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# INTEGRAL COMPUTATION OF MAGNETIC FIELD OF SHIELDED THREE-PHASE LINE

ING. PAVEL KARBAN, PROF. ING. IVO DOLEŽEL, CSC.,
DOC. ING. BOHUŠ ULRYCH, CSC.

**Abstract**: *The paper deals with numerical modeling of a three-phase harmonic-current carrying shielded line. In most similar cases, computation of magnetic field and other associated quantities is realized using the finite element method that is effective, reliable and the results obtained correspond to the physical reality. Nevertheless, the method may become problematic when particular subregions (conductors, insulation, shielding elements) are geometrically incommensurable, which is even the case of thin shielding shells. That is why the authors use the integral approach for modeling of the relevant effects. Presented is its basic continuous mathematical model that is solved numerically. The theoretical analysis is supplemented with a typical example.*

**Key words**: *Integral approach, numerical analysis, shielded system of conductors*

## INTRODUCTION

One of the structural elements of heavy-current lines are often the shielding shells, whose main purpose is to reduce the magnetic field in their vicinity. Modeling of the effects of the shielding shell is not easy and usually must be carried out numerically. Reliable and robust for this purpose are various differential methods, particularly FE analysis. Nevertheless, application of these methods may become problematic when the individual subregions of the system (conductors, insulation, shielding elements) are geometrically incommensurable, which may lead to problems associated with the generation of the discretization mesh.

The first attempts to numerically solve problems of dynamic shielding started appearing in the seventies and the eighties of the last century. The relevant papers usually present methods consisting of the combination of analytical methods and various numerical techniques. The methodology was principally applied to the nonmagnetic cylindrical single or multiple shells (see, for example, [1]–[5]).

The authors offer a method based on an integral approach, which may sometimes prove to be quite a useful and suitable alternative. This approach, of course, is not new, see, for example, [6]–[8], but in the past it was not used too frequently because of their drawbacks (necessity of working with dense or fully populated matrices, computation of various complicated proper and improper integrals etc.).

The paper presents the basic continuous integrodifferential mathematical model of the problem that is solved numerically. The theoretical analysis is supplemented with a typical example whose results are discussed.

## 1 FORMULATION OF THE TECHNICAL PROBLEM

Consider an arrangement in Fig. 1 containing three nonmagnetic phase conductors of any cross-section that are surrounded by a thin, well electrically conductive shielding shell that is also nonmagnetic. The phase conductors carry harmonic currents of the same amplitude $I$, but mutually shifted by 120°. The task is to determine the distribution of the current densities induced in the shielding shell and their influence on the resultant magnetic field.

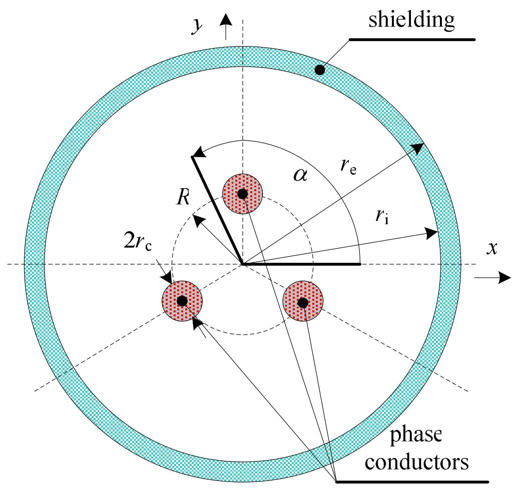


*Fig. 1. Basic arrangement of the solved task*

## 2 Continuous Mathematical Model

First we will present a general mathematical model valid for any linear system, whose parts can, moreover, move.

Let the system contain $n$ nonmagnetic metal bodies $\Omega_j,\ j=1,\ldots,n$ (Fig. 2) whose electrical conductivities are in turn $\gamma_j,\ j=1,\ldots,n$. The bodies carry currents $i_j(t),\ j=1,\ldots,n$ and can move at time variable velocities $\boldsymbol{v}_j(t),\ j=1,\ldots,n$. The system is homogeneous and its permeability is $\mu_0$.

