

# The Retrieval of NURBS-surface by Genetic Algorithm on the Basis of Point Cloud

Eugene Vladimirovich Popov  
Professor of NNSACEU  
Ilyinskaya Street 65  
603950, Nizhny Novgorod,  
Russia  
popov@sandy.ru

Sergej Igorevich Rotkov  
Professor of NNSACEU  
Ilyinskaya Street 65  
603950, Nizhny Novgorod,  
Russia  
rotkov@nngasu.r

## ABSTRACT

The approach to the geometrical modeling problem solution is described in this report. The approach is dedicated to the approximation of the cloud of points by a NURBS-curve or NURBS-surface and is based on the inheritance mechanism or on the so-called Genetic Algorithm. Genetic Algorithm is the heuristic search and optimization technique that mimics the process of natural evolution. The mechanisms of evolution seem well suited for some of the most pressing scientific problems in many fields. Therefore, the concept of evolution can be applied to solve different computational problems and NURBS-surface retrieval including. The efficiency of the approach is demonstrated by the retrieval of a human face and ship hull surface.

## Keywords

Genetic algorithm, NURBS-surface, ship hull design, point cloud retrieval

## 1. INTRODUCTION

The Genetic Algorithm (GA) is a modern adaptive technique for the solution of functional optimization problems that are frequently used nowadays. It is based on the genetic pattern of biological organisms, namely biological populations that evolve over generations subjecting to the laws of natural selection and the principle "the fittest survive" formulated by Charles Darwin [1]. Similar to these processes the genetic algorithm is able to "solve" real-world problems, if they are properly coded [2]. For example, the GA can be applied to the design of the bridge to find the maximum strength/weight ratio or to determine the most economical pattern when cutting out the fabric cloth. Another example is searching for a set of rules or equations that will predict the ups and downs of a financial market, such as that for foreign currency. Usually such search problems can benefit from an effective use of parallelism, in which many different possibilities are explored simultaneously in an efficient way.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

Genetic algorithm (GA) was invented by John Holland [3] and it is still thoroughly investigated in many studies. In contrast to the natural evolution of living organisms the GA only simulates evolutionary processes in populations that are essential for their development. However, there is not any exact answer to the question which biological processes are essential for the development, and which are not [3].

In wilderness individuals are competing with each other within the population for a set of resources such as food or water. Besides, usually members of the population of one species compete for a mate. Those individuals who are best suited for the environment will be relatively more successful to produce offspring [1]. Poorly adapted individuals either will not produce offspring or their offspring will be very few [3]. It means that the genes of highly suited or adapted species will be available in an increasing number of children of each successive generation. Therefore, a new species will become more and more adaptable to the environment [4].

GA uses a direct analogy of such a mechanism. It works with a group of "individuals" (or "genome") i.e. with a population. Each individual represents a possible problem solution and is assessed according to its "fitness" to how "good" the corresponding problem solution is [3]. In nature a "good solution" means that an individual is capable of competing for resources in the most efficient way [1]. The fittest individuals have the opportunity to "reproduce" the breed by "crossover" with other species of the

population. This leads to the appearance of new species that combine some of the characteristics inherited from their parents. The least adapted individuals are less likely to be able to produce offspring, which in its turn leads to the gradual disappearance of their quality from the population in the process of evolution [1], [3].

In this way the whole new population of feasible solutions is reproduced by selecting the best representatives of the previous generation, crossing them and getting a new set of individuals. This new generation has better characteristics than the good members of the previous generation. Therefore, good features are distributed throughout the population from generation to generation. Hybridization of the fittest individuals leads to the fact that the most promising areas of the search are explored. Eventually, the population converges to an optimal solution [1].

## 2. THE SURFACE RECONSTRUCTION AND GA

Many practical surface reconstruction techniques based on measured data points require the solution of optimization problems in fitting surface data. In general, we need determining the necessary and sufficient conditions for the possibility of GA application to this problem solution.

