Propagation of plane waves in a rotating magneto-thermoelastic fiber-reinforced medium under G-N theory

N. Maity[a], S. P. Barik[b,*], P. K. Chaudhuri[c]

[a]*Department of Mathematics, Nabagram K D Paul Vidyalaya, 27 G. T. Road, Serampore – 712 203, India*
[b]*Department of Mathematics, Gobardanga Hindu College, Khantura, 24-Parganas (N), West Bengal, India*
[c]*Department of Applied Mathematics, University of Calcutta, 92, A. P. C. Road, Kolkata – 700 009, India*

Abstract

The article is concerned with the possibility of plane wave propagation in a rotating elastic medium under the action of magnetic and thermal fields. The material is assumed to be fibre-reinforced with increased stiffness, strength and load bearing capacity. Green and Nagdhi's concepts of generalized thermoelastic models II and III have been followed in the governing equations expressed in tensor notation. The effects of various parameters of the applied fields on the plane wave velocity have been shown graphically.
© 2017 University of West Bohemia. All rights reserved.

Keywords: fibre-reinforced media, plane wave, magnetic permeability and electric conductivity, Green-Nagdhi models II and III

1. Introduction

The concept of fibre-reinforced solids stems from the necessity of generating more strength, stiffness and load bearing capacity of solids through the process of suitably mixing certain kind of materials with such solids. In situations where light material with high strength and stiffness is essential, as for example, in manufacturing aircraft components, fibre-reinforced material can be a good option. Fibre-reinforcing is not any new idea. In fact, fibre-reinforcing in its crude form was a common practice followed in hardening soil by mixing straw dust or rice husk with the soil in early stages of civilization and is still followed in some rural areas. Another very common instance is the use of iron wire/rods in manufacturing concrete structures. With the advancement of technology and innovation of modern methods different kinds of materials are manufactured as per need using fibre-reinforcing concept. Different methods are adopted in fibre-reinforcing process, viz., long fibres are arranged in parallel in the matrix, short fibres are randomly arranged in the matrix, or fibres in the form of cloth are set in the matrix. Obviously, the strength and the load bearing capacity of the fibre-reinforced material will depend upon the fibre arrangement in the matrix.

In the study of deformation in solids great care and attention are necessary for bodies subjected to flow or generation of heat in it due to the fact that thermal loading causes deformation different from that caused by mechanical loading. Thermoelasticity takes care of the deformations and stresses produced by thermal loadings as well as those produced by mechanical loadings. The equations governing thermoelastic behavior consist of the modified stress strain relations, the stress equations of motion and the heat conduction equation. But the parabolic type of heat conduction equation as was used initially in the study of thermoelastic behavior was

---

*Corresponding author. Tel.: +91 94 33 470 465, e-mail: spbarik1@gmail.com.

found to yield some unrealistic situation in the sense that the velocity of heat wave propagation was infinite. Modified version of the Fourier law of heat conduction as was suggested by Lord and Shulman [15] in 1967 yielded a finite velocity of heat wave propagation by introducing relaxation time into the energy equation, rendering the new thermoelastic field equations fully hyperbolic. In 1972 Green and Lindsay [12] proposed a theory of thermoelasticity keeping the Fourier law of heat conduction unchanged and modifying the classical energy equation and stress-strain-temperature relations. Investigation of various problems characterizing the two theories based on totally independent ideas, have been discussed by Chandrasekharaiah [5, 6]. Modifications of the constitutive equations of thermoelasticity done afterwards by Green and Nagdhi [12, 13] paved the way to accommodate a wider class of heat flow problems. In G-N model II constitutive equations are derived using thermal displacement gradient among the constitutive variables and reduced energy equation. The model considers no energy dissipation. G-N model III is a modification of G-N model II in the sense that the temperature gradient has also been considered in the constitutive equation in addition to those considered in model II. This theory allows energy dissipation and permits the propagation of thermal signals at both finite and infinite speeds.

Study of the effects of magnetic field on thermoelastic medium is another area of interest. The field of study known as magneto thermoelasticity has applications in several areas, particularly in nuclear devices, biomedical engineering and geomagnetic investigations. Some of the works related to this field are Abd-Alla and Al-Dawy [1, 2], Ezzat and Othman [10], Ezzat [9], Ezzat et al. [11], Wang et al. [25], Othman and Song [22], Othman [19], Othman and Said [23]. A number of discussions relating wave propagation in rotating isotropic or anisotropic media were reported in literature, some of which are the works of RoyChoudhuri [7], Gupta and Gupta [14], Singh [24], Maity et al. [16, 17]. Roychoudhuri and Banerjee [8] studied the propagation of time-harmonic coupled electromagnetoelastic dilatational thermal shear waves using the thermoelasticity theory of type II (G-N model) [13]. Thermoelastic plane waves in a rotating isotropic medium has been studied by Ahmad and Khan [3]. A number of discussion relating to fibre-reinforced materials were discussed by Belfield [4], Othman [20], Othman [21], Markham [18], Zorammuana [26].

The present discussion aims at the study of the propagation of plane waves in a rotating thermoelastic fiber-reinforced medium with and without energy dissipation under Green-Nagdhi model. A magnetic field of uniform magnitude is supposed to be acting on the medium but there is no body force. Fiber-reinforcing of general type has been considered and the governing equations of motion are framed taking into account of the thermoelastic characteristics of the material, rotational effects and the applied magnetic field. Equations have been presented using tensor notations. Possibilities of plane wave propagation in the medium have been studied in this discussion. Effects of rotation, applied magnetic field, and temperature of the material on plane wave propagation have been examined severally and jointly. Finally, some graphical presentations have been made to assess the effects of various parameters in the plane wave propagation in fiber-reinforced media of different nature.