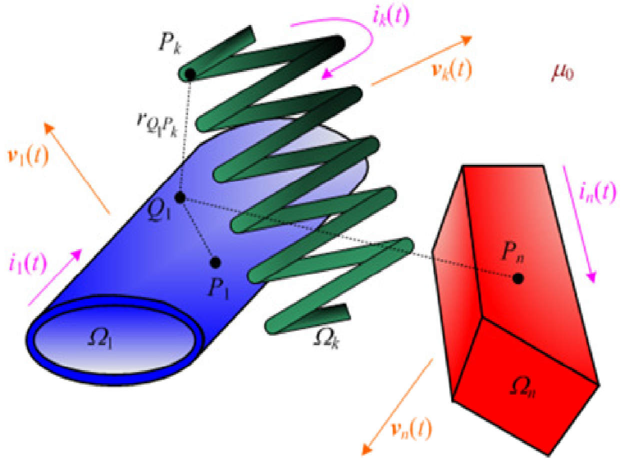


*Fig. 2. A general system with $n$ nonmagnetic bodies*

Generally, the basic task is to determina

- the distribution of current densities in the bodies that is a function of space and time,
- the forces acting on the individual bodies of the system,
- the corresponding volume Joule losses in the bodies and their temperature rise (when necessary).

The knowledge of the above quantities is a must for the following thermal, mechanical and other calculations, for example in various coupled problems.

The magnetic vector potential $\boldsymbol{A}$ at any reference point $Q_k \in \Omega_k$ in the system in Fig. 2 is given by formula

$$\boldsymbol{A}(Q_k,t)=\frac{\mu_0}{4\pi}\sum_{j=1}^{n}\int_{\Omega_j}\frac{\boldsymbol{J}(P_j,t)}{r_{Q_kP_j}(t)}\cdot \mathrm{d}V+\boldsymbol{A}_0(t)\,. \tag{1}$$

Here $\boldsymbol{J}(P_j,t)$ denotes the total current density at a general integration point $P_j$ whose distance from the reference point $r_{Q_kP_j}(t)$ is (due to possible mutual velocity between the bodies $j$ and $k$) generally a function of time. The function $\boldsymbol{A}_0(t)$ is not known in advance and must be determined indirectly, as will be shown later on. The total current density $\boldsymbol{J}(P_j,t)$ at any integration point $P$ consists of two parts: the first one denoted as $\boldsymbol{J}_{\text{ext}}(P_j,t)$ comes from the field current (external sources) and the second one $\boldsymbol{J}_{\text{eddy}}(P_j,t)$ comes from the time variation of the magnetic field.

The second Maxwell equation for fields with movement reads

$$\operatorname{curl}\boldsymbol{E}=-\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t}=-\frac{\mathrm{d}(\operatorname{curl}\boldsymbol{A})}{\mathrm{d}t}=-\operatorname{curl}\left(\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}\right) \tag{2}$$

and its formal solution reads

$$\boldsymbol{E}=-\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}-\operatorname{grad}\varphi \tag{3}$$

where $\varphi$ is usually interpreted as a scalar electric potential.

After multiplying (3) by electrical conductivity $\gamma$ of the involved media we have

$$\gamma\boldsymbol{E}=-\gamma\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}-\gamma\operatorname{grad}\varphi\,, \tag{4}$$

whose particular terms can be interpreted as

$$\gamma\boldsymbol{E}=\boldsymbol{J},\quad -\gamma\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}=\boldsymbol{J}_{\text{eddy}},\quad -\gamma\operatorname{grad}\varphi=\boldsymbol{J}_{\text{ext}}\,. \tag{5}$$

Combining (1), (4) and (5) at point $Q_k \in \Omega_k$ and time $t$ provides

$$\boldsymbol{J}(Q_k,t)-\boldsymbol{J}_{\text{ext}}(t)=\boldsymbol{J}_{\text{eddy}}(Q_k,t)=-\gamma_k\frac{\mathrm{d}\boldsymbol{A}(Q_k,t)}{\mathrm{d}t}=$$
$$-\frac{\mu_0\gamma_k}{4\pi}\sum_{j=1}^{n}\int_{\Omega_j}\frac{\boldsymbol{J}(P_j,t)}{r_{Q_kP_j}(t)}\cdot\mathrm{d}V-\gamma_k\frac{\mathrm{d}\boldsymbol{A}_0(t)}{\mathrm{d}t}$$

which, after performing the time derivative, gives

$$\boldsymbol{J}(Q_k,t)+\frac{\mu_0\gamma_k}{4\pi}\cdot\sum_{j=1}^{n}\int_{\Omega_j}\frac{\dfrac{\partial\boldsymbol{J}(P_j,t)}{\partial t}\cdot\mathrm{d}V}{r_{P_jQ_k}(t)}-$$
$$-\frac{\mu_0\gamma_k}{4\pi}\cdot\sum_{j=1}^{n}\int_{\Omega_j}\frac{\left(\boldsymbol{v}_{kj}(t)\cdot\boldsymbol{r}_{P_jQ_k}(t)\right)\boldsymbol{J}(P_j,t)\,\mathrm{d}V}{r^3_{P_jQ_k}(t)}+ \tag{6}$$
$$+\boldsymbol{J}_{k0}(t)=\boldsymbol{0},\quad k=1,\ldots,n$$