Let us suppose there is a finite set of geometric parameters  $G = \{g_1, g_2, \dots, g_n\}$ . Let us also assume that a given finite set of conditions  $C = \{c_1, c_2, \dots, c_k\}$  exists. Then we will need to build another finite set of geometric parameters  $T = \{t_1, t_2, \dots, t_m\}$  that satisfies each element of the set of conditions  $C$ . The algorithm of finding set  $T$  is unknown. We should also find such finite set  $\tilde{T}$  the elements of which are the set of geometrical parameters  $\tilde{T}_i$  that meet *at least one condition* of set  $C$ . If they meet *all the conditions* of set  $C$  the problem is solved. Therefore, set  $\tilde{T}$  should meet the following requirements to solve the problem:

- a certain numerical value  $q_i$  must be assigned to each element of set  $\tilde{T}$  ( $\tilde{T}_i$ ) that should indicate the degree of satisfaction of this element to the set of conditions  $C$ ,
- the *mutation* operation over any element of set  $\tilde{T}$  must be defined (see [1], [4]). This operation is aimed at mapping the required element to another element from the same set  $\tilde{T}$ ,

- the *crossover* operation must be defined between any two elements of set  $\tilde{T}$  [4] the result of which is also an element of set  $\tilde{T}$ .

The finitude of set  $\tilde{T}$  is a necessary condition, but for the determination of each element of this set it is sufficient to formulate an appropriate rule. The crossover and mutation operations of GA are similar to the original genetic operations. Therefore, the product of the GA operations can be called "descendant" or "child".

Let us now find the optimal arbitrary problem solution. Suppose there is a one-to-one correspondence between a string of characters and some of the desired solutions of the problem (it is a binary string in GA). Any solution can be encoded as a string and any string can be considered as a solution. By analogy with wildlife these strings can be called *genes*.

The next step is to assign a quality factor to each gene. This means that there is a criterion for choosing the most appropriate problem solution. Then the process of evolution consisting of two major operations on genes crossover and mutation begins. The crossover of two genes is to produce a new gene by gluing pieces of parental genes (see fig. 1)

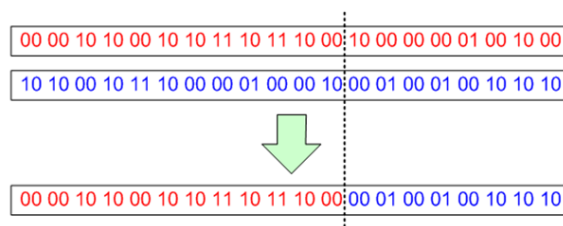


Figure 1. Scheme of crossover between two genes

The "child" partly inherits the properties of parental genes in terms of fitness both negative and positive ones. The replacement of one or more characteristics in a gene at random or by inverting the bit will be called gene mutation (see Fig. 2). As a result of a gene mutation an entirely new problem solution can be obtained (either better or worse than the initial one).

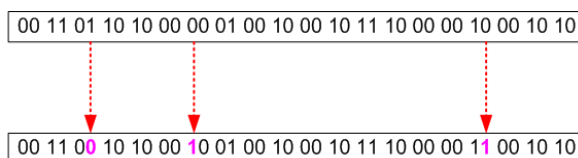


Figure 2. Scheme of gene mutation

After the crossover and mutation are carried out a new generation is re-evaluated and then a new stage of evolution starts. The optimal problem solution here includes the following points:

1. Forming the initial population, i.e. filling a random string array. The size of the initial population is one of the parameters and can vary depending on the problem.
2. Assigning some quality factor to each individual of a given population.
3. Sorting the population in descending order of a quality factor.
4. If the first gene of the population (the best solution) satisfies the problem conditions the evolution stops. Otherwise, crossing the first quarter of population to the second quarter and applying the mutation operator to the second half of population is needed [5].
5. Obtaining a new generation of solutions that are used when going to step two.