2. Field equations

Following Belfield et al. [4] the stress-strain relations for linearly fibre-reinforced elastic medium may be expressed in tensor form as

$$
\begin{aligned}
\tau_{ij} = {} & \lambda\epsilon_{kk}\delta_{ij} + 2\mu_T\epsilon_{ij} + \alpha^*(a_k a_m \epsilon_{km}\delta_{ij} + a_i a_j \epsilon_{kk}) + \\
& 2(\mu_L - \mu_T)(a_i a_k \epsilon_{kj} + a_j a_k \epsilon_{ki}) + \beta^*(a_k a_m a_i a_j \epsilon_{km}) - \nu T\delta_{ij},
\end{aligned} \tag{1}
$$

where τ_{ij} are the cartesian components of the stress tensor;

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

are the strain components, related to the displacement vector u_i; λ, μ_T are elastic constants; α^*, β^*, $(\mu_L - \mu_T)$ are reinforcing parameters; ν is a temperature parameter and $\mathbf{a} = (a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1$.

For a rotating elastic medium the equation of motion, in absence of body force, can be written as

$$\tau_{ij,j} = \rho[\ddot{u}_i + \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\}_i]. \tag{2}$$

In (2), ρ denotes the material density, $\mathbf{\Omega}$ is the angular velocity vector, overhead dot represents differentiation with respect to time and the suffix i after second bracket represent the ith component of the vector inside.

If, in addition, the solid is under the action of magnetic field $\mathbf{H}$, then the governing field equations involving the displacement $\mathbf{u} = u_i(\mathbf{x}, t)$ and the temperature $T(\mathbf{x}, t)$, for a fiber-reinforced material with rotation, in absence of body force, may be written as

$$(\lambda + \mu_T)u_{k,ki} + \mu_T u_{i,kk} + \alpha^*(a_k a_m u_{k,mi} + a_i a_j u_{k,kj}) +$$
$$(\mu_L - \mu_T)[a_i a_k u_{k,jj} + a_i a_k u_{j,kj} + a_j a_k(u_{k,ij} + u_{i,kj})] - \nu T_{,i} +$$
$$\beta^* a_i a_j a_k a_m u_{k,mj} + (\mathbf{J} \times \mathbf{B})_i = \rho[\ddot{u}_i + \{\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\}_i] \tag{3}$$

and

$$\alpha K \dot{T}_{,kk} + K^* T_{,kk} = \rho c_e \ddot{T} + \nu T^* \ddot{u}_{k,k}, \tag{4}$$

where $\alpha = 1$ and $\alpha = 0$ respectively correspond to heat conduction with and without energy dissipation. The term $\mathbf{J} \times \mathbf{B}$ in (3) arises from the presence of the applied magnetic field. Due to the application of the initially applied magnetic field $\mathbf{H}_0$, an induced magnetic field $\mathbf{h}$, an induced electric field $\mathbf{E}$ and a current density $\mathbf{J}$ are developed. For a slowly moving homogeneous electrically conducting medium, the simplified system of linear equations of electrodynamics are

$$\nabla \times \mathbf{h} = \mathbf{J} + \epsilon_0 \dot{\mathbf{E}},$$
$$\nabla \times \mathbf{E} = -\mu_0 \dot{\mathbf{h}},$$
$$\nabla \cdot \mathbf{h} = 0, \tag{5}$$
$$\mathbf{E} = -\dot{\mathbf{u}} \times \mathbf{B},$$

where ϵ_0 is the electrical conductivity and μ_0 is the magnetic permeability so that $\mathbf{B} = \mu_0 \mathbf{H}$ is the magnetic field in the medium due to total magnetic field $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$, arising from applied field $\mathbf{H}_0$ and induced field $\mathbf{h}$.

If we assume that $\mathbf{H}_0 = (H_{01}, H_{02}, H_{03})$ and $\mathbf{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$, then utilizing relations (5) and neglecting products of $\mathbf{h}$ with $\mathbf{u}$ and its derivatives, the governing equations of motion (3) and (4) for a medium in thermoelasticity with and without energy dissipation under the action of applying magnetic field and rotation may be written in tensor notation as

$$(\lambda + \mu_T)u_{k,ki} + \mu_T u_{i,kk} + \alpha^*(a_k a_m u_{k,mi} + a_i a_j u_{k,kj}) +$$
$$(\mu_L - \mu_T)[a_i a_k u_{k,jj} + a_i a_k u_{j,kj} + a_j a_k(u_{k,ij} + u_{i,kj})] +$$
$$\beta^* a_i a_j a_k a_m u_{k,mj} + \mu_0 H_0^2 u_{j,ji} - \mu_0 H_{0k} H_{0i} u_{j,jk} - \mu_0 H_{0m} H_{0k} u_{k,im} + \mu_0 H_{0m} H_{0k} u_{i,km} -$$
$$\mu_0^2 \epsilon_0 H_0^2 \ddot{u}_i + \mu_0^2 \epsilon_0 H_{0k} H_{0i} \ddot{u}_k - \nu T_{,i} = \rho(\ddot{u}_i + \Omega_k \Omega_i u_k - \Omega^2 u_i + 2\epsilon_{ijk}\Omega_j \dot{u}_k), \tag{6}$$
$$\alpha K \dot{T}_{,kk} + K^* T_{,kk} = \rho c_e \ddot{T} + \nu T^* \ddot{u}_{k,k}, \tag{7}$$

3

where T^* is the reference temperature, $\nu = (3\lambda + 2\mu)\alpha_t$, K is the coefficient of thermal conductivity, K^* is the additional material constant, ρ is the mass density, c_e is the specific heat of the solid at constant strain, α_t is the coefficient of linear thermal expansion, λ and μ are Lame' constants. In (6), ϵ_{ijk} represents the Levi-civita tensor which has a non-zero value only if i, j, k are all distinct and has a value of 1 if i, j, k are in cyclic order, whereas it has a value of -1 when they are acyclic.

