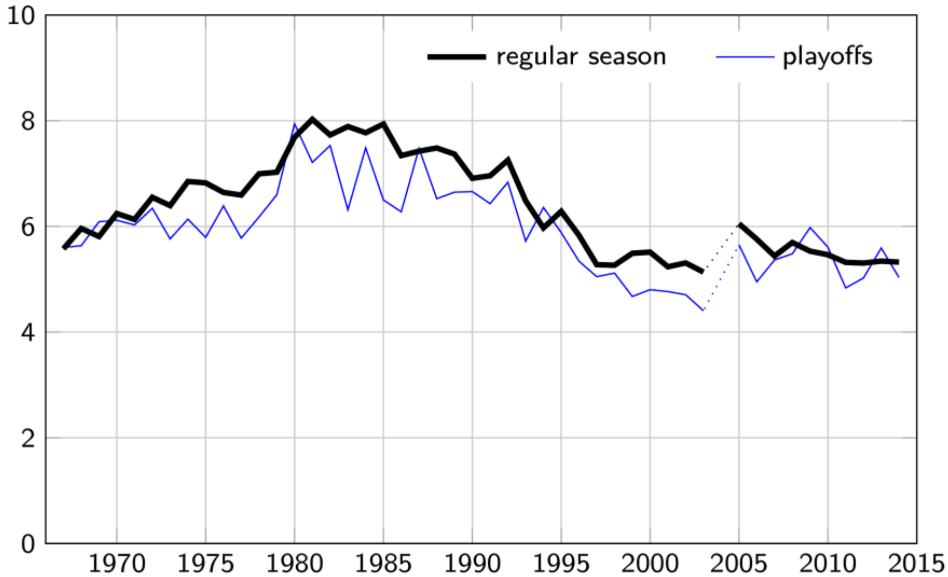
[10] The former change may be disliked by some fans (for example, those seated behind their own goal), whereas the latter change carries the logistic cost of having to move all of the equipment twice over the course of the game, and teams being unable to access their own bench directly from their dressing room. More related discussion appears in the Conclusions section.

[11] Kaplan et al. (2014) use per-second scoring to great effect in modelling ice hockey data.

Figure 2: The average number of goals per game in the NHL.



Notes. The thick black line shows the series for the regular season and the thin blue line shows the series for the playoffs. Both series include regulation time as well as overtime goals, but exclude the notional goal awarded for winning the shootout. The dotted lines covering the 2004-2005 period represent the 'lockout' season. Source: www.quanthockey.com/TS/TS_GoalsPerGame.php.

Specifically, one needs to take account of the rule changes implemented after the 2004-2005 lockout season, designed (successfully) to increase scoring.[12] Splitting our data into the periods before and after, we obtain a statistically significant downward linear trend for both sub-samples. For the 1995-2004 period, the regression yields the slope estimate of -0.113 goals/match in the regular season ($p<0.001$), and for the 2005-2015 period the estimate is -0.068 ($p<0.001$).

The continued efforts of NHL administrators to address the downward trend are driven by their belief that higher scoring enhances the game's attractiveness. For example, NHL.com (2015) discusses some measures such as reducing the size of the goalie's equipment, enlarging the goals, and reducing the number of players in overtime (for more see Gretz, 2015). In relation to the latter measure, implemented in the 2015-2016 season, it argues that team General Managers "*…are pleased with how scoring in overtime has increased because of the 3-on-3 format*", and quotes the Dallas Stars

[12] To name the key changes: (i) the two-line offside rule was repealed; (ii) introduction of a penalty for the defensive team shooting the puck over the glass deliberately; (iii) no line change allowed following icing; (iv) reduction of the area where the goalie is allowed to play. There were also additional changes in the interpretation of existing rules, relating mainly to stricter penalty calls on fouls.

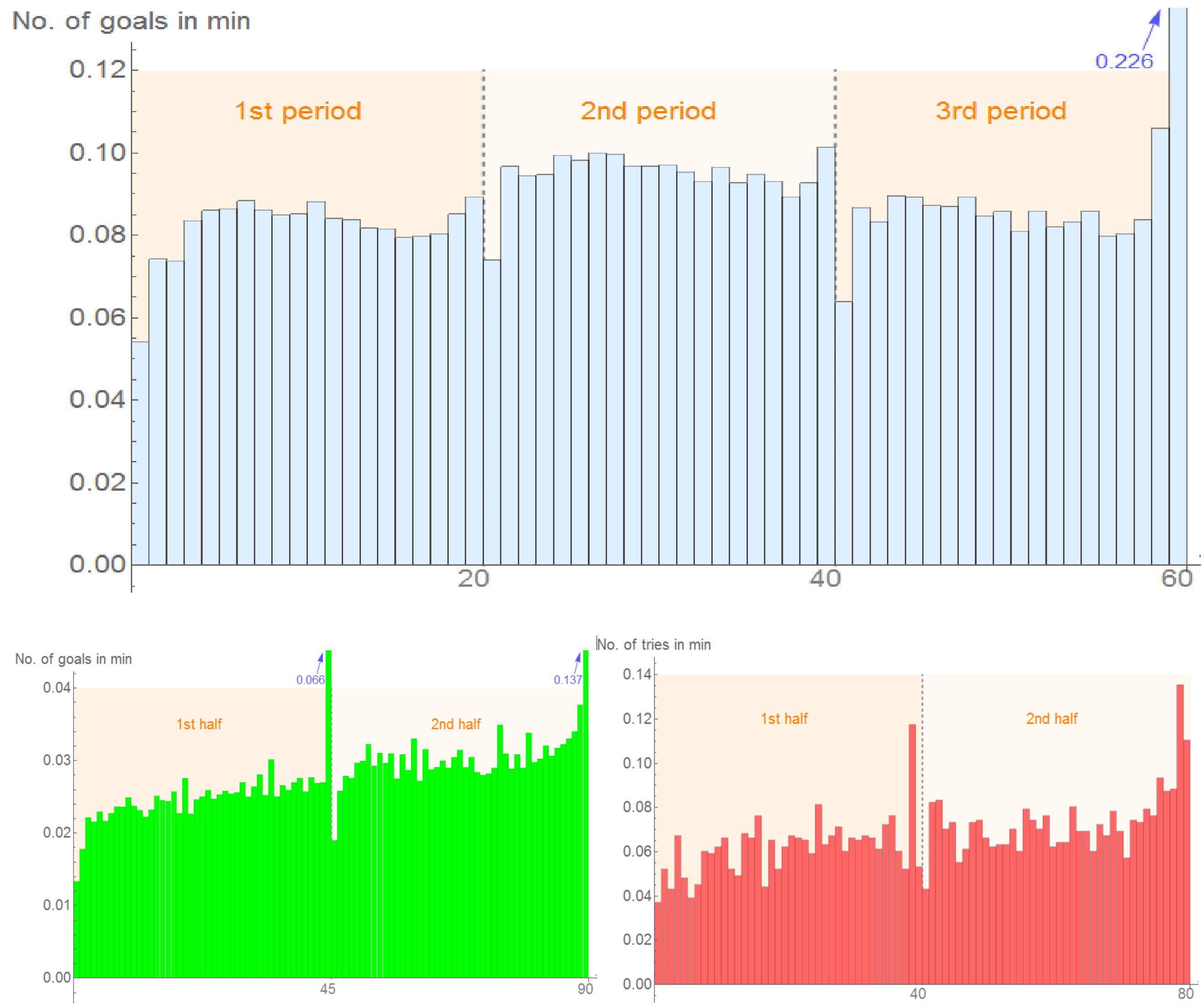
General Manager as saying: "*The fans love it. It's entertainment and we're in the entertainment business*."