where $\boldsymbol{v}_{kj}(t)=\boldsymbol{v}_k(t)-\boldsymbol{v}_j(t)$ denotes the mutual velocity between the $k-$th and $j-$th body (so that $\boldsymbol{v}_{kk}(t)=0$) and $\boldsymbol{J}_{k0}(t)$ is an unknown function given as

$$\boldsymbol{J}_{k0}(t)=\boldsymbol{J}_{\text{ext}}(t)+\gamma_k\frac{\mathrm{d}\boldsymbol{A}_0(t)}{\mathrm{d}t}\,. \tag{7}$$

These functions must be determined from some supplementary conditions. One of them is the indirect conditions in the form

$$\int_{S_j}\boldsymbol{J}_j(Q_j,t)\cdot\mathrm{d}\boldsymbol{S}=i_j(t) \tag{8}$$

where the integration in (8) has to be carried out over a suitable cross-section of the $j-$th body of the system (the selection of such a cross-section in case of geometrically complicated bodies is, however, not easy and usually depends on the task solved).

In case of a 2D arrangement without motion the basic equation (6) may be simplified considerably. Moreover, when the field currents are harmonic, all current densities and also magnetic vector potential may be expressed in terms of their phasors.

Consider an ideal 2D system containing generally several active and passive parts that is depicted in Fig. 3. Both magnetic vector potential and current densities have only one nonzero component in the direction of axis $z$.

Now the modified equations describing these quantities read

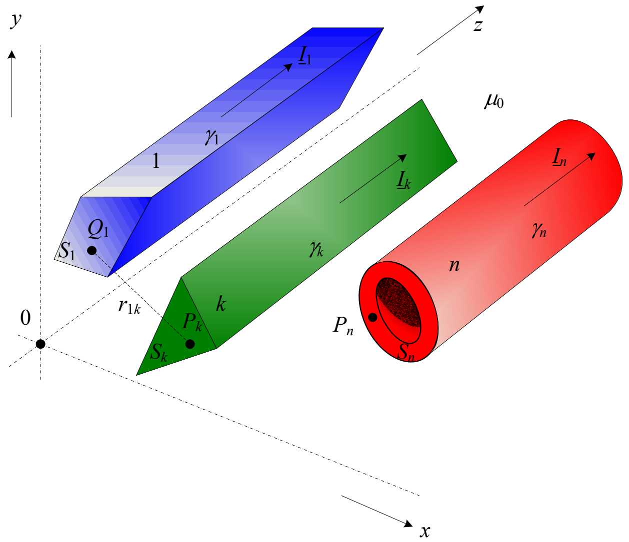


*Fig. 3. A general 2D arrangement*

$$\underline{A}_z(Q_k) = -\frac{\mu_0}{2\pi}\sum_{i=1}^{n}\int_{S_i}\underline{J}_{zi}(P_i)\ln r_{ik}\,\mathrm{d}S + \underline{A}_0,$$
$$\underline{J}_{zk}(Q) = -\mathrm{j}\cdot\gamma\omega\underline{A}_z(Q) + J_{zk,\mathrm{ext}}, \quad (9)$$
$$\int_{S_i}\underline{J}_{zi}(P)\,\mathrm{d}S = \underline{I}_i,\ i = 1,\ldots,n$$

where the significance of particular symbols is the same as in (6). The integration is now performed not over the volumes of the individual bodies, but just over their cross-sections.

## 3 NUMERICAL SOLUTION OF THE CONTINUOUS MODEL

The solution of the system (9) has to be carried out numerically. First, all the surfaces $S_i$, $i = 1,\ldots,n$ have to be discretized. Then we can proceed in two ways. The classical way consists in approximating the actual current density in every cell by a constant value at its midpoint. The number of the degrees of freedom (DOFs) is now equal to the number of cells + number of the indirect conditions. A more sophisticated and efficient, but also much more laborious algorithm starts from a suitable modification of the Galerkin technique. This technique is based on replacing the actual distribution of current density in every element by a linear combination of the trial functions (here in two variables $x, y$). The most convenient is to create an orthonormal system of the trial functions whose usage improves the conditionality of the system matrix. The trial functions are usually polynomials of a selected degree, whose coefficients must be determined by specific integration.