3. Plane wave propagation

In order to examine the possibility of a plane wave propagation in the medium under consideration we shall assume a solution of governing equations (6) and (7) in the form

$$(u_i, T) = (Ap_i, B) \exp\{\tau(qn_sx_s - \omega t)\}, \quad i = 1, 2, 3, \quad \tau = \sqrt{-1}. \tag{8}$$

The speed of the wave is

$$c_n = \frac{\omega}{Re(q)}. \tag{9}$$

The direction of plane wave propagation is represented by the unit vector $\mathbf{n} = (n_1, n_2, n_3)$, while the direction of particle displacement is denoted by the unit vector $\mathbf{p} = (p_1, p_2, p_3)$. A and B appearing in (8) are constants.

Substituting (8) into (6) and (7), using $\tau^2 = -1$, $n_kn_k = 1$ and setting $B/A = D$, we get

$$\begin{aligned}
&(\lambda + \mu_T)p_kq^2n_in_k + \mu_Tp_iq^2 + \alpha^*(a_ka_mn_in_m + a_ia_jn_kn_j)p_kq^2 + \\
&\quad (\mu_L - \mu_T)[a_ia_kp_kq^2 + a_ia_lp_kq^2n_kn_l + a_ja_k(n_in_jp_kq^2 + n_kn_jp_iq^2)] + \\
&\quad \beta^*a_ia_ja_ka_mp_kq^2n_j + \mu_0H_0^2p_jq^2n_jn_i - \mu_0H_{0k}H_{0i}p_jq^2n_jn_k - \\
&\quad \mu_0H_{0m}H_{0k}p_kq^2n_in_m + \mu_0H_{0m}H_{0k}p_iq^2n_kn_m - \mu_0^2\epsilon_0H_0^2p_i\omega^2 + \\
&\mu_0^2\epsilon_0H_{0k}H_{0i}p_k\omega^2 - \nu D\tau qn_i = \rho[p_i\omega^2 - \Omega_k\Omega_ip_k + \Omega^2p_i + 2\epsilon_{ijk}\Omega_jp_k\tau\omega], \quad i = 1, 2, 3, \tag{10}
\end{aligned}$$

$$K^*q^2 - \rho c_e\omega^2 = D^{-1}\nu T^*\omega^2q\tau p_kn_k. \tag{11}$$

Eliminating D from (10) and (11) we get

$$\begin{aligned}
&(\lambda + \mu_T)p_kq^2n_in_k + \mu_Tp_iq^2 + \alpha^*(a_ka_mn_in_m + a_ia_jn_kn_j)p_kq^2 + \\
&\quad (\mu_L - \mu_T)[a_ia_kp_kq^2 + a_ia_lp_kq^2n_kn_l + a_ja_k(n_in_jp_kq^2 + n_kn_jp_iq^2)] + \\
&\quad \beta^*a_ia_ja_ka_mp_kq^2n_j + \mu_0H_0^2p_jq^2n_jn_i - \mu_0H_{0k}H_{0i}p_jq^2n_jn_k - \mu_0H_{0m}H_{0k}p_kq^2n_in_m + \\
&\quad \mu_0H_{0m}H_{0k}p_iq^2n_kn_m - \mu_0^2\epsilon_0H_0^2p_i\omega^2 + \mu_0^2\epsilon_0H_{0k}H_{0i}p_k\omega^2 + \\
&\frac{T^*\nu^2\omega^2q^2p_kn_kn_i}{\rho c_e\omega^2 - (K^* - K\tau\omega\alpha)q^2} = \rho[p_i\omega^2 - \Omega_k\Omega_ip_k + \Omega^2p_i + 2\epsilon_{ijk}\Omega_jp_k\omega], \quad i = 1, 2, 3. \tag{12}
\end{aligned}$$

Equation (12) is a set of three linear homogeneous equations in p_k of the form

$$p_k\eta_{ik} - p_i\theta = 0. \tag{13}$$

Also

$$\eta_{ik} = q^2[F_{ik} + M_{ik} + T_{ik}] + R_{ik}, \tag{14}$$

where

$$F_{ik} = (\lambda + \mu_T)n_i n_k + \alpha^*(a_k a_m n_i n_m + a_i a_j n_k n_j) +$$
$$(\mu_L - \mu_T)(a_i a_k + a_i a_l n_k n_l + a_j a_k n_i n_j) + \beta^* a_i a_j a_k a_m n_m n_j,$$
$$\theta = -[\mu_T + (\mu_L - \mu_T)a_j a_k n_k n_j] + \mu_0^2 \epsilon_0 \omega^2 H_0^2 - q^2 \mu_0 H_{0m} H_{0k} n_m n_k + \rho(\Omega^2 + \omega^2),$$
$$M_{ik} = \mu_0 H_0^2 n_i n_k - \mu_0 H_{0i} H_{0j} n_j n_k - \mu_0 H_{0m} H_{0k} n_i n_m + \frac{\mu_0^2 \epsilon_0 \omega^2 H_{0k} H_{0i}}{q^2},$$
$$R_{ik} = \rho \Omega_k \Omega_i - 2\rho \epsilon_{ijk} \omega_j \tau \Omega, \tag{15}$$
$$T_{ik} = \frac{T^* \nu^2 \omega^2 n_i n_k}{\rho c_e \omega^2 - (K^* - K\tau\omega\alpha)q^2}.$$