In assessing possible ways to partly reverse the downward trend and increase scoring, our sample is drawn from bookmakers' data provided by Trefik. It contains 24,498 NHL matches taking place between January 1995 and June 2015, with 93% of these from the regular season and 7% from the playoffs. For all these games, we have exact (by-second) scoring times and additional details for most (e.g. bookmakers' odds for 81%). Approximately 77% of the games in our sample finished after the regulation 60 minutes with one team winning, whereas the remaining matches were tied and went into overtime. We include these games, but take into account only their regulation-time scoring (unlike the data in Figure 2, which include overtime goals). The specifics of overtime are discussed in the final section of the paper.

The panels in Figure 3 plot the distribution of regulation-time goals scored in ice hockey, soccer and rugby (tries). Before discussing the headline scoring patterns, let us clarify that the increase in ice hockey scoring during the final two minutes (59'-60') of the game is due to goals scored to the empty net after the trailing team "pulls the goalie" (the OR literature on this phenomenon dates back to Morrison and Wheat, 1986). Further, the sharp increases in soccer scoring in minutes 45' and 90' are due to stoppage time (which is usually 1-2 minutes in the first half and 3-4 minutes in the second half). This is because in our sample, all goals scored during time added to compensate for delays are assigned to the final minute of regulation time in each half. In rugby, the stoppage time tends to be even longer, and the time data in Figure 3 are based on 'count-up' – we scale them to the 40-minute halves.[13]

[13] Our rugby data come from ProZone for the 2002-2012 seasons of Super Rugby (958 regular-season matches), which (unlike most tournaments) awards a bonus-point to either team for scoring at least four tries. Lenten and Winchester (2015) show that this incentivizes more tries to be scored late-on in matches. In addition, there also exists a 'narrow-loss' bonus, which could conceivably have a similar effect. Our soccer data are drawn from the Trefik database and contain 26,429 matches from six major European leagues, domestic cups, as well as European and World Championships. For more details about this dataset, see Lenten et al. (2013).

Figure 3: Scoring frequency, i.e. the number of goals in each minute, in ice hockey (top), soccer (bottom left) and rugby (tries, bottom right).



The findings of Figure 3 on soccer and rugby, unlike the findings on ice hockey, are in line with other sports; e.g. Merritt and Clauset (2014) demonstrate how the scoring dynamics in the NBA (basketball) and CFL/NFL (gridiron football) exhibit similar statistical properties to soccer and rugby. In particular, in all these sports we observe lower-scoring levels at the start of each period, small linear increases in the middle stages and an accelerating increase in scoring towards the end. Such within-game scoring pattern is intuitively uncontroversial. More action tends to happen in sporting contests as the match timeline progresses, because the equation becomes increasingly clear (i.e. uncertainty diminishes).[14] This process gathers further momentum towards the end as the trailing team (if still within reach of

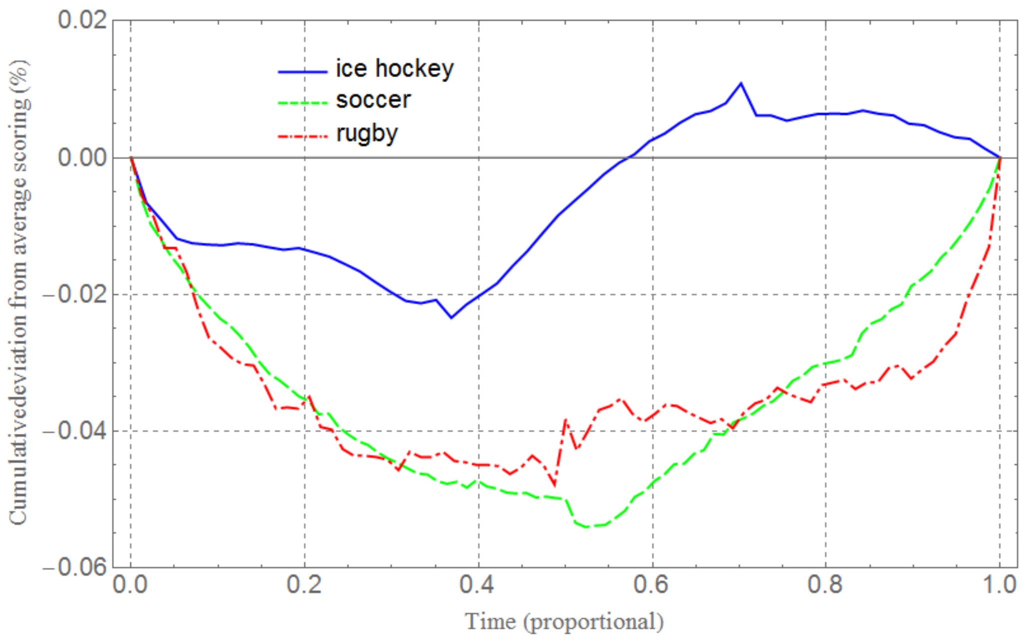
[14] For more on outcomes in various sports using dynamic learning for stochastic processes, see Percy (2015).

victory) “goes for broke” and strategically re-allocates team resources from defense to attack, as well as increases effort exertion. This generally boosts both teams’ scoring.