A more detailed description of the possibilities can be found in some previous papers of the authors (see, for example, [9] and [10])

## 4 ILLUSTRATIVE EXAMPLE

The methodology was tested on several examples In this paper we present the computation of magnetic field in an arrangement of a three-phase system of conductors in a shielding shell. The system has the following parameters (see Fig. 1): $I = 400$ A, $f = 1000$ Hz, $r_i = 0.05$ m, $r_e = 0.055$ m, $R = 0.013$ m. The phase conductors have square cross-sections.

The calculation is based on the following facts:

- The distribution of current densities in the phase conductors is not affected by the shielding shell and is the same is if were not for the shell.
- On the contrary, the distribution of the current densities in the shell is affected by the instantaneous distribution of the current densities in the phase conductors.
- The total current in the shielding shell is (due to symmetry) equal to zero.

Starting from these statements, the calculation consists of two steps. In the first one we calculate the distribution of the current densities in the phase conductors (that includes both skin and proximity effects), while in the second step we calculate the distribution of the current densities in the shell.

The system was solved numerically using the zero-order technique by the code developed and written by the authors.

Fig. 4 shows the distribution of the force lines near the system of the conductors without shielding shell.

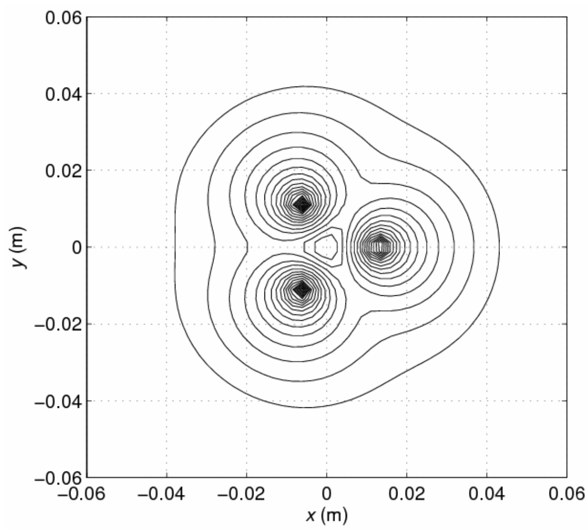


*Fig. 4. Distribution of the phasors of the force lines in the system without the shielding shell*

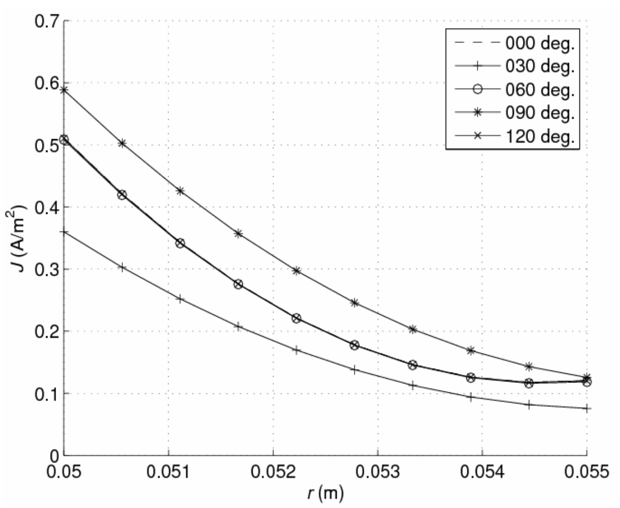


*Fig. 5. Distribution of eddy current densities along the thickness of the shielding shell for varying angle $\alpha$ (see Fig. 1)*

Fig. 5 shows the distribution of modules of the current densities induced in the shell (along the thickness of the shell for varying angle $\alpha$, see Fig. 1). The distribution is repeated after 120°.

## 5 CONCLUSION

The algorithm provides results that agree with the data obtained by other already validated methods (finite element analysis etc.). Its further development can be seen in using elements of higher-order of accuracy.

## 6 ACKNOWLEDGMENT

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Ing. Pavel Karban, Doc. Ing. Bohuš Ulrych, CSc.
University of West Bohemia
Faculty of Electrical Engineering
Univerzitní 26
306 14 Plzeň
E-mail: {karban, ulrych}@kte.zcu.cz

Prof. Ing. Ivo Doležel, CSc.
Academy of Sciences of the Czech Republic
Institute of Thermomechanics
Dolejškova 5
182 02 Praha 8
E-mail: dolezel@iee.cas.cz