Rewriting (13) as

$$p_k(\eta_{ik} - \theta\delta_{ik}) = 0 \tag{16}$$

and noting that not all p_ks are zero, it follows that

$$|\eta_{ik} - \theta\delta_{ik}| = 0. \tag{17}$$

Equation (17) with a third order determinant on its left hand side yields an algebraic equation in q^2 with complex coefficients which will determine the wave speed c_n in (9). It is clear that the velocity of the plane wave propagation depends on the terms F_{ik} arising from the elastic behavior of the material and the direction n_i of propagation of the wave, the terms M_{ik} arising from the applied magnetic field, the terms R_{ik} arising from the rotation of the medium and the terms T_{ik} arising from the thermal character of the material.

4. Particular Case

Let us consider a fibre-reinforced elastic body with fibre-reinforcing direction $\mathbf{a} = (a_1, a_2, a_3)$ is rotating with uniform angular velocity $\mathbf{\Omega} = \Omega(0, 0, 1)$ (Fig. 1). Let us suppose that a uniform magnetic field $\mathbf{H}_0 = H_0(0, 1, 0)$ is applied to the body. We investigate propagation of a plane wave in the medium in a direction specified by the unit vector $\mathbf{n} = (0, n_2, n_3)$.
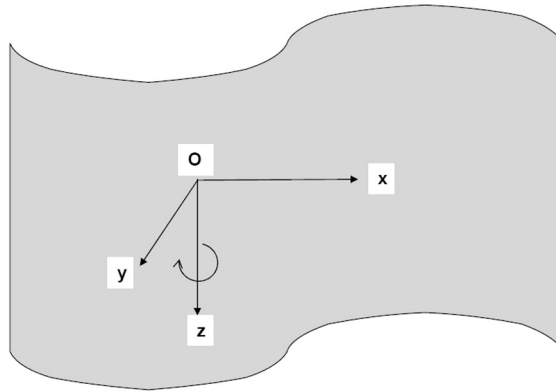

Fig. 1. Geometry of the problem

In this case we rewrite (16) in the form

$$p_k D_{ik} = 0, \quad i = 1, 2, 3, \tag{18}$$

where $D_{ik} = \eta_{ik} - \theta\delta_{ik}$.

Writing $x = q/\omega$, we find that, in this particular case

$$D_{11} = a_{11}x^2 - R, \qquad D_{12} = a_{12}x^2 + 2\rho\Gamma\tau, \qquad D_{13} = D_{31} = a_{13}x^2,$$
$$D_{21} = a_{12}x^2 - 2\rho\Gamma\tau, \qquad D_{22} = x^2[a_{22} + n_2^2\gamma(x^2)] - R,$$
$$D_{23} = D_{32} = x^2[a_{23} + n_2n_3\gamma(x^2)], \qquad D_{33} = x^2[a_{33} + n_3^2\gamma(x^2)] - R_1,$$

where

$$a_{11} = (\mu_L - \mu_T)a_1^2 + \beta^*a_1^2(a_2n_2 + a_3n_3)^2 + \mu_T,$$
$$a_{12} = (\alpha^* + \mu_L - \mu_T)a_1n_2(a_2n_2 + a_3n_3) + (\mu_L - \mu_T)a_1a_2 + \beta^*a_1a_2(a_2n_2 + a_3n_3)^2,$$
$$a_{13} = (\alpha^* + \mu_L - \mu_T)a_1n_3(a_2n_2 + a_3n_3) + (\mu_L - \mu_T)a_1a_3 + \beta^*a_1a_3(a_2n_2 + a_3n_3)^2,$$
$$a_{22} = (\lambda + \mu_T)n_2^2 + \{2\alpha^* + 3(\mu_L - \mu_T)\}a_2n_2(a_2n_2 + a_3n_3) + (\mu_L - \mu_T)a_2^2 +$$
$$\beta^*a_2^2(a_2n_2 + a_3n_3)^2 + \mu_T,$$
$$a_{23} = (\lambda + \mu_T)n_2n_3 + (\alpha^* + \mu_L - \mu_T)(a_2n_2 + a_3n_3)(a_2n_3 + a_3n_2) +$$
$$(\mu_L - \mu_T)a_2a_3 + \beta^*a_2a_3(a_2n_2 + a_3n_3)^2,$$
$$a_{33} = (\lambda + \mu_T)n_3^2 + \{2\alpha^* + 3(\mu_L - \mu_T)\}a_3n_3(a_2n_2 + a_3n_3) + (\mu_L - \mu_T)a_3^2 +$$
$$\beta^*a_3^2(a_2n_2 + a_3n_3)^2 + \mu_T + \mu_0H_0^2n_3^2,$$
$$R = \epsilon_0\mu_0^2H_0^2 + \rho(1 + \Gamma^2), \qquad R_1 = \epsilon_0\mu_0^2H_0^2 + \rho,$$
$$\gamma(x^2) = \frac{T^*\nu^2}{\rho c_e - (K^* - K\tau\omega\alpha)x^2} \quad \text{and} \quad \Gamma = \frac{\Omega}{\omega}.$$