Such inverse correlation between uncertainty and effort is analogous to that discussed by Prendergast (1999), whose model shows how greater uncertainty in the labor-market environment requires employers to offer higher incentive pay to maintain effort levels from their workers.

In order to highlight the differences between ice hockey on one hand, and soccer/rugby on the other, Figure 4 offers an alternative perspective. It differs from Figure 3 in that it plots *cumulative* deviations from the average time-dependent scoring probability for each sport, whereby the duration of the game has been converted into a unit interval for comparability. The scoring surge in the second period of ice hockey is remarkably stable over time. For example, the sinusoidal shape in Figure 4 occurs for every single NHL season in our sample.

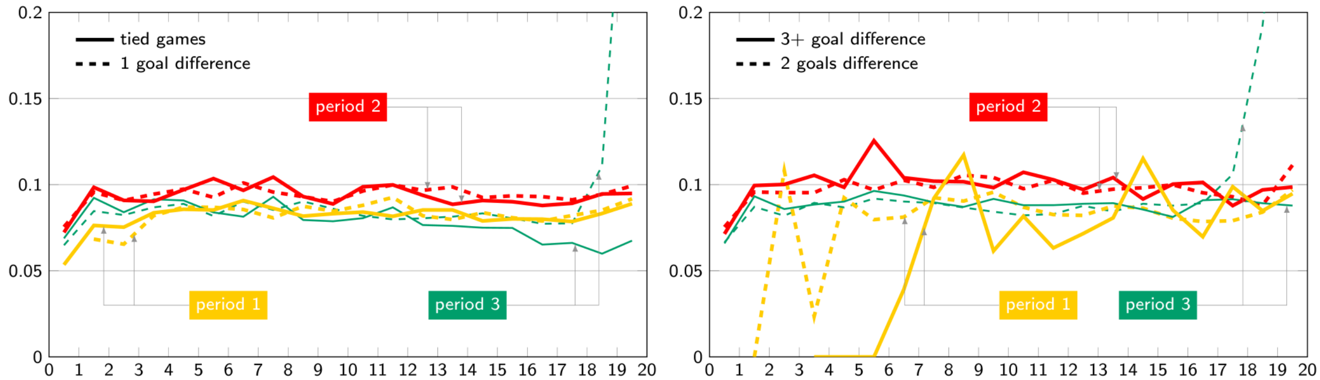
Figure 4: Cumulative deviation from average scoring in ice hockey (solid blue line), soccer (dashed green line) and rugby (chain red line).



Notes. Final minutes were excluded for reasons discussed In Section 1, specifically 59’-60’ in hockey, 45’ and 90’ in soccer, and 39’-40’ and 79’-80’ in rugby.

One possible explanation of the differences among the sports could be distinct degrees of competitive balance, i.e. differences in the distribution of the quality of teams within a competition. It is conceivable that a less-competitively balanced sport has more games effectively decided earlier in the game timeline, and thus the players of both teams could – in principle – reduce their effort levels. This could under some circumstances (defense being less effort-intensive than attack) lead to a reduction in scoring in the third period. However, that does not seem to be the case, as the increased scoring probability in ice hockey's second period occurs irrespective of prior scoring developments. To demonstrate this claim visually (in addition to its formal treatment in our Models 2 and 3), Figure 5 plots four types of games based on the goal difference (0, 1, 2, or 3+) at given minutes. It shows that all four types of games feature an increased probability of scoring in the second period.[15]

Figure 5: Average number of goals in each minute of each period for alternative scoring differences.



Notes. The yellow, red and green colors report periods 1, 2, and 3 respectively. The solid lines report games with goal differences of 0 (left panel) and 3 or more goals (right); the dashed lines show goal differences of 1 (left) and 2 (right).

The figure also brings to light two auxiliary results worth mentioning. First, the substantial increase in scoring in the last two minutes of the game is due purely to the team trailing by one goal (or possibly two) taking the risk and pulling the goalie. Specifically, in games featuring a one-goal difference with two minutes remaining, the number of goals increases 1.4 times in 59' and 4.6 times in 60' compared to the average number of goals in 57'-58'.

Second, Figure 5 reveals that the number of goals in tied games decreases in the final nine minutes of the third period, and particularly so in the final five minutes. Part of this scoring decline in drawn games may be due the players' "loss aversion" (a phenomenon extensively studied in behavioral

[15] In addition to these indirect pieces of evidence, a possible way to consider the direct effect of existing sides/bench locations on the players' behavior is through the differences in the forwards' and defenders' shift-lengths across the periods. Our explanation of the second-period scoring surge implies that in the second period, defenders may sometimes find it harder to substitute on the fly and may thus have a longer shift than in the other two periods. Knowing this, however, they may try to substitute at an earlier opportunity than usual, which may offset this effect. Therefore, while the joint effect on the average duration of the defenders' second-period shift is ambiguous, the effect on the variance is not. In particular, the variance of the length-shift should be higher for defenders in the second period than in the first and third periods, and vice-versa for forwards. Unfortunately, per-period data on shift-length that would enable us to test our conjecture are not readily available.

economics; for an illustration, see Pope and Schweitzer, 2011).[16] Nevertheless, as Banerjee et al. (2007) and many others have argued, the decline is primarily a consequence of the existing points structure in the NHL, which differs from other elite ice hockey leagues. Specifically, in the NHL the winning team always receives two points; whereas the losing team receives zero points after the 60 minutes regulation time, but one point if the game is drawn and the loss occurs subsequently in overtime or a penalty shootout. The fact that aggregate points increase from two to three following a regulation time draw unsurprisingly leads to more cautious play in the second half of period 3 in games that are level. This is not the case in European ice hockey leagues, where three points (rather than two) are awarded for a regulation time win.

