Multiple-Input Multiple-Output Least-Squares Constant Modulus Algorithms

P. Sansrimahachai, D. B. Ward and A. G. Constantinides
Imperial College London
Department of Electrical and Electronic Engineering
Exhibition Road, London SW7 2BT, UK

Abstract— **In this paper, we propose novel blind separation techniques for multiple-input multiple-output systems based on least-squares constant modulus algorithm (LSCMA). To ensure that each output signal is extracted from different input signals, the proposed algorithms have been derived by using successive interference cancellation and Gram-Schmidt orthogonalization procedure. The performance is observed through computer simulations and compared with the multitarget LSCMA (MT-LSCMA) and LS multi-user CMA (LS-MU-CMA). Simulation results have shown that the proposed algorithms exhibit better performance in terms of both bit error rate (BER) and convergence speed.**

I. Introduction

Blind source separation (BSS) algorithms for instantaneous mixtures have been popularly investigated. These systems of instantaneous mixtures include the Bell Labs layered space-time (BLAST) architecture [1], [2], [3] where a number of antenna elements are used at both the transmitter and receiver. BLAST has been proved theoretically and experimentally that it has a high transmission capacity. In practical situations where training can be expensive or impossible, blind separation techniques are therefore considered to detect transmitted signals. In this paper, blind algorithms based on least-squares constant modulus algorithms are of interest.

Least-squares constant modulus algorithm (LSCMA) was first proposed in [4] for a two-sensor array. The basic concept of the LSCMA is to combine the well-known least-squares estimator (LSE) and constant modulus (CM) properties to blindly extract communication signals. This algorithm is rapidly convergent and globally stable for any linearly-independent set of input signals. Therefore, the algorithm was later modified in [5] and [6] for using in a more general system, i.e. a multi-user system.

In order to prevent two or more output signals extracted from the same input signal, multitarget LSCMA (MT-LSCMA) proposed in [5] uses a soft-orthogonalized technique which includes the Gram-Schmidt orthogonalization method followed by a softening procedure. To satisfy the same purpose, least-squares multi-user CMA (LS-MU-CMA) [6] exploits the cross correlation between output signals. In addition, this term together with the LS concept has been used in [7] for the similar purpose, but the algorithm has been derived in a different way.

Due to the slow convergence speed of the above mentioned algorithms, we propose in this paper new blind separation techniques based on the LSCMA. The first algorithm is formulated from the concept of successive interference cancellation, whereas the second algorithm is derived from the orthogonality constraint which can be done by using the Gram-Schmidt orthogonalization procedure. These novel techniques exhibit faster convergence speed and have better performance in terms of bit error rate (BER).

The paper is organised as follows. Sections II and III explain some notations used in this paper and the vertical-BLAST (V-BLAST) system. A revision of least-squares techniques based on the constant modulus algorithm is presented in section IV. We formulate the proposed algorithms, namely the successive interference cancellation LSCMA (SIC-LSCMA) and the Gram-Schmidt LSCMA (GS-LSCMA), in section V and VI. Simulation results are given in section VII and the paper is finally concluded in section VIII.

II. Notations

- k is the sample index.
- m is the block iterative index.
- ξ is a small stopping parameter, i.e. 10^{-4}.
- $(\cdot)^T$ is the transpose operation.
- $(\cdot)^*$ is the conjugate operation.

 0-7803-7974-8/03/$17.00 © 2003 IEEE

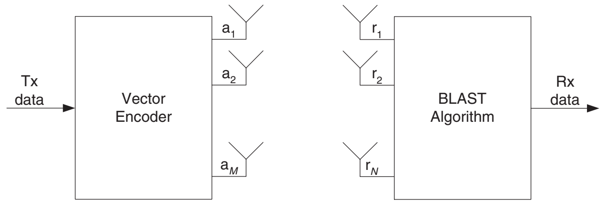

Fig. 1. The BLAST system consisting of *N* receive antennas and *M* transmit antennas.

- $(\cdot)^H$ is the conjugate transpose operation.
- $\langle |x|^2 \rangle_L = \frac{1}{L}\sum_{k=1}^{L} x(k)x(k)^*$.