Since all the values of p_k are not zero, the equation $|D_{ik}| = 0$ obtained from (18) yields two algebraic equations

$$D_1x^{10} + D_2x^8 + D_3x^6 + D_4x^4 + D_5x^2 + D_6 = 0 \tag{19}$$

and

$$D_7x^4 + D_8x^2 + D_9 = 0, \tag{20}$$

where

$$D_1 = \mu_T A_2(a_{22}a_{33} - a_{23}^2),$$
$$D_2 = -\mu_T A_2(a_{22}R_1 + a_{33}R) + (a_{22}a_{33} - a_{23}^2)(\mu_T B_2 - RA_2) +$$
$$\mu_T A_1(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3),$$
$$D_3 = \mu_T A_2 RR_1 - (a_{22}R_1 + a_{33}R)(\mu_T B_2 - RA_2) + (a_{22}a_{33} - a_{23}^2)(\mu_T C_2 - RB_2) -$$
$$\mu_T A_1(R_1n_2^2 + Rn_3^2) + (\mu_T B_1 - RA_1)(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) - 4a_{33}A_2\rho^2\Gamma^2,$$
$$D_4 = \mu_T B_2 RR_1 - (a_{22}R_1 + a_{33}R)(\mu_T C_2 - RB_2) - RC_2(a_{22}a_{33} - a_{23}^2) - A_2R^2R_1 -$$
$$(\mu_T B_1 - RA_1)(R_1n_2^2 + Rn_3^2) - RB_1(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) +$$
$$4\rho^2\Gamma^2(R_1A_2 - n_3^2A_1 - a_{33}B_2),$$
$$D_5 = \mu_T C_2 RR_1 - B_2R^2R_1RC_2(a_{22}R_1 + a_{33}R) + RB_1(R_1n_2^2 + Rn_3^2) +$$
$$4\rho^2\Gamma^2(R_1B_2 - n_3^2B_1 - a_{33}C_2),$$
$$D_6 = R_1C_2(4\rho^2\Gamma^2 - R^2),$$
$$D_7 = \mu_T(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3),$$
$$D_8 = R(a_{33}n_2^2 + a_{22}n_3^2 - 2a_{23}n_2n_3) + \mu_T(R_1n_2^2 + Rn_3^2),$$
$$D_9 = R(R_1n_2^2 + Rn_3^2) - 4\rho^2\Gamma^2n_3^2,$$
$$A_1 = -T^*K^*\nu^2, \qquad A_2 = K^{*2} + \alpha K^2\omega^2,$$
$$B_1 = \rho c_e T^*\nu^2, \qquad B_2 = -2\rho c_e K^*, \qquad C_2 = \rho^2c_e^2.$$

A closer look at equations (19) and (20) will reveal that (19) is the same for both the models II and III, whereas (20) is an additional equation for the model III. A real and positive root of (19) and (20) will lead to the determination of the speed of plane wave propagation in the medium. The speed of wave propagation is actually $1/|x|$.

Furthermore if the wave is propagating in the direction of the axis of symmetry, i.e., if $n_2 = 0$, $n_3 = 1$ and $\mathbf{a} = (0, 0, a_3)$, then equation (17) becomes

$$\begin{vmatrix} D'_{11} & 2\tau\rho\Gamma & 0 \\ -2\tau\rho\Gamma & D'_{22} & 0 \\ 0 & 0 & D'_{33} \end{vmatrix} = 0, \tag{21}$$

where

$$D'_{11} = D'_{22} = x^2\mu_T - \epsilon_0\mu_0^2 H_0^2 - \rho(1 + \Gamma^2),$$
$$D'_{33} = x^2[(\lambda + 2\mu_T) + 2\alpha^* a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta^* a_3^4 + \mu_0 H_0^2 + \gamma(x^2)] - \epsilon_0\mu_0^2 H_0^2 - \rho.$$

The equation $|D_{ik}| = 0$ yields four different velocities of plane wave propagations $(V_{\mathrm{com}})_i$, $i = 1, 2, 3, 4$ common to models II and III and an additional velocity V_2 for model III only, given by

$$(V_{\mathrm{com}})_{1,2} = \sqrt{\frac{\Upsilon \pm \sqrt{\Upsilon^2 - 4A_5\rho c_e R_1 K^*}}{2\rho c_e R_1}}, \tag{22}$$

which clearly depends upon the reinforcing direction, applied magnetic field and temperature parameters of the medium

$$(V_{\mathrm{com}})_{3,4} = \sqrt{\frac{\mu_T}{\epsilon_0\mu_0^2 H_0^2 + \rho(1 \pm \Gamma)^2}}, \tag{23}$$

which depends upon the applied magnetic field, rotation of the medium but not the direction of reinforcing parameter and temperature parameters and

$$V_2 = \sqrt{\frac{A_5}{R_1}}, \tag{24}$$

which depends upon the reinforcing direction, applied magnetic field and rotation parameter of the medium, where

$$\Upsilon = (A_5\rho c_e + K^* R_1 + T^*\nu^2)$$

and

$$A_5 = (\lambda + 2\mu_T) + 2\alpha^* a_3^2 + 4(\mu_L - \mu_T)a_3^2 + \beta^* a_3^4 + \mu_0 H_0^2, \qquad R_1 = \epsilon_0\mu_0^2 H_0^2 + \rho.$$

5. Numerical results and discussions

The present study focuses on the effects of fibre-reinforcing, rotation, magnetic field and temperature on the propagation of plane waves in a solid. To observe the effects numerically we have adopted three sets of values of relevant parameters from the works of Othman et al. [19],