3 Estimates of the Effect of Changes in Sides/Bench Positions on Goal Scoring

The previous section implies two alternative changes to the NHL's operations that could be made in pursuit of its objective to increase the amount of attacking play and promote scoring. We call them Starting Long and Always Long – see the depiction of both in Figure 1. Presuming our explanation of ice hockey scoring dynamics in Section 2 is correct, we estimate the effect of both reform proposals using three empirical setups. In the benchmark specification (Model 1 of Section 3.1) we construct a simple ANOVA-type model combined with a linear regression. This allows for distinct levels of scoring across the periods as well as for a linear change of scoring within each period. The model uses per-minute scoring data and removes both the "idiosyncratic" first four and the last two minutes of each period.[17] We have estimated an additive as well as a multiplicative (Poisson) version of this model.

In our second specification (Model 2 of Section 3.2), we construct a regression model that estimates the average number of goals in each minute of the game including two important factors, namely whether it is a regular season game or a playoff game, and a measure of competitive balance using the bookmakers' odds. In addition, the different scoring dynamics of the beginning and end of each period is controlled for formally. Our final specification (Model 3 of Section 3.3) offers an even more

[16] Intuitively, the teams of a tied game are more concerned about not losing the one point that a draw currently guarantees them than about not securing the additional point for winning the game in regulation time.

[17] The latter exclusion is to remove the high-scoring effect of the goaltender's substitution and empty nets, the former is to remove the low scoring at the start of each period (largely due to the initial face-off occurring in the middle of the ice rink and initial caution during each line's first shift). Both effects are exhibited clearly in Figures 3 and 5.

refined perspective on the start and finish of each period, exploiting exact scoring times in seconds, rather than in minutes.

3.1 Model 1: ANOVA-type model

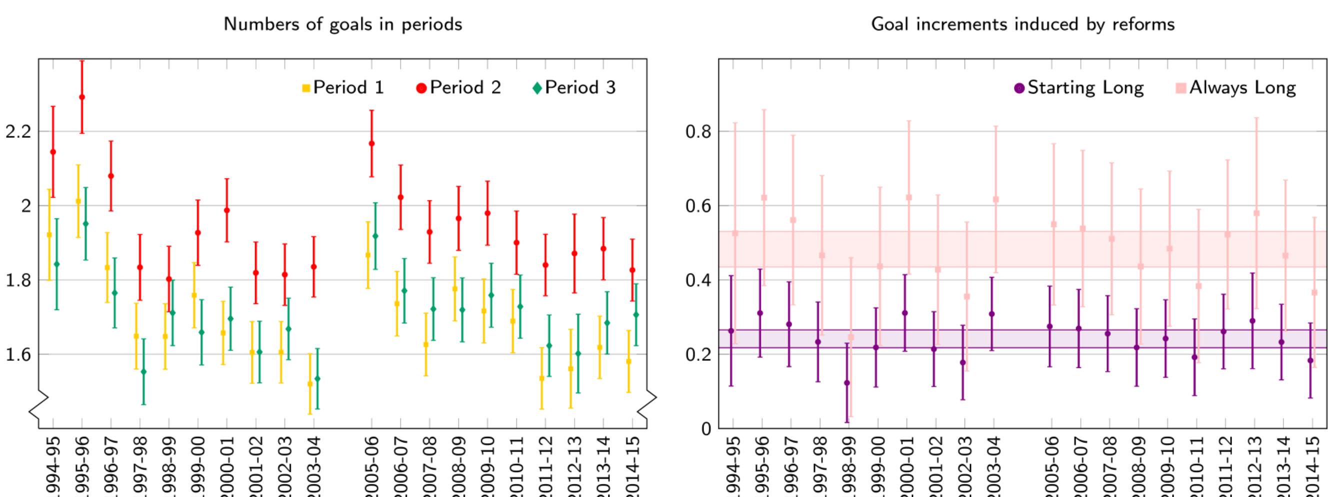
The additive specification assumes that the expected number of goals scored in the t-th minute, t=1,2,…,20, of period P (denoted GM)[18] can be written as:

$$GM(P,t) = \mu_P + \beta t, \quad (1)$$

where μ_P expresses the number of goals in period P and β is a linear scoring trend within each period. Figures 3 and 5 show that while assuming a linear trend may not be realistic for any entire period, it becomes plausible when the starting and final minutes of each period are removed. To this end, we disregard the initial four and the final two minutes of each period in this specification.[19]

Figure 6 shows results of the ANOVA-type model (1) applied successively to each season. Although average scoring varies across the seasons, a clear pattern emerges: the number of goals is higher in period 2 than in the remaining periods.

Figure 6: Estimates from Model 1 graphically.



Notes. The predicted number of goals per period by season (left) and their increases under the alternative proposals (right). The dots show the means, the lines are the 95% confidence intervals (period 1 is yellow, period 2 red, and period 3 green). Starting Long is in purple and Always Long is in pink.

[18] Throughout the paper we consider goals scored at time 0:00-0:59 to be scored in the 1st minute, i.e., t=1, etc.

[19] The multiplicative specification we have estimated is of the form: $GM(P,t) = \exp(\mu_P + \beta t)$, where the interpretation of the coefficients differs somewhat (for Poisson-type modeling of NHL scoring, see Buttrey et al., 2011). As the results are very similar to the additive specification, we do not report them here, but they are available upon request.

Statistically, in all seasons except 1998-99, the values of μ_P cannot be regarded equal – the hypothesis $\mu_1 = \mu_2 = \mu_3$ is rejected by an F-test with p-values of less than 0.001 for all of these seasons. Furthermore, in all seasons except 2014-15, the hypothesis of equality of μ_1 and μ_3 cannot be rejected at the 5% significance level. In terms of the slope coefficient β, it is significantly different from zero (at the 5% level) in nine out of the 20 seasons and in all of these cases the estimates are negative.

The estimated effects of the Starting Long and Always Long reforms are depicted graphically in the right panel of Figure 6, and summarized in Table 1. Essentially, the model calculates the difference between the number of goals scored in period 2 and in periods 1 or 3, whereby the former can be written as: $20\left(\mu_2 - \frac{\mu_1+\mu_3}{2}\right)$.

Table 1: Scoring predictions from Model 1.