III. System Model

We consider the Bell Labs Layered Space-Time (BLAST) architecture [1], [2], [3] as a system model in this paper. The architecture, as shown in Fig. 1, consists of *N* receive antennas and *M* transmit antennas. Denoting $\mathbf{a}(k)$, $\mathbf{v}(k)$ and $\mathbf{H}$ the $M \times 1$ transmitted signal vector, the $N \times 1$ noise vector and the $N \times M$ channel matrix, respectively, the corresponding $N \times 1$ received signal vector is given as

$$\mathbf{r}(k) = \mathbf{H}\mathbf{a}(k) + \mathbf{v}(k). \tag{1}$$

It is assumed that each component of $\mathbf{a}(k)$ has the same statistical properties and is drawn from the same constellation, i.e. QAM [2]. The channel matrix $\mathbf{H}$ can be modelled by a matrix having independent identically distributed (iid), complex, zero-mean, unit-variance entries. We assume that $\mathbf{H}$ is unitary. If it is not unitary, the received signals are then prewhitened. The noise at the receiver is assumed to be a complex additive white Gaussian noise. Each component of the noise vector is statistically independent and of identical power at the antenna outputs.

Here we consider the data of block N_B and the $N \times N_B$ received signal matrix can be formulated as

$$\begin{aligned} \mathbf{X} &= [\mathbf{r}(1) \cdots \mathbf{r}(N_B)] \\ &= \mathbf{HA} + \mathbf{V} \end{aligned} \tag{2}$$

where $\mathbf{A}$ and $\mathbf{V}$ are formulated in the similar fashion as $\mathbf{X}$. This problem is regarded as a problem of blind source separation (BSS). It is therefore possible to recover the transmitted signals $\mathbf{A}$ using only the received signals $\mathbf{X}$.

Denoting $\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_M]$ the $N \times M$ equalizer matrix, the estimate of $\mathbf{A}$ is obtained by

$$\begin{aligned} \mathbf{Y} &= [\mathbf{y}(1) \cdots \mathbf{y}(N_B)] \\ &= \mathbf{W}^H\mathbf{X}. \end{aligned} \tag{3}$$

It is necessary to note that the matrix $\mathbf{W}$ is feasible to separate the sources, except for a possible permutation of $\mathbf{Y}$ and an arbitrary scaling of each source signal. In the non-blind case, $\mathbf{W}$ can be estimated from the classical least-squares estimator (LSE) via some training symbols. The equalizer matrix $\mathbf{W}$ is therefore obtained by minimizing a least-squares criterion

$$\begin{aligned} \mathbf{W}_{LS} &= \arg\min_{\mathbf{W}} \; N_T^{-1} \sum_{k=1}^{N_T} |\mathbf{a}(k) - \mathbf{W}^H\mathbf{r}(k)|^2 \\ &= \mathbf{R}_T^{-1}\mathbf{P}_T \end{aligned} \tag{4}$$

where N_T denotes the length of training, $\mathbf{R}_T = \mathbf{X}_T\mathbf{X}_T^H$ and $\mathbf{P}_T = \mathbf{X}_T\mathbf{A}_T^H$.

IV. Least-Squares Techniques

A. Least-Squares Constant Modulus Algorithm

The least-squares constant modulus algorithm (LSCMA) [4] was developed based on the Gauss's method of complex-argument cost functions. Consider the constant modulus algorithm whose cost function is represented in a form

$$J(\mathbf{W}) = \sum_{k=1}^{N_B} \phi_k^2(\mathbf{W}) = \|\Phi(\mathbf{W})\|_2^2 \tag{5}$$

where $\Phi(\mathbf{W}) = |\mathbf{W}^H\mathbf{X}| - 1$. Its partial Taylor-series expansion with sum-of-squares component can be approximately given as

$$J(\mathbf{W} + \Delta) \approx \|\Phi(\mathbf{W}) + Q(\mathbf{W})^H\Delta\|_2^2. \tag{6}$$

Gauss's method updates $\mathbf{W}$ by the offset Δ which minimizes $J(\mathbf{W} + \Delta)$ resulting in

$$\begin{aligned} \mathbf{W}_{m+1} &= \mathbf{W}_m - \Delta \\ &= \mathbf{W}_m - (R_Q)^{-1}Q(\mathbf{W}_m)\Phi^H(\mathbf{W}_m) \end{aligned} \tag{7}$$

where $Q(\mathbf{W}) = \nabla_{\mathbf{W}} J(\mathbf{W})$ and $R_Q = Q(\mathbf{W}_m)Q(\mathbf{W}_m)^H$. By assuming that the transmitted signals are linearly independent, the update equation for LSCMA is therefore obtained as

$$\begin{aligned} \mathbf{W}_{m+1} &= \mathbf{W}_m - (\mathbf{X}\mathbf{X}^H)^{-1}\mathbf{X}(\mathbf{Y}_m - \Psi_m)^H \\ &= (\mathbf{X}\mathbf{X}^H)^{-1}\mathbf{X}\Psi_m^H \end{aligned} \tag{8}$$

where $\psi_i(k) = y_i(k)/|y_i(k)|, \quad i = 1, .., M$. The block update algorithm can be summarised as follows. Find