Markham [18], Zorammuana [26] as given below

$$\lambda = 9.40 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \mu_T = 1.89 \times 10^9 \, \mathrm{N \, m^{-2}},$$
$$\mu_L = 2.45 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \rho = 1.70 \times 10^3 \, \mathrm{kg \, m^{-3}},$$

$$\lambda = 5.65 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \mu_T = 2.46 \times 10^9 \, \mathrm{N \, m^{-2}},$$
$$\mu_L = 5.66 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \rho = 2.26 \times 10^3 \, \mathrm{kg \, m^{-3}},$$

$$\lambda = 7.59 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \mu_T = 3.50 \times 10^9 \, \mathrm{N \, m^{-2}},$$
$$\mu_L = 7.07 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \rho = 1.60 \times 10^3 \, \mathrm{kg \, m^{-3}},$$

$$\alpha = 3.668 \times 10^{-4} \, \mathrm{N}, \qquad \beta = 1.138\,49 \times 10^{11} \, \mathrm{N \, m^{-2}},$$
$$\xi = 1.475 \times 10^{12} \, \mathrm{N \, m^{-2}}, \qquad \bar{k} = 1.753 \times 10^{-15} \, \mathrm{N \, m^{-2}},$$

$$\alpha^* = -1.28 \times 10^9 \, \mathrm{N \, m^{-2}}, \qquad \beta^* = 0.32 \times 10^9 \, \mathrm{N \, m^{-2}}.$$

Although our analysis is quite general in respect of fibre-reinforcing direction, direction of the axis of rotation and the direction of the applied magnetic field, for simplicity of calculations and for a quick view on the trend of behavior of the plane wave velocity under the actions of rotation, thermal and magnetic effects, we shall limit our numerical computation for the fibre-reinforced medium which

(a) has fibre-reinforcing in the direction $(0, 0, 1)$,

(b) is under the applied magnetic field in the direction $(0, 1, 0)$,

(c) is rotating about an axis having direction cosines $(0, 0, 1)$.

Furthermore, we shall assume that a plane wave is propagating in the direction $(0, 0, 1)$. Under such conditions, the wave velocity values are given by equations (22), (23) and (24).

Adopting the parameter as listed above in our numerical computation, we have tried to analyse the trends of the behavior of plane wave propagating in the medium under different media conditions and load conditions with respect to two G-N models. In Figs. 2–9 we have plotted the velocity of plane wave propagation against applied magnetic field H_0 under different states of rotation, reinforced and thermal conditions. One interesting observation from the figures indicate that in absence of applied magnetic field the wave velocity has its maximum value under thermal, rotational and fibre-reinforced conditions. In other words, increase of magnetic field under any of these conditions depresses the wave velocity.

Figs. 2–4 show that for a fixed applied magnetic field, the velocity of propagation decreases with the increase of reinforcing parameter values $\mu_L - \mu_T$.
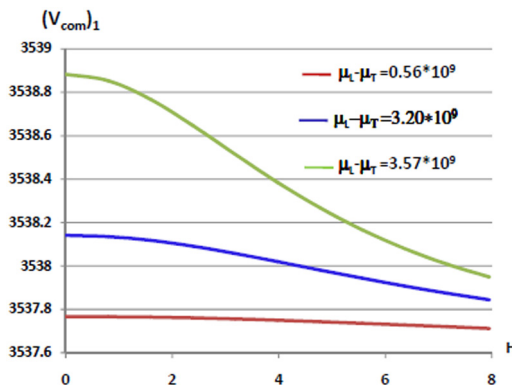

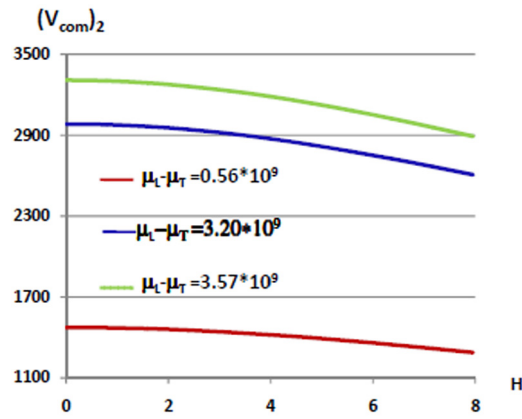Fig. 2. Effect of reinforcing parameter on wave velocity $(V_{\mathrm{com}})_1$

Fig. 3. Effect of reinforcing parameter on wave velocity $(V_{\text{com}})_2$
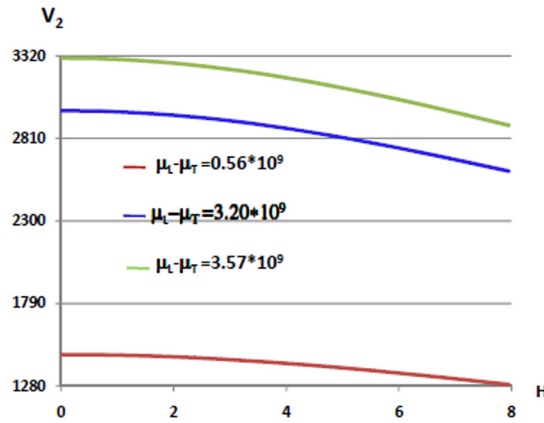

Fig. 4. Effect of reinforcing parameter on wave velocity (V_2)