All stated above can be summarized in the following aggregative algorithm:

**Step 1.** Select some random sample  $P_0 = \{\tilde{T}_i, \tilde{T}_j, \tilde{T}_k, \dots, \tilde{T}_z\}$  from set  $\tilde{T}$ . The size of the sample is not fixed and for each specific problem it can be set experimentally. This sample will be zero population in the GA evolution and selected elements of set  $\tilde{T}$  are individuals of zero population. Make zero population as current population.

**Step 2.** Sort out all individuals of the current population in descending order of individual quality  $q_i$ .

**Step 3.** If the best individual of the current population satisfies all the conditions of set  $C$  this individual is a problem solution and we quit. Otherwise the next Step is executed.

**Step 4.** Allocate a certain percentage of the best individuals and build the next generation in the population according to the following rules: the next generation consists of the best individuals, their children and mutated bad individuals (mutants).

**Step 5.** Consider a new generation as a current generation and go to Step 2.

Finally, the output will contain the solution that satisfies all the conditions of set  $C$ .

NURBS is one of the most employed surface fitting models, provided that it is a standard representation of curves and surfaces and is widely supported by modern standards like OpenGL and IGES, which are used for graphics and geometric data exchange. In addition, the NURBS surface model has stability, flexibility, local modification properties and is robust to noise.

The NURBS surface fitting problem is usually considered as a non-linear optimization problem. In order to find a good NURBS model from a large number of data, generally the knots, control points and weights are respected as variables [6]. In [7] binary-coded Genetic Algorithm is used for control point optimization and then knot values optimization and the error minimization of parametric surfaces as a global optimization problem is shown. In a similar way using GA, in [8], [9], optimization of both the knots and the weights corresponding to the control points for curve and surface fitting is done. In this study we consider a cubic NURBS surface dependent on control points and corresponding weights.

When using GA for the surface reconstruction on the basis of the point cloud by NUSBS it is necessary to define the basic terms of GA: gene, quality criterion, crossover and mutation. Let there be some surface found by the array of control points and corresponding weights. The explanation in this case is acceptable without loss of generality on the basis of NURBS-curve (see Fig.3).

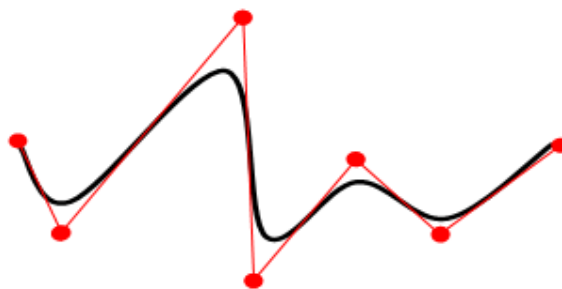


Figure 3. An arbitrary NURBS-curve

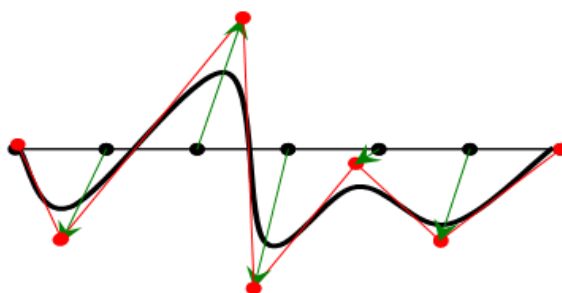


Figure 4. The vector defining the genome curve

We assign the array of co-ordinates of control points and weights to a surface on which each control point and weight is a vector corresponding to the deviation from the reference point at a fixed zero position (see Fig. 4). This can help to establish a correspondence between any of such surfaces and line vectors that can be called a genome. In this study weights are set to one to make NURBS uniform.