 0-7803-7974-8/03/$17.00 © 2003 IEEE

$\mathbf{R} = \mathbf{X}\mathbf{X}^H$ and its inverse $\mathbf{R}_{inv} = \mathbf{R}^{-1}$. Then initialize $\mathbf{W}_0 = \mathbf{I}$.

for $m = 0, 1, ..$

$$\begin{aligned} \mathbf{Y}_m &= \mathbf{W}_m^H \mathbf{X} \\ \psi_{m,i}(k) &= y_{m,i}(k)/|y_{m,i}(k)|, \quad i = 1, .., M \\ \mathbf{P}_m &= \mathbf{X}\Psi_m^H \\ \mathbf{W}_{m+1} &= \mathbf{R}_{inv}\mathbf{P}_m \end{aligned}$$

until $\|\mathbf{W}_{m+1} - \mathbf{W}_m\|/\|\mathbf{W}_m\| < \xi$

B. Multitarget Least-Squares Constant Modulus Algorithm

The multitarget least-squares constant modulus algorithm (MT-LSCMA) was proposed in [5] as an extension of the LSCMA to blindly adapt a narrowband beamformer. The algorithm separates and captures multiple communication signals by exploiting only their modulus variation. The original MT-LSCMA consists of three principle components:

- a soft-orthogonalized LSCMA
- a set of sorting and classification algorithms
- a fast acquisition algorithm.

Only the first component will be considered here and we will observe the iterative update version of this algorithm. The summary of the algorithm can be given as follows. At the end of the LSCMA, the soft orthogonalization is proceeded through the following steps.

- Compute the hard-orthogonalized equalizer matrix $\hat{\mathbf{W}}$ by using a standard Gram-Schmidt Orthogonalization procedure and find its corresponding output signals

$$\hat{\mathbf{Y}} = \hat{\mathbf{W}}^H \mathbf{X} \quad (9)$$

- Compute a softening parameter λ_i, $i = 1, ..., M$ by initializing $\widetilde{y}_{0,i} = y_i$ and calculating

$$\epsilon_i = \frac{\langle |y_i(k) - \hat{y}_i(k)|^2 \rangle_{N_B}}{\langle |y_i(k)|^2 \rangle_{N_B}}. \quad (10)$$

for $p = 0, 1, ...$

$$\widetilde{\epsilon}_{p,i} = \frac{\langle |\widetilde{y}_{p,i}(k) - \hat{y}_i(k)|^2 \rangle_{N_B}}{\langle |\widetilde{y}_{p,i}(k)|^2 \rangle_{N_B}} > 0 \quad (11)$$

$$\lambda_{p,i} = \sqrt{\frac{\widetilde{\epsilon}_{p,i}(1 - \epsilon_i)}{\epsilon_i(1 - \widetilde{\epsilon}_{p,i})}} \quad (12)$$

$$\widetilde{y}_{p,i}(k) = \lambda_{p,i}\, y_i(k) + (1 - \lambda_{p,i})\, \hat{y}_i(k) \quad (13)$$

until $\epsilon_{p,i}$, $\widetilde{\epsilon}_{p,i} < \epsilon_{\max}$.

C. Least-Squares Multi-User Constant Modulus Algorithm

Another technique based on the LSCMA was proposed in [6], namely least-squares multi-user constant modulus algorithm (LS-MU-CMA). This algorithm combines the LSCMA and MU-CMA [8], and serves as a block CM type algorithm that efficiently performs blind spatial co-channel interference mitigation. The LS-MU-CMA was derived from the MU-CMA cost function defined as

$$J_i = \mathbf{E}\{(|y_i| - d)^2\} + 2\sum_{l=1}^{i-1} |y_i^* y_l|^2. \quad (14)$$

where d is a constant modulus, i.e. $d = 1$. After separating and expanding the expectation in J_i, the resulting weight update equation is obtained in a form identical to the least-squares estimator. Similar to the previous algorithm, we will consider the iterative update version of this algorithm which can be given as follows. Calculate $\mathbf{R}$ and then initialize $\mathbf{W}_0 = \mathbf{I}$. For $m = 0, 1, ..$ until $\|\mathbf{W}_{m+1} - \mathbf{W}_m\|/\|\mathbf{W}_m\| < \xi$, we perform the following steps at each m.