Figs. 5–6 display the rotational effects on the wave velocity. It is seen that for a fixed value of applied magnetic field, the wave velocity decreases as rotation parameter increases.
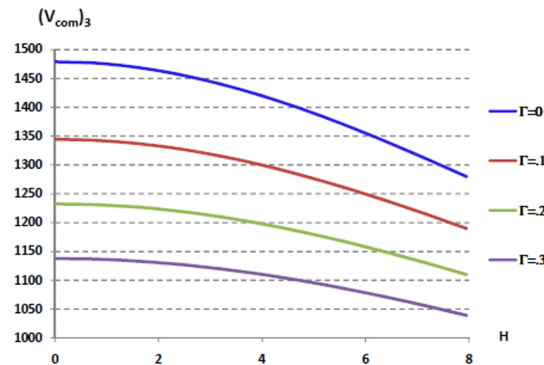

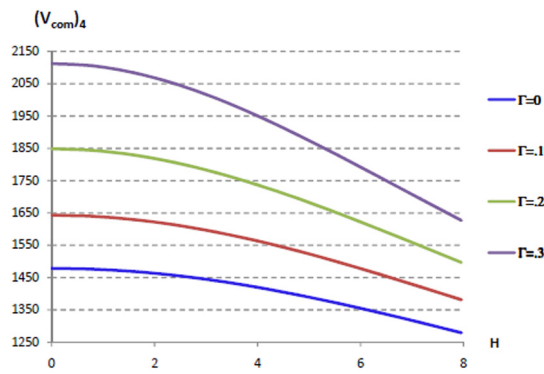Fig. 5. Effect of rotation parameter Γ on wave velocity $(V_{\text{com}})_3$

Fig. 6. Effect of rotation parameter Γ on wave velocity $(V_{com})_4$

The effects of reinforcing parameter μ_T on the wave velocity are shown in Figs. 7–8. The wave velocity increases with the increase of this parameter values.
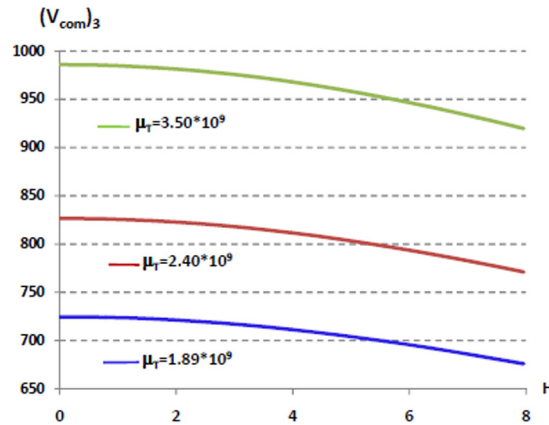

Fig. 7. Effect of reinforcing parameter μ_T on wave velocity $(V_{com})_3$
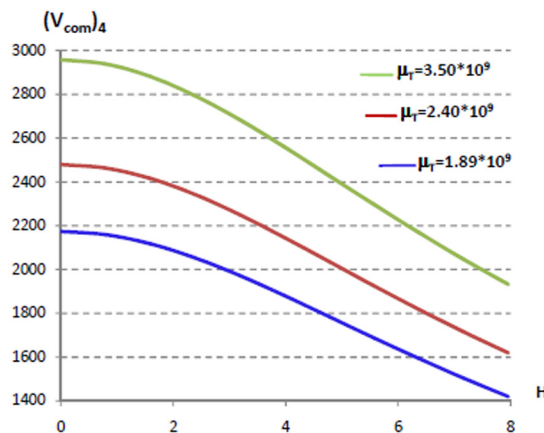

Fig. 8. Effect of reinforcing parameter μ_T on wave velocity $(V_{com})_4$

Fig. 9 shows the thermal effects on the wave velocity indicating that the velocity decreases with the increase of temperature. This figure also shows that at a fixed temperature the increase of a magnetic field decreases the wave velocity.
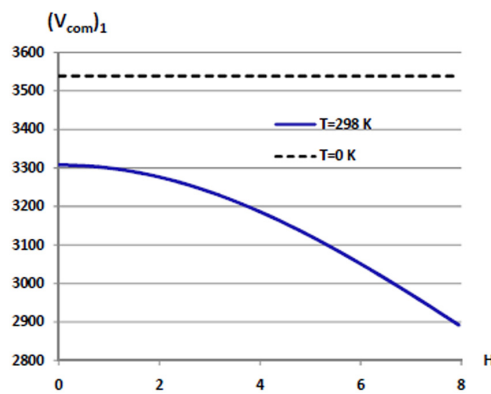
10

Fig. 9. Effect of temperature on wave velocity $(V_{\text{com}})_1$

6. Conclusion

The present study aiming for examining the possibility of plane wave propagation in a fibre-reinforced medium under the action of rotation, magnetic and thermal loadings, shows that the plane wave propagation is significantly influenced by the parameter values of the medium as well as other conditions imposed on the medium. Study shows that fibre-reinforcing direction in the medium plays an important role on the wave velocity. Effects of rotation, magnetic and thermal fields applied jointly or severally are also very significant. Numerical evaluation of plane wave velocity based on available parameter values of fibre-reinforced materials shows that keeping the other parameter values unchanged, the increase of applied magnetic field in a certain direction on the medium depresses the wave velocity. Also, for a fixed value of applied magnetic field, wave velocity decreases with the increased values of rotation and fibre-reinforcing parameters. It is also found that G-N model III predicts two more possible velocities of plane wave propagation in the medium in comparison with those in G-N model II.

References

[1] Abd-Alla, A. N., Relaxation effect on reflection of generalized-thermoelastic plane waves, Mechanics Research Communications 27 (2000) 591–600.
https://doi.org/10.1016/S0093-6413(00)00134-8

[2] Abd-Alla, A. N., Al-Dawy, A. A. S., The reflection phenomena of SV waves in a generalized thermoelastic medium, International Journal of Mathematics and Mathematical Science 23 (2000) 529–546.