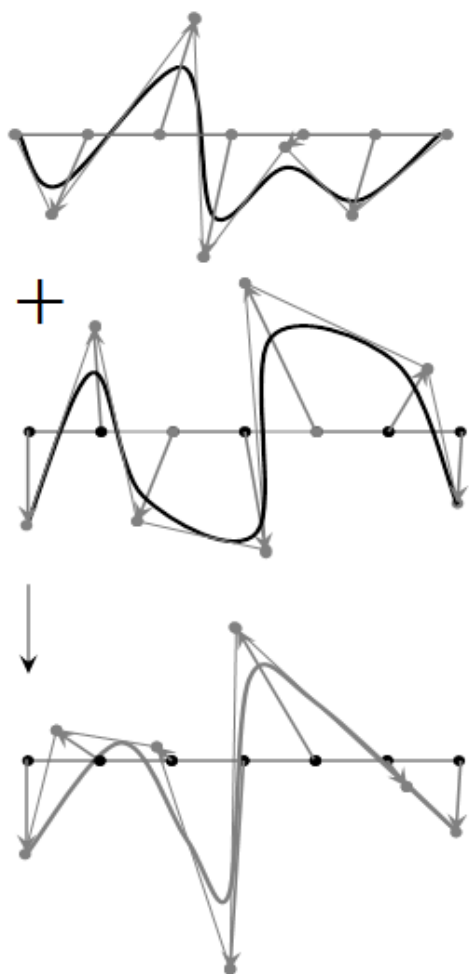


Figure 5. The crossover operation

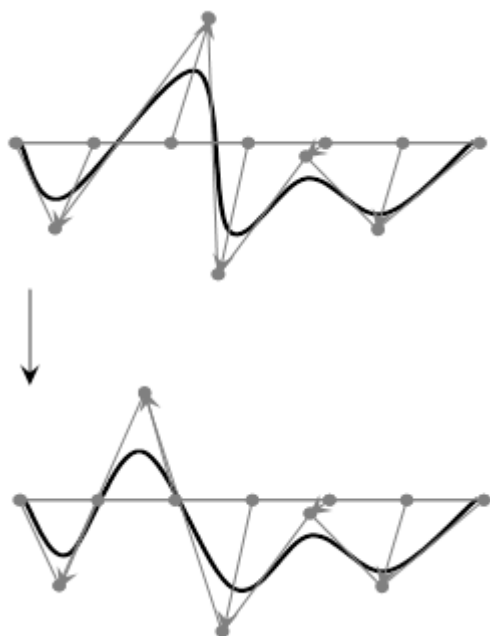


Figure 6. The mutation operation

Next, let us introduce the concept of the gene quality. The criterion can be developed in such a way that the surface reconstructed from the point cloud is as close to a given point as possible. Therefore, the criterion is the minimal total (sum) distance between the point cloud and the NURBS. The smaller is the distance, the better corresponding surface satisfies the problem solution.

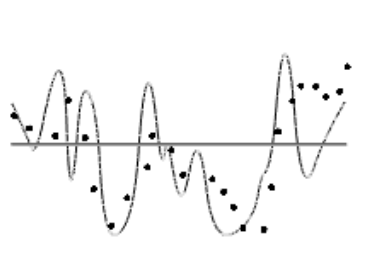
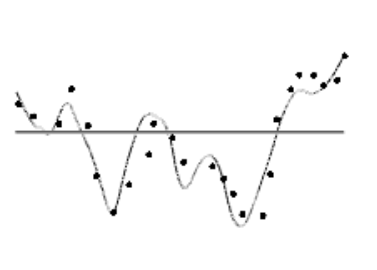
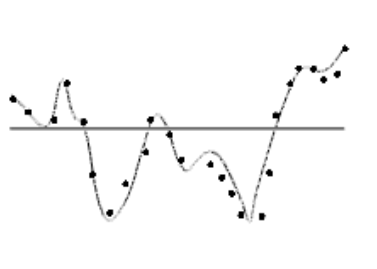
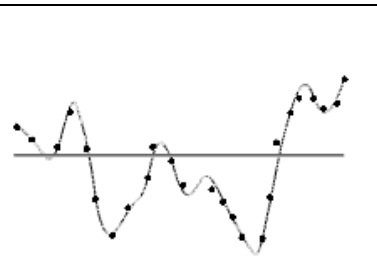
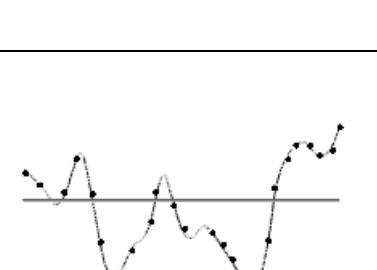
The gene of a descendant is formed on the parent genes basis by adding deviation vectors of the surface control points. This defines the crossover operation (Fig. 5). The mutation operation is done in the following way: any deviation vector in the mutating gene is replaced by a random vector (see Fig. 6).