for $i = 1, .., M$

$$\begin{aligned} \mathbf{R}_i &= \mathbf{R} \\ \phi_{ii} &= \mathbf{w}_{m,i}^H \mathbf{R} \mathbf{w}_{m,i} \\ \text{for } j = 1 : i-1 & \\ u_j &= \mathbf{w}_{m,j}^H \mathbf{R} \\ \phi_{ji} &= u_j \mathbf{w}_{m,i} \\ \text{if } |\phi_{ji}| > \gamma|\phi_{ii}| \text{ do} \quad & \mathbf{R}_i = \mathbf{R}_i + u_j^H u_j \\ \mathbf{P}_{m,i} &= \mathbf{X}\Psi_{m,i}^H \\ \mathbf{w}_{m+1,i} &= \mathbf{R}_i^{-1}\mathbf{P}_{m,i} \end{aligned}$$

V. Successive Interference Cancellation Least-Squares Constant Modulus Algorithm

In this section, we propose the first new technique that could serve as an alternative way for blind signal separation. The proposed algorithm combines the LSCMA and the separation technique namely the successive interference cancellation to blindly separate the received signals. This algorithm is therefore named successive interference cancellation least-squares constant modulus algorithm (SIC-LSCMA).

Consider the cost function of the SIC-LSCMA given as

$$J(\mathbf{w}_i) = \sum_{k=1}^{N_B} (|\mathbf{w}_i^H \mathbf{r}_i(k)| - 1)^2 = \|\Phi(\mathbf{w}_i)\|_2^2 \quad (15)$$

where $\Phi = |\mathbf{w}_i^H \mathbf{X}_i| - 1$ and

$$\mathbf{X}_i = \mathbf{X}_{i-1} - \widetilde{\mathbf{h}}_{i-1}\Psi_{i-1} \quad (16)$$

represents the interference cancellation procedure. The channel $\widetilde{\mathbf{h}}_{i-1}$ used in this procedure can be estimated from [9]

$$\widetilde{\mathbf{h}}_{i-1} = \frac{\mathbf{E}\{\mathbf{r}(k)\psi_{i-1}^*(k)\}}{\mathbf{E}\{|\psi_{i-1}|^2\}}. \quad (17)$$

The proposed algorithm can be summarised as follows. Find $\mathbf{R} = \mathbf{X}\mathbf{X}^H$ and its inverse $\mathbf{R}_{inv} = \mathbf{R}^{-1}$. Then initialize $\mathbf{W}_0 = \mathbf{I}$. For $m = 0, 1, ..$ until $\|\mathbf{W}_{m+1} - \mathbf{W}_m\|/\|\mathbf{W}_m\| < \xi$, we perform the following successive interference cancellation at each m by first setting $\widetilde{\mathbf{X}}_1 = \mathbf{X}$.

for $i = 1, ..., M$

- $\mathbf{y}_{m,i} = \mathbf{w}_i^H \widetilde{\mathbf{X}}_i = i^{th}$ row of $\mathbf{Y}_m$
- $\psi_{m,i}(k) = y_{m,i}(k)/|y_{m,i}(k)|$
- $\widetilde{\mathbf{h}}_{m,i} = \widetilde{\mathbf{X}}_i \Psi_{m,i}^H (\Psi_{m,i}\Psi_{m,i}^H)^{-1}$
- $\widetilde{\mathbf{X}}_{i+1} = \widetilde{\mathbf{X}}_i - \widetilde{\mathbf{h}}_{m,i}\Psi_{m,i}$

and then update

$$\mathbf{W}_{m+1} = \mathbf{R}_{inv}\mathbf{X}\Psi_m^H.$$

The SIC-LSCMA requires $O(NMN_B)$ flops for the successive interference cancellation and another $O(NMN_B)$ flops to update $\mathbf{W}$.