Number of Goals	Period 1	Period 2	Period 3	Game Total
Actual NHL scoring	1.64	1.90	1.85	5.39
Model 1 predicted scoring under *Starting Long*	1.88 (+14.73%)	1.66 (-12.67%)	2.09 (+13.04%)	5.63 (+4.48%)
Model 1 predicted scoring under *Always Long*	1.88 (+14.73%)	1.90 (+0.00%)	2.09 (+13.04%)	5.87 (+8.95%)

Table 1 shows that scoring increases in periods 1 and 3 under the proposed reform by 14.73% and 13.04%, respectively.[20] It should be stressed, however, that the simplicity of this approach is likely to come at the cost of not accounting accurately for the effects at the start and end of each period. This is rectified in the remaining two specifications.

3.2 Model 2: Per-Minute Regression

The approach in this section will enable us to better deal with the scoring dynamics at the beginning and end of each period, as well as with the effect of additional factors. In contrast to the ANOVA-type model, we now include the goals in the starting and final minutes, too. Table 2 describes the variables included in the regression, whereby capital letters *P* and *M* stand for period and minute, respectively.

[20] For comparable estimates using back-of-the-envelope calculations, see Willis (2014).

Table 2: Definitions of variables and summary statistics from Model 2.

Variable	Variable description	Values	Mean	Standard deviation
GM	Dependent variable: the number of goals scored in each *minute* of the game	0-3	0.089	0.293
tM	Time within a period in minutes	1-20	10.500	5.766
P2	Period 2	0/1	0.333	0.471
P3	Period 3	0/1	0.333	0.471
M1	1st minute of a period	0/1	0.050	0.218
M2	2nd minute of a period	0/1	0.050	0.218
M3	3rd minute of a period	0/1	0.050	0.218
M19_P12	19th minute of periods 1 and 2	0/1	0.033	0.180
M20_P12	20th minute of periods 1 and 2	0/1	0.033	0.180
M19_P3	19th minute of period 3	0/1	0.017	0.128
M20_P3	The last minute of the game	0/1	0.017	0.128
PLAYOFF	A playoff game as opposed to a regular-season game	0/1	0.067	0.250
UNBAL	Measure of the teams' unbalancedness as per equation (2)	0-1.53	0.323	0.264

Our dependent variable, *GM*, is the number of goals scored in each minute of the game, whereas our key independent variable is *P2*, capturing the second-period scoring surge due to the sides/bench positions favoring attack over defense.

In addition to time dummies designed to control for the irregularities in the initial and final minutes of each period, the *PLAYOFF* dummy filters out possible scoring differences between the knock-out stage and the regular season (apparent in Figure 2). Finally, *UNBAL* is a measure of the teams' relative strength, which is derived from the decimalized bookmakers' odds as follows:

$$UNBAL = \, absolute \; value \left(ln \left(\frac{probability \; of \; team \; 1 \; "win"}{probability \; of \; team \; 2 \; "win"} \right) \right). \quad (2)$$

A higher *UNBAL* value occurs if there are less evenly-matched teams for which the respective probabilities of 'winning' (captured by the odds) are more disparate.[21] The estimates of Model 2 are reported in Table 3.

[21] Model 2 uses a slightly smaller sample compared to the other models; 4,620 games had to be excluded as they did not have odds data available to calculate the *UNBAL* variable. The number of observations in this model is therefore 1,192,860; that is, each of the 60 minutes of 19,878 games.

Table 3: Estimates from Model 2.

Variable	Estimate	Standard Error	P-Value
CONSTANT	0.08734	0.00099	<1E-6
tM	-0.00052	0.00007	<1E-6
P2	0.01328	0.00066	<1E-6
P3	0.00281	0.00063	0.00004
M1	-0.02736	0.00143	<1E-6
M2	-0.00578	0.00139	0.00003
M3	-0.00840	0.00136	<1E-6
M19_P12	0.00503	0.00162	0.00186
M20_P12	0.00879	0.00164	<1E-6
M19_P3	0.03218	0.00221	<1E-6
M20_P3	0.13912	0.00223	<1E-6
PLAYOFF	-0.00854	0.00107	<1E-6
UNBAL	0.00387	0.00102	0.00014

These estimates are in line with the findings of Model 1. Most importantly, we observe a statistically-significant increase in scoring in period 2 (to obtain the total second-period scoring surge, the estimated *P2* parameter must be multiplied by 20). The effect is fairly large quantitatively, e.g. it is 55.6% greater in magnitude than the well-known reduction in scoring in playoff games observed almost every season – see Figure 2. Regarding the latter effect, our estimates suggest, for example, that for evenly-matched opponents there are 10.65% more goals in the NHL's regular season than in playoff games.[22]

As expected, *UNBAL* is positive, implying that matches between evenly-matched teams tend to produce fewer goals. For example, we estimate that in a match featuring a 2:1 favourite (i.e. if predicted winning probabilities of the teams from bookmakers' odds are 2/3 and 1/3), there are 3.02% more goals than if the teams are evenly-matched (i.e. their winning probabilities are 1/2 and 1/2). Substantiating our reform proposals, note that the second-period scoring surge is 243.6% greater than this phenomenon. Finally, the lower (higher) scoring in the initial (final) minutes of each period is also statistically significant, and so is the slight downward scoring trend in the middle stages of each period.

