Discrete-time and digital chaotic systems synthesis

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Abstract **This paper describes new method of high order discrete-time chaotic systems synthesis. Proposed method is based on a special system structure called dissipation normal form and leads to chaos in conservative systems. A study of quantization effects indicates that presented method is suitable for digital chaotic systems design.**

Keywords **digital chaotic systems, discrete-time chaotic systems, dissipation normal form, chaos synthesis**

I. INTRODUCTION

During last decades, the chaos theory evolved into discipline which intervenes in many branches. Among others, chaotic systems describe fluid flows, dynamics of populations, vibrations in nonlinear mechanic systems etc.

From the beginning some researchers speculate about using chaotic signals in cryptography. From this point of view, it is necessary to develop methods for systematic design of high order chaotic systems which could be used as chaotic signals and chaotic carriers generators.

Recently a new method of high order continuous and discrete-time chaotic systems synthesis was published [1], [2]. The method is based on special system structure called dissipation normal form. When some suitable nonlinearity is brought in the system parameters, the system shows chaotic oscillations which can be characterized by non-periodical changing of the system stability and instability intervals. Another method is usage of direct discretization of the continuous chaotic system based on dissipation normal form [3].

In the following text, another possibility of application of this structure is introduced. Chaotic oscillations are achieved by the system parameters sign switching. The system stays conservative with constant energy given by initial conditions.

II. DISCRETE-TIME DISSIPATION NORMAL FORM

Dissipation normal form is a special systems structure which can be derived from natural requirements for abstract energy conservation law validity [4].

Consider a state representation of linear, strictly causal n-order system with scalar input and scalar output:

$$\begin{aligned}\mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k)+\mathbf{B}u(k)\\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k)\end{aligned} \tag{1}$$

Let us define an abstract system energy and output signal power:

$$E(\mathbf{x},k)=\frac{1}{2}\|\mathbf{x}(k)\|^2 \qquad P=-\|\mathbf{y}(k)\|^2 \tag{2}$$

It is assumed that the system is dissipative. Considering zero input we can write an equation expressing a power balance condition.

$$\Delta E = E(\mathbf{x},k+1)-E(\mathbf{x},k)=-\Delta P = y(k+1)-y(k) \tag{3}$$

After some manipulations it can be derived that it must hold

$$\left(\mathbf{A}^T\mathbf{A}-\mathbf{I}\right)=-\mathbf{C}^T\mathbf{C} \tag{4}$$

Dissipation normal form is the system structure which corresponds to the equation (5). For a 4th-order system the **A**, **B** and **C** matrices have following structure

$$\mathbf{A}=\begin{bmatrix}-\Delta_3\Delta_4 & \delta_3 & 0 & 0\\ -\Delta_2\delta_3\Delta_4 & -\Delta_2\Delta_3 & \delta_2 & 0\\ -\Delta_1\delta_2\delta_3\Delta_4 & -\Delta_1\delta_2\Delta_3 & -\Delta_1\Delta_2 & \delta_1\\ \delta_1\delta_2\delta_3\Delta_4 & \delta_1\delta_2\Delta_3 & \delta_1\Delta_2 & \Delta_1\end{bmatrix} \tag{5}$$

$$\mathbf{B}=\begin{bmatrix}\beta_1 & \beta_2 & \beta_3 & \beta_4\end{bmatrix}^T \tag{6}$$

$$\mathbf{C}=\begin{bmatrix}\gamma & 0 & 0 & 0\end{bmatrix} \tag{7}$$

Parameters δ_i and their complements Δ_i in **A** matrix must follow following requirements:

$$\forall i,\, i\in\{1,2,\dots,n\}:\ 0<\delta_i\le 1,\ |\Delta_i|<1,\ \delta_i^2+\Delta_i^2=1 \tag{8}$$

and for parameters in the **B** and **C** matrices it holds:

$$\beta_i\ne 0,\qquad i\in\{1,2,\dots,n\},\ \gamma=\delta_n\ne 0 \tag{9}$$

When $\Delta_i > 1$, the system is unstable. When the parameter $\delta_n = 0$, the system is conservative, $\Delta_n = 1$.

III. DISCRETE-TIME CHAOTIC SYSTEMS DESIGN

New approach for discrete-time chaotic systems design uses linear structures. Nonlinearity in the system is presented in the form of parameter sign switching. The sign is changed in certain number of iterations.

A. Example of the 4th-order system

Consider system with presented structure with following parametrization:

$$\Delta_1=\pm 1;\ \Delta_2=\Delta_3=-0.9;\ \Delta_4=-1; \tag{10}$$

$$\delta_i=\sqrt{1-\Delta_i^2}\ ;\ i\in\{1,2,3,4\} \tag{11}$$

In this example the parameter Δ_1 sign is switched by 15 iterations. Results are depicted in the figures (1), (2), (3).