VI. Gram-Schmidt Least-Squares Constant Modulus Algorithm

By adding a constraint to the cost function of the LSCMA given in equation (5), the problem of the Gram-Schmidt LSCMA (GS-LSCMA) can be formulated as

$$\begin{aligned} \min \quad & J(\mathbf{W}) = \|\Phi(\mathbf{W})\|_2^2 \\ \text{subject to} \quad & \mathbf{W}^H\mathbf{W} = \mathbf{I} \end{aligned} \quad (18)$$

where $\mathbf{I}$ is the identity matrix. This orthogonality constraint ensures that the output signals are recovered from different input signals and can be implemented by using the well-known Gram-Schmidt orthogonalization technique.

The LSCMA is therefore simply modified by performing the Gram-Schmidt orthogonalization procedure after $\mathbf{W}_{m+1}$ is calculated. The Gram-Schmidt orthogonalization procedure is implemented as

$$\begin{aligned} \mathbf{w}_1^{\text{new}} &= \mathbf{w}_1/\|\mathbf{w}_1\| \\ P &= \sum_{l=1}^{i-1} \left((\mathbf{w}_l^{\text{new}})^H \mathbf{w}_i\right) \mathbf{w}_l^{\text{new}} \\ \mathbf{w}_i^{\text{new}} &= \frac{\mathbf{w}_i - P}{\|\mathbf{w}_i - P\|}, \; i = 2, \ldots, M. \end{aligned} \quad (19)$$

This procedure is the hard orthogonalization step in the MT-LSCMA which has been claimed in the same paper that the excessive misadjustment can occur if this technique is used to separate the output signals. In this mentioned algorithm, the Gram-Schmidt procedure is performed after $\mathbf{W}$ of the LSCMA has converged. It is therefore possible that this procedure may degrade the performance of the MT-LSCMA.

In the GS-LSCMA, however, the Gram-Schmidt orthogonalization procedure is performed immediately after the update of $\mathbf{W}$. The output of this procedure converges simultaneously as the LSCMA converges. The algorithm could therefore be more robust and exhibit better performance than the MT-LSCMA. In addition, when the channel is assumed to be unitary, this procedure has been adopted in an adaptive blind source separation algorithm, namely the multi-user kurtosis (MUK) algorithm proposed in [10], and provides a promising separation performance.

Unfortunately, the Gram-Schmidt orthogonalization procedure is computationally expensive. The modified version of this procedure, that yields less computational complexity of $2NM^2$ flops [11], is then used. As a consequence, the GS-LSCMA requires $O(NMN_B)$ flops to obtain $\mathbf{Y}$ and another $O(NMN_B)$ flops to update $\mathbf{W}$.

VII. Simulation Results

Performance comparison is observed through simulations. A V-BLAST system consisting of $N = M = 2$ was considered and a random instantaneous channel $\mathbf{H}$ was assumed to be unitary. We compared the bit error rate (BER) performance of MT-LSCMA, LS-MUCMA and SIC-LSCMA by varying the signal-to-noise ratio (SNR). In addition, we also observed the BER performance versus the number of block iterations when SNR was fixed at 12 dB.

Parameters were set as follows. The stopping parameter ξ was chosen to be 10^{-4}. N_B was chosen to be 20. The parameters in the algorithms were defined as follows: $\epsilon_{\max} = 0.1$, $\gamma = 1$ and $\delta = 2 \times 10^{-3}$. In

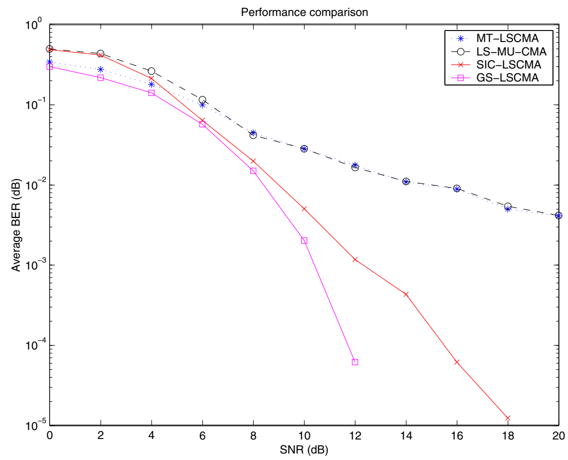

Fig. 2. Bit error rate (BER) comparison (MT-LSCMA, LS-MU-CMA, SIC-LSCMA, GS-LSCMA) for N_B = 20.