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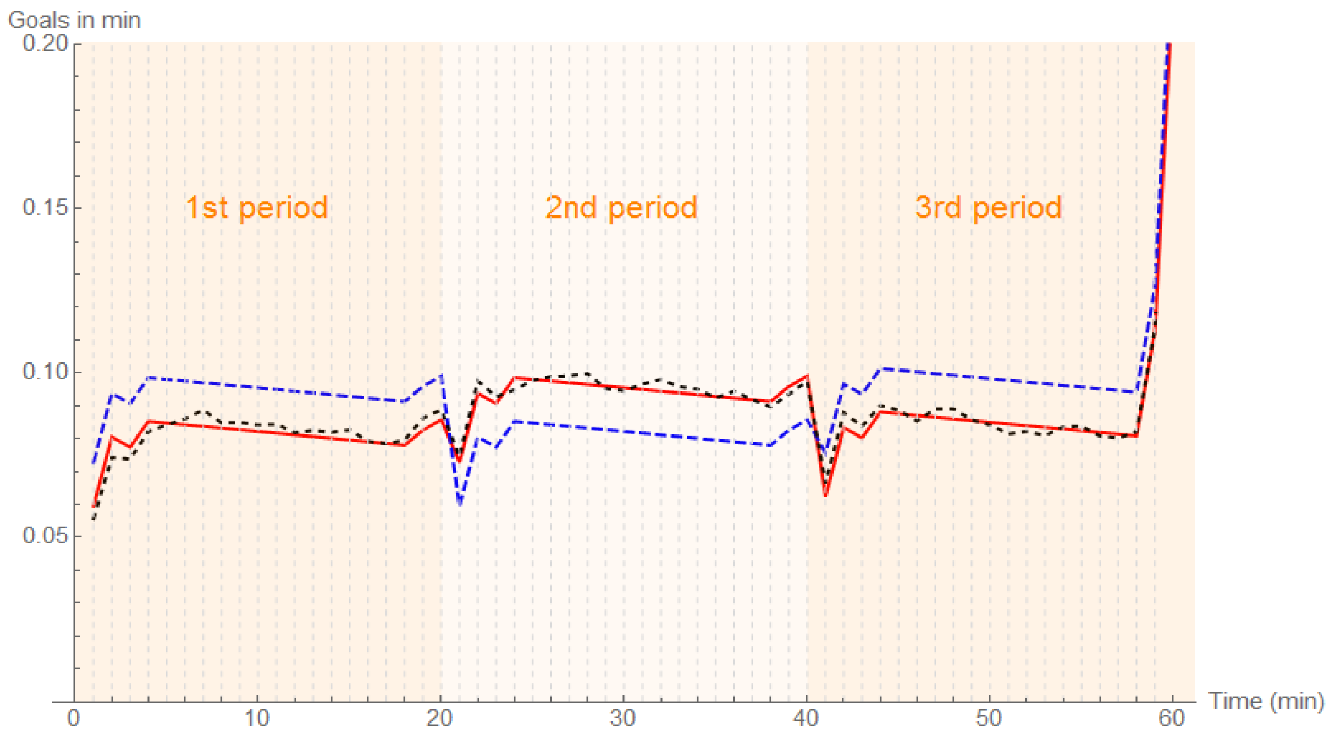
[22] Similarly, in our Super Rugby sample, aggregate scoring falls from an average of 48.43 points per match in regular-season games to 44.41 points per game in playoff matches.

Table 4 reports the number of goals implied by Model 2 and the marginal effect of the two possible operational reforms. The table is supplemented by Figure 7 in which the results are presented graphically.

Table 4: Scoring predictions from Model 2.

Number of Goals	Period 1	Period 2	Period 3	Game Total
Actual NHL scoring	1.62	1.89	1.84	5.35
Model 2 predicted scoring under the status quo	1.61	1.88	1.84	5.33
Model 2 predicted scoring under *Starting Long*	1.88 (+16.50%)	1.61 (-14.16%)	2.10 (+14.46%)	5.59 (+4.99%)
Model 2 predicted scoring under *Always Long*	1.88 (+16.50%)	1.89 (+0.00%)	2.10 (+14.46%)	5.86 (+9.98%)

Figure 7: Per-minute scoring implied by Model 2 under the status quo and considered alternatives.



Notes. The black dotted line shows actual scoring, the red solid line shows estimated levels in Model 2 under the status quo, and the blue dashed line shows predicted scoring in Model 2 under Starting Long.

Table 4 shows that our estimates are almost identical to those from real data, which gives credibility to our empirical approach. The estimates imply that both reforms would raise the number of goals scored in period 1 by 16.50% and in period 3 by 14.46% (assuming no change in scoring in the final minute). Under the more logistically appealing Starting Long, a predicted 14.16% drop in second period scoring would occur, so overall scoring would rise by 4.99% for the game. Under Always Long, the overall increase in scoring would be 9.98%.

3.3 Model 3: Per-Second Regression

The final specification takes advantage of the fact that our dataset includes exact scoring times in seconds. The dependent variable in this model therefore becomes the number of goals in each second. However, we face computational limitations since the dataset consists of approximately 88 million data points (whether a goal was scored in one of the 3,600 seconds of the game for each of the 24,498 games). For that reason, we cannot include the *PLAYOFF* and *UNBAL* variables featured in Model 2. Nevertheless, we extend the earlier specification by allowing for a distinct scoring trend in the middle section of each period and a more nuanced treatment of the initial and final sections.

Naturally, in the per-second specification the number of possible models is very large as one could consider a wide range of the initial and final seconds of each period to control for. We use a simple model selection approach based on the AIC measure to identify the relevant time intervals for inclusion, gradually eliminating the initial and final intervals of five seconds' length. These are the variables *SX-Y*, *SX-Y_P12* and *SX-Y_P3* reported in Table 5 – together with the other variables included in Model 3. The estimated values of all the parameters of our optimal model are then presented in Table 6.

Table 5: Definitions of variables and summary statistics from Model 3.

Variable	Variable description	Values
GS	Dependent variable: the average number of goals scored in each *second* of the game	
tPS	Time within period *i* in seconds (centered to the middle of the period, e.g. *tP2S=0* at time 10:00)	-420-580 (P1/2) -420-470 (P3)
P2	Period 2	0/1
P3	Period 3	0/1
SX-Y	Time interval [X,Y] within each period	0/1
SX-Y_P12	Time interval [X,Y] within periods 1 and 2	0/1
SX-Y_P3	Time interval [X,Y] within period 3	0/1

Table 6: Estimates from Model 3.

Variable	Estimate	Standard Error	P-Value
CONSTANT	0.00139	0.00001	<1E-6
tP2S	0.0000001	<1E-6	0.00011
tP3S	0.0000002	<1E-6	<1E-6
P2	0.00023	0.00001	<1E-6
P3	0.00005	0.00001	0.00002
S0-10	-0.00150	0.00005	<1E-6
S10-20	-0.00062	0.00005	<1E-6
S20-180	-0.00007	0.00001	<1E-6
S1180-1190_P12	0.00014	0.00006	0.02321
S1190-1200_P12	0.00050	0.00006	<1E-6
S1070-1110_P3	0.00016	0.00004	0.00020
S1110-1140_P3	0.00083	0.00005	<1E-6
S1140-1150_P3	0.00157	0.00009	<1E-6
S1150-1160_P3	0.00189	0.00009	<1E-6
S1160-1170_P3	0.00218	0.00009	<1E-6
S1170-1180_P3	0.00228	0.00009	<1E-6
S1180-1195_P3	0.00251	0.00007	<1E-6
S1195-1200_P3	0.00333	0.00012	<1E-6

The reported results are consistent with those from Models 1 and 2. There is again a statistically-significant scoring surge in period 2, and it is – in fact – slightly larger.[23] Table 7 offers the number of goals as implied by Model 3, showing that our estimates are virtually identical to the raw data.