[3] Ahmad, F., Khan, A., Effect of rotation on wave propagation in a transverseley isootropic medium, Mathematical Problem in Engineering 7 (2001) 147–154.

[4] Belfield, A. J., Gers, T. G., Spencer, A. J. M., Stress in elastic plates reinforced by fibres lying in concentric circles, Journal of the Mechanics and Physics of Solids 31 (1983) 25–54.
https://doi.org/10.1016/0022-5096(83)90018-2

[5] Chandrasekharaiah, D. S., Hyperbolic thermoelasticity: A review of recent literature, Applied Mechanics Reviews 51 (1998) 705–729. https://doi.org/10.1115/1.3098984

[6] Chandrasekharaiah, D. S., Thermoelasticity with second sound: A review, Applied Mechanics Reviews 39 (1986) 355–376. https://doi.org/10.1115/1.3143705

[7] Choudhuri, S. K. R., Electro-magneto-thermo-elastic plane waves in rotating media with thermal relaxation, International Journal of Engineering Science 22 (1984) 519–530.
https://doi.org/10.1016/0020-7225(84)90054-5

[8] Chodhuri, S. K. R., Banerjee, M., Magneto-thermoelastic plane waves in a rotating media in thermoelasticity of type II (G-N Model), International Journal of Mathematics and Mathematical Sciences 71 (2004) 3917–3929.

[9] Ezzat, M. A., Fundamental solution in generalized magneto-thermoelasticity with two relaxation times for perfect conductor cylindrical region, International Journal of Engineering Science 42 (2004) 1503–1519. https://doi.org/10.1016/j.ijengsci.2003.09.013

[10] Ezzat, M. A., Othman, M. I. A., Electro-megneto-thermoelastic plane waves with relaxation times in a medium of perfect conductivity, International Journal of Engineering Science 38 (2000) 107–120. https://doi.org/10.1016/S0020-7225(99)00013-0

[11] Ezzat, M. A., Othman, M. I. A., Karamany, A. S., Electromagneto-thermoelastic plane waves with thermal relaxation in a medium of perfect conductivity, Journal of Thermal Stresses 24 (2001) 411–432.

[12] Green, A. E., Naghdi, P. M., On undamped heat waves in an elastic solid, Journal of Thermal Stresses 15 (1992) 253–264. https://doi.org/10.1080/01495739208946136

[13] Green, A. E., Naghdi, P. M., Thermoelasticity without energy dissipation, Journal of Elasticity 31 (1993) 189–209. https://doi.org/10.1007/BF00044969

[14] Gupta, R. R., Gupta, R. R., Effect of rotation on propagation of waves in transversely isotropic thermoelastic half-space, Indian Journal of Materials Science 2014 (2014) 6–13.

[15] Lord, H. W., Shulman, Y., A generalized dynamic theory of thermoelasticity, Journal of the Mechanics and Physics of Solids 15 (1967) 299–309. https://doi.org/10.1016/0022-5096(67)90024-5

[16] Maity, N., Barik, S. P., Chaudhuri, P. K., Plane wave propagation in a rotating anisotropic medium with voids under the action of a uniform magnetic field, International Journal of Computational Materials Science and Engineering 5 (2016) 1650015 (1–20).

[17] Maity, N., Barik, S. P., Chaudhuri, P. K., Reflection of plane waves in elastic half space with void pores under the action of uniform magnetic field, International Journal of Engineering Science 71 (2013) 62–73. https://doi.org/10.1016/j.ijengsci.2013.05.007

[18] Markham, M. F., Measurement of the elastic constants of fibre composites by ultrasonics, Composites 1 (1969) 145–149. https://doi.org/10.1016/0010-4361(69)90059-7

[19] Othman, M. I. A., Generalized thermoelastic plane waves in rotating media with thermal relaxation under the temperature dependent properties, Mechanics and Mechanical Engineering 9 (2005) 89–100.

[20] Othman, M. I. A., Lotfy, K., Said, S. M., Anwar Bég, O., Wave propagation in a fiber-reinforced micropolar thermoelastic medium with voids using three models, International Journal of Applied Mathematics and Mechanics 8 (2012) 52–69.

[21] Othman, M. I. A., Said, S. M., Two dimensional problem of thermally conducting fibre-reinforced medium under Green-Nagdi theory, Journal of Thermoelasticity 1 (2013) 13–20.

[22] Othman, M. I. A., Song, Y., The effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation, Acta Mechanica 184 (2006) 189–204. https://doi.org/10.1007/s00707-006-0337-4

[23] Othman, M. I. A., Tantawi, R. S., The effect of magnetic field on an infinite conducting thermoelastic rotating medium under G-N theory, International Journal for Innovative Research in Science & Technology 4 (2015) 2301–2309.

[24] Singh, B., Yadev, A., Reflection of plane waves in a rotating transversely isotropic magneto-thermoelastic solid half space, Journal of Theoretical and Applied Mechanics 42 (2012) 33–60.

[25] Wang, X., Lu, G., Guillow, S. R., Magneto-thermoelastic stress and perturbation of magnetic field vector in a solid cylinder, Journal of Thermal Stresses 25 (2002) 909–926. https://doi.org/10.1080/01495730290074397

[26] Zorammuana, C., Singh, S. S., SH-wave at a plane interface between homogeneous and inhomogeneous fibre-reinforced elastic half-spaces, Indian Journal of Materials Science 2015 (2015) 1–8.