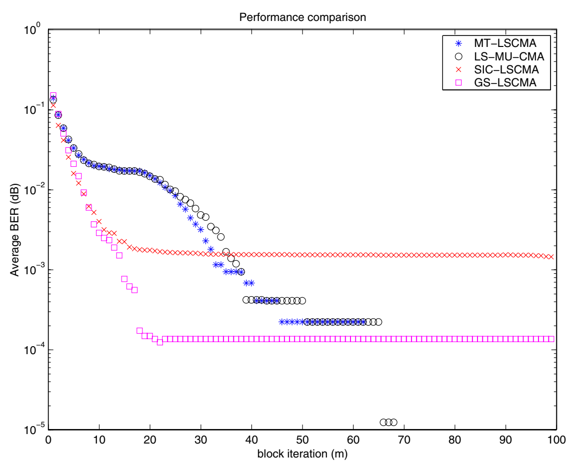

Fig. 3. Bit error rate (BER) comparison (MT-LSCMA, LS-MU-CMA, SIC-LSCMA, GS-LSCMA) for SNR = 12 dB.

addition to these settings, the number of block iterations was limited to 100.

Figures 2 and 3 show the comparison of the blind algorithms (MT-LSCMA, LS-MU-CMA, SIC-LSCMA and GS-LSCMA). It can be observed that the proposed algorithms exhibit better performance and these algorithms need 20 block iterations to achieve its minimum BER at SNR = 12 dB. In addition, the GS-LSCMA outperforms the other algorithms by having the best BER performance.

VIII. Conclusions

We have presented two novel algorithms based on the least-squares constant modulus algorithm (LSCMA). The first algorithm can be referred to as multi-stage blind signal detection since the interference cancellation is performed after each output is detected. The second algorithm has been derived from the orthogonality constraint which ensures that each output is obtained from different inputs. This algorithm is simply performed by implementing the Gram-Schmidt orthogonalization procedure after the LSCMA equalizer matrix is updated. The performance has been observed through simulations and compared with the multitarget LSCMA (MT-LSCMA) and LS multi-user CMA (LS-MU-CMA). The proposed algorithms have exhibited better performance in terms of both bit error rate (BER) and convergence speed with reasonable computational complexity.

References

[1] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas”, *Bell Labs Technical Journal*, Vol. 1, No. 2, pp.41-59, 1996.

[2] P. W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, “V-BLAST: An architecture for realizing very high data rates over the Rich-scattering wireless channel”, invited paper, *Proc. ISSSE-98*, Pisa, Italy, Sept. 29, 1998.

[3] G. D. Golden, C. J. Foschini, R. A. Valenzuela, P. W. Wolniansky, “Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture”, *Electronics Letters*, vol. 35, no. 1, January 1999.

[4] B. G. Agee, “The least-squares CMA: a new technique for rapid correction of constant modulus signals,” *Proc. ICASSP*, vol. 11, pp. 953 - 956, 1986.

[5] B. G. Agee, “Blind separation and capture of communication signals using a multitarget constant modulus beamformer,” *Proc. IEEE Military Communications Conference*, vol. 2, pp. 340 - 346, 1989.

[6] J. Leary, “Least-squares multi-user CMArray: a new algorithm for blind adaptive beamforming,” *Proc. Thirty-First Asilomar Conference on Signals, Systems and Computers*, vol. 1, pp. 902 - 905, 1998.

[7] F. R. P. Cavalcanti, J. M. T. Romano, A. L. Brandão, “Least-squares CMA decorrelation for fast blind multiuser signal separation,” *Proc. ICASSP*, vol. 5, pp. 2527 - 2530, 1999.

[8] C. B. Papadias, A. J. Paulraj, “A constant modulus algorithm for multiuser signal separation in presence of delay spread using antenna arrays,” *IEEE Signal Processing Letters*, vol. 4, issue 6, pp. 178 - 181, 1997.

[9] C.-Y. Chi, C.-Y. Chen, C.-H. Chen, C.-C. Feng, “Batch processing algorithm for blind equalization using higher-order statistics,” *IEEE Signal Processing Magazine*, pp. 25 - 49, 2003.

[10] C. B. Papadias, “Globally convergent blind source separation based on a multiuser kurtosis maximization criterion”, *IEEE Transactions on Signal Processing*, vol. 48, no. 12, pp. 3508 - 3519, 2000.

[11] G. H. Golub, C. F. Van Loan, *Matrix Computations*. Baltimore, MD: John Hopkins University Press, 1990.