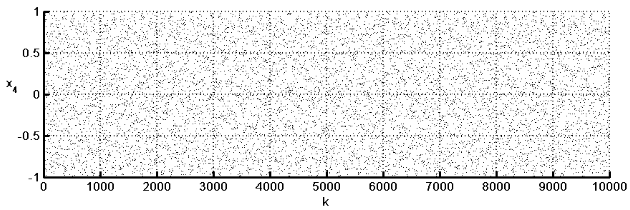

Fig. 1. Time evolution of the state variable x_4

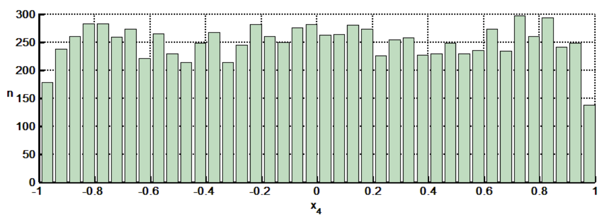

Fig. 2. Histogram of the state variable x_4, computed over 10000 iterations

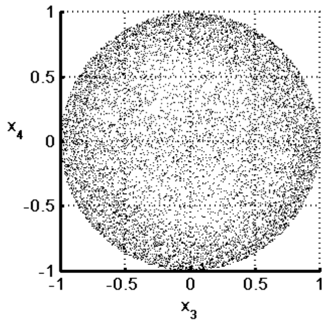

Fig. 3. 2D-projection of the chaotic attractor, full-precision computation

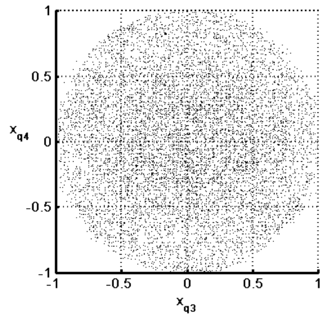

Fig. 6. 2D-projection of the attractor, 8-bit fixed-point calculations

The state trajectory moves along the hyper-sphere surface, the system energy is constant and given by the initial conditions (in the example: $\mathbf{x}_0 = [1,0,0,0]^T$). All those computations were performed in full-precision.

IV. Digitalization and fixed-point computation

B. Example of the 4^{th}-order system, 16-bit quantization

Now let us quantize all the parameters and arithmetic operations. The word-length is 16 bits; the fraction-length is 15 bits. Presented results were obtained by using of MATLAB with `fix-point toolbox`.

Figures of the state variables time evolution and state trajectory attractor are almost undistinguishable from the figures (1) and (3). At the figure (4) there is histogram for the x_4 state variable, computed over 10000 iterations.

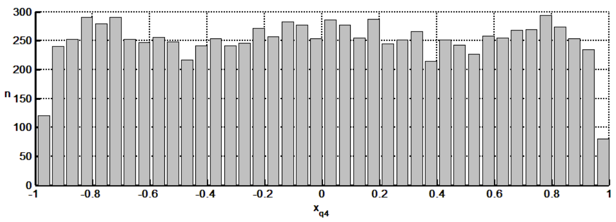

Fig. 4. Histogram of the state variable x_4, 16-bit fixed-point calculations

The energy time evolution is very interesting. As we can see at the figure (5), quantization caused dissipativity.

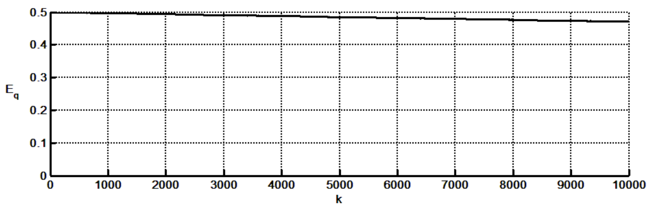

Fig. 5. The state-space energy time evolution

To keep the conservativity it would be necessary to slightly increase the parameter Δ_1 value. In full-precision it would cause instability.

C. Example of the 4^{th}-order system, 8-bit quantization

In following experiment the system is quantized in 8 bits. Some results are depicted at the figures (6), (7). There is no perceptible periodicity in 30 000 iterations. The figures correspond to 10 000 iterations.

The energy is not constant but chaotic and its average value is given by the initial conditions.

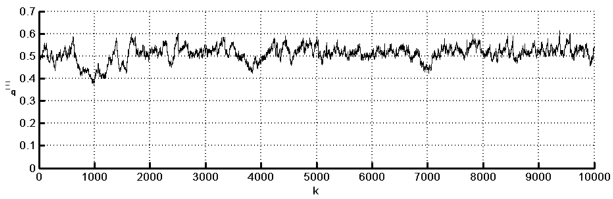

Fig. 7. The state-space energy time evolution

D. Example of the 4^{th}-order system, 5-bit quantization

In this experiment extremely raw quantization was used. When we quantize in 5 bits the system is periodic with period T = 2280 iterations. Contingent periodicity in examples *A*, *B*, *C* presented above was not identified.

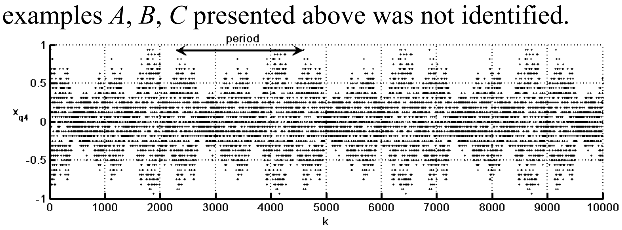

Fig. 8. Time evolution of the variable x_4, 5 bit fixed-point calculations

V. Conclusion

Presented method of discrete and digital chaotic systems synthesis is applicable for generating long pseudo-random and chaotic sequencies including sequencies with very raw quantization. Implementation of this method is very easy because it is not necessary to realize any complicated nonlinearity. Switching of the parameter sign is very easy from the implementation point of view. The digital system is conceived as conservative and the state trajectory motion range is given by initial conditions.

VI. References

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