The developed algorithm is tested for the fitting of the NURBS curve to the set of points. The process of fitting is shown in Table 1. The quality factor in Table 1 means the dimensionless sum distance between the point cloud and the NURBS curve normalized by the point cloud length at a horizontal axis. After 30 generations the evolutionary strategy reached the minimum, i.e. the quality factor decreased 4895 times in comparison with the non-optimized NURBS curve. The quality factor reached the value at about 0.12% of overall point cloud length that complies with requirements.

**The best NURBS**

**Table 1**

Generation #	Best NURBS	Quality factor
0		---
1		5.8746

5		3.5214
15		1.2341
20		0.3294
25		0.1546
30		0.0012

### 3. THE RETRIVAL OF A HUMAN FACE

In Fig. 7 the result of the retrieval process of a human face surface is presented. It is based on the points cloud obtained by 3D laser scanning. The best representatives of some

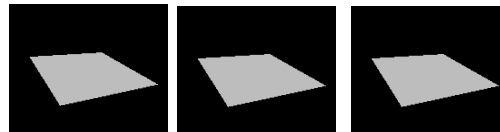
generations are shown in Fig. 7 arranged in triserial order. The criterion to choose the representatives is the minimum of total distance from all the points to the surface.

As we can see in Fig. 7 the quality of the NURBS representation of a human face rises from generation to generation. However, the best representatives at 1500th generation exhibit several artifacts (near the nose and the eyes, and at the boundary of the face). The artifacts at the boundary of the face can be easily eliminated by a trimming option. The artifacts inside the NURBS surface can be also eliminated by further steps of evolution. Generally the quality of the resulting surfaces can be very high. Besides, there are a lot of GA children in the evolution but it is possible to select the most appropriate for use depending on the goals.

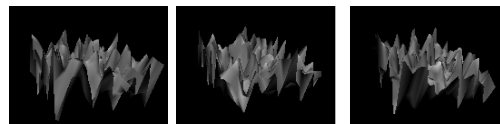
**Initial set of points:**



**0- generation**



**5th generation:**



**900th generation:**



**1500th generation:**



**Figure 7. The best human face representatives of some generations arranged in triserial order**

Thus, this approach allows us to model relatively complex objects such as a whole human face only by one NURBS-surface. A multivariate solution that is especially important in design can be considered as a

positive result. The example described above plays rather a testing role aimed at developing the approach.

#### 4. THE SURFACE OF THE SHIP HULL

The use of GA is also proposed as a method for improving ship hull design through more effective exploration of the design space. The problem of creating fair ship hull surface is of major importance in Computer Aided Ship Design environment. The fairness of these surfaces is one of the most important conditions for the ship structure accuracy. It results in time saving and in improvement of the assembly and hull welding quality. It also reduces the cost of the structure significantly [10], [11], [12]. Currently several methods to control ship surface quality are used:

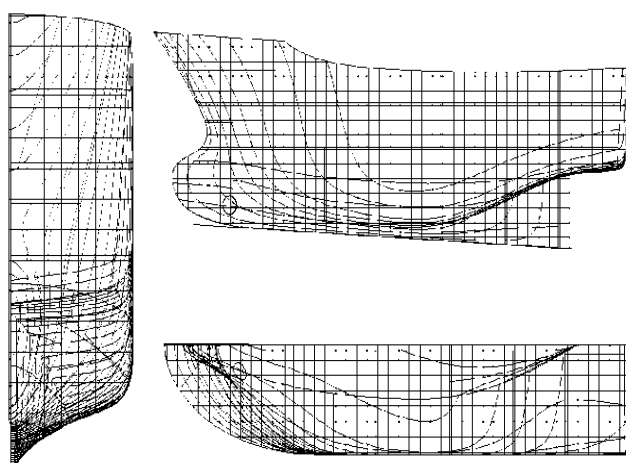
1. The Gaussian curvature visualization which allows identifying the problematic areas of the ship hull surface.
2. Visualization of the curvature radii of curves, surfaces and sections.
3. Visualization of the inflection points and inflection lines of sections and surfaces.
4. Dynamic change of the inflection lines and lines of curvature by the manual surface editing.
5. Visualization of surfaces and sections in a compressed form at one of the coordinate axes. It is a very important feature for modeling surfaces strongly elongated at one of the coordinate axes (wings, ruder etc.).
6. Automatic control of the surface deviation from the original data.

The described means of the surface quality control allows us to abandon the paper drawings completely, to reduce the simulation time sufficiently and to increase the quality of the simulated surface.

When designing the ship hull surface by means of shipbuilding CAD-systems the main purpose is to reduce manual labor. The advantages of GA use in this case are the following. First, genetic algorithm (GA) is a highly effective tool for the exploration of large-scale, nonlinear design spaces and, when combined with gradient based search techniques, may provide a more computationally efficient means of identifying near optimal designs. Second, GA method may provide a high utility tool that can enhance the ship design process. Third, the current design choice method of weighted objective measures of effectiveness can mask potentially useful areas of the design space. Therefore, the approach for the NURBS-surfaces retrieval on the boundaries of parametric quadrilateral defined by diametric buttock has been developed. The approach is based on GA and permits half of the hull surface to be designed without subdividing it into separate patches. Besides,

the design can be based on the general hull curves as well as on a point cloud that can be obtained by different ways including 3D scanning.

NURBS-surface retrieval on points cloud obtained on the basis of the main shipbuilding curves (buttocks, frames and waterlines, see Fig. 8) is carried out by means of the genetic algorithm with the scheme described in Section 2. Usually the main shipbuilding curves contain the information not only about the surface form, but also the information about the smoothness at key hull points and derivatives of any order. When forming a set of points on the basis of these curves all this information is lost. It means that the locus of points remains in a space only through which the surface should pass under design.



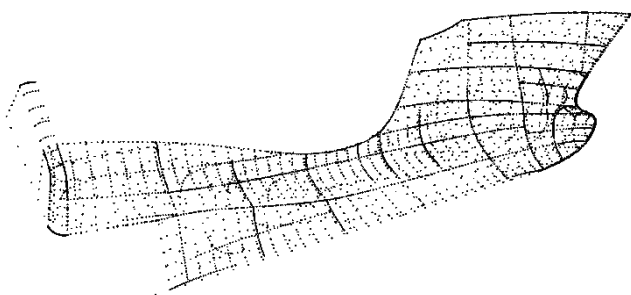
**Figure 8. The theoretical drawing of seagoing trawler hull**

The smoothness of the final NURBS-surface is determined by the degree of consistency of the source of the original theoretical drawing and smoothness of the main hull lines. An example of such points set to build a final NURBS-surface is presented in Fig.9.

Let us consider the process of the surface retrieval for the seagoing trawler as an example. The theoretical drawing of the trawler is shown in Fig.8. [13] The initial data in the form of a point cloud formed on the basis of hull curves are shown in Fig.9. Given that the smoothness of the surface depends on the smoothness of the source hull lines the latter have been smoothed by the Gauss algorithm. This allows obtaining the consistency with the main hull lines.

The Genetic Algorithm was used from the starting point for the half portion of the ship hull design space as described in Section 2. For the initial population we choose a set of NURBS-surfaces with the size of

the box bounding set of points and random deviations of nodes at the Y-axis. Default settings utilized initially included floating point genes. Crossover operations utilized simple procedure, i.e. genes were added between parents. All crossover rates are defined as