Table 7: Scoring predictions of Model 3.

Number of Goals	Period 1	Period 2	Period 3	Game Total
Actual NHL scoring[24]	1.64	1.90	1.85	5.39
Model 3 predicted scoring under the status quo	1.64	1.90	1.85	5.39
Model 3 predicted scoring under *Starting Long*	1.91 (+16.71%)	1.63 (-14.36%)	2.12 (+14.78%)	5.66 (+5.09%)
Model 3 predicted scoring under *Always Long*	1.91 (+16.71%)	1.90 (0.00%)	2.12 (+14.78%)	5.94 (+10.16%)

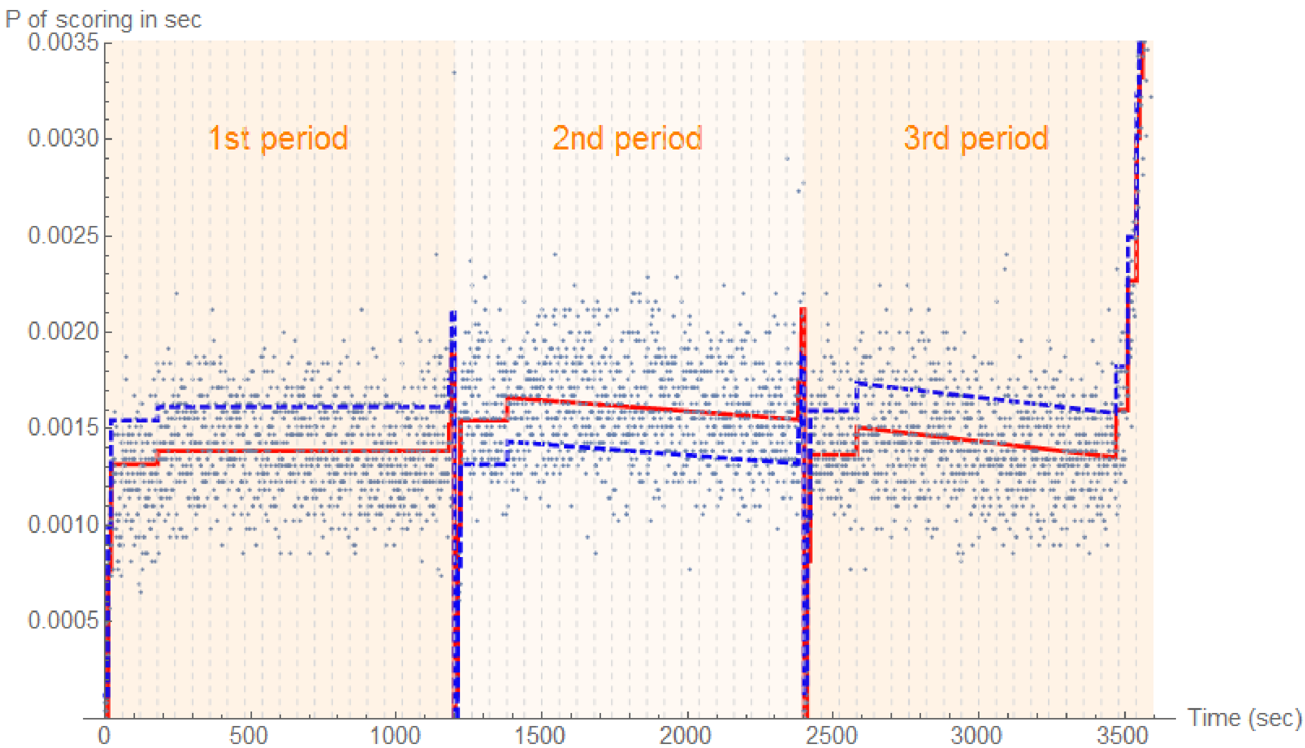
The table shows that the proposed Starting Long reform is predicted to increase the number of goals scored in period 2 by 16.71% and in period 3 by 14.78%. The predicted 14.36% decline in period 2

[23] The values of (per-second) parameters within Model 3 can be compared to the (per-minute) estimates of Model 2 when multiplied by 60.

[24] Note that the actual scoring reported in Table 7 differs slightly from its counterpart in Table 4 due to the somewhat larger sample.

scoring under Starting Long thus implies an overall scoring increase of 5.09% in this setup, whereas Always Long would see the total number of goals (over the full course of the game) increase by 10.16%. The findings of Table 7 are plotted in Figure 8 for greater clarity.

Figure 8: Per-second scoring implied by Model 3 under the status quo and considered alternatives.



Notes. The black dotted line shows actual scoring, the red solid line shows estimated levels in Model 3 under the status quo, and the blue dashed line shows predicted scoring in Model 3 under Starting Long.

Table 6 and Figure 8 also reveal some auxiliary results. The decreasing scoring dynamics within periods is insignificant in period 1, and it is weaker in period 2 than in period 3. The reasons for the latter, primarily the NHL points system, were discussed earlier.

4 Summary and Conclusions

The world's oldest and most prestigious ice hockey league, the NHL, has seen a declining trend in goal scoring since the early 1980s. In an attempt to ensure the game's attractiveness and withstand competition from other major sports, the League's authorities have been searching for ways to boost scoring (see e.g. NHL.com, 2015 or Gretz, 2015). This paper considers two alternative operational changes in this respect relating to the physical location of teams' on-ice sides and off-ice benches. These proposals would alter player behaviour towards more attack primarily through direct logistic means making it relatively easier (harder) for forwards (defenders) to substitute during the action. Hence, they are firmly within the realm of operations research rather than economics, which focuses on the indirect effects through altering incentives for players (see e.g. McCannon, 2011).[25]

We consider two types of reforms, which we call: (i) *Starting Long*; and (ii) *Always Long*. In each of our three model specifications (ANOVA-type, per-minute regression and per-second regression), the estimated impacts of the two proposals were found to be significant. This is both from the statistical point of view and from the policy point of view. Both proposals imply an increase in scoring in periods 1 and 3 by around 16.5% and 14.5% respectively, whereby under proposal (i) there would also be a scoring decline of around 14.0% in period 2. For the NHL as a whole, reforms (i) and (ii) would translate, respectively, into approximately 350 and 700 extra goals every season, using the averages of all our variables (including the odds). These estimates were shown also to be highly robust, evidenced by the similarity between the three analyses.

We acknowledge that our examination does not capture how the effect of the sides/bench swap would combine with other effects influencing scoring dynamics such as "scoring momentum", loss aversion, power plays, or skater fatigue. For example, it is likely that higher scoring in period 1 could create a momentum and greater scoring in the remainder of the game (as reported for soccer by Lenten et al., 2013). In such a case, our results could be interpreted as lower-bound estimates of the actual effects.

On the other hand, accounting for the fact that the number of penalty calls tends to be higher in period 2 than in period 3 (referees are more hesitant to influence the end of the game) could, in principle, reduce our estimate. This effect is arguably minor; even if we assume that penalty calls are independent of the sides/bench position.

A rough calculation is as follows. Lopez and Snyder (2013) report a difference of 0.50 between the mean number of penalty calls in periods 2 and 3 in the regular season, and the average number of 7.88 penalties in a game (assuming the number of penalty calls in period 1 is comparable to period 3). Using the proportion 21.9% of uneven strength minutes in the game from Buttrey' et al. (2011), the

[25] There also exist numerous parallels of this study to Hurley (2009), which similarly uses operations research techniques to revise group allocation methods to optimize an objective – fairness of competition according to athlete birthdate. For a a comprehensive survey of the operations research literature relating to sport, see Wright (2009).

duration of power-play per penalty is 0.219 x 60 (minutes) / 7.88 = 1.67 minutes (i.e. many penalty calls overlap so the teams end up playing with even strength). We can then use Buttrey's et al. (2011) who report a difference in goal rates of 2.61 goals per 60 minutes between 5-on-4 and 5-on-5 situations. Assuming that penalty calls are not affected by the scoring dynamics, the increase in the number of goals due to 0.50 more power-plays in period 2 would be approximately (0.50 x 1.67 x 2.61)/60 = 0.0363 goals.

Put differently, under these circumstances the penalty calls dynamics across periods would explain only about 13.5% of the second period scoring increase. And even less if we take into account the fact that a (potentially large) part of this increase is endogenous to the sides/bench positions we examine. In particular, the higher number of penalty calls in the second period may be due to the long change, whereby defenders commit more fouls as a consequence of a more challenging substitution. In such case our estimates above would be largely unaffected.

Other possible unaccounted for factors could include the interaction between the size of the rink and defensive versus attacking strategies of the teams. For example, the European rink is larger than the North American, which seems to affect substitutions and thus scoring.[26]

In addition, it is important to carefully assess the possible costs of the proposed changes. The Starting Long reform does not incur any major logistic costs. The starting sides can be changed at no cost and most ice hockey arenas are fairly symmetric in regards to the seating disposition in case some fans prefer to shift sides following the reform.[27] There are nevertheless two possible non-logistic type disadvantages of this proposal that need to be considered, apart from the possible adjustment cost on the players.

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[26] Specifically, in many European leagues, the three forwards about to come onto the ice sit (in all periods) closer to the opponent's goal, whereas in the NHL they sit closer to their own goal. This may reflect a somewhat greater relative focus on defense in the NHL, and in principle, could determine the magnitude of the effects of our proposed reforms across the elite leagues. We have been unable to obtain the data on how substitution strategies depend upon the time remaining and the score, and what the proportion of substitutions on the fly is. In principle, the latter could differ across the periods, and/or change as a consequence of players executing more enforced stoppages under the long change. These considerations could thus have some quantitative effect on scoring dynamics under the proposed reforms.

[27] In many European arenas even a bench swap would be straightforward as both teams enter the benches through the same corridor in-between the two benches.

First, having a long change in period 3 makes it slightly harder for the goaltender to substitute on the fly in the final minute(s) of the game. The second drawback may be related to overtime. Should the sides of tied games be switched before overtime, as has been the case since the 2014-15 season, having a short change could partly reduce the number of goals in overtime, and increase the proportion of games decided by penalty shootouts. This could partly offset the increase in overtime scoring due to the 2015-16 move from 4-on-4 to 3-on-3 skaters, see Vollman (2015). Arguably, this potential downside is relatively minor as it only applies to the minority of games that are not decided in regulation time.[28] Furthermore, the rules could stipulate that sides are not switched before overtime, which (in fact) was the case until the 2013-14 season.

The Always Long proposal would be associated with some (possibly greater) drawbacks. Switching sides is standard practice in sports involving goals at opposite ends that have multiple periods. While the main reasons, such as asymmetric weather conditions, are generally absent in ice hockey due to roofed stadiums, not switching sides may still be perceived as problematic for perceptions of integrity and fairness. Further, some fans sitting behind the goals may prefer to see both teams attacking from a close range (although those who prefer to only see their team attacking would welcome this reform). The variant of the proposal in which benches are switched every period circumvents these drawbacks, but would be associated with the logistic inconvenience of having to move all equipment on the benches twice during the game, which seems impractical.

In summary, Starting Long appears the favorable reform in terms of cost-benefit considerations. It is likely to enhance social welfare in professional ice hockey via greater fan interest and ultimately result in higher League revenues. This is also because the proposal does not risk common unintended consequences of rule changes, such as the limited effectiveness of soccer's three-point-for-a-win rule (Mehrez and Pliskin, 1987), or Major League Baseball's Designated Hitter rule (Bradbury and Drinen, 2006).[29] As such, the reform seems more appealing in comparison to alternative proposals to

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[28] Let us mention that both proposals are likely to slightly decrease the proportion of games going to overtime. We have estimated that the probability of the game finishing in a draw after regulation time would decline by about 0.6-0.7 percentage points under Starting Long and 1.3 percentage points under Always Long. These estimates were obtained by simulations of 100,000 games, assuming the second-period surge estimate from Model 1.

[29] For a comprehensive discussion of case studies from sports involving perverse unintended consequences, see Kendall and Lenten (2017).

stimulate goal scoring in ice hockey, for example, moving the icing line back or increasing goal size. There is, therefore, a compelling case to try the Starting Long proposal in the NHL, or at least initially in a lower-tier ice hockey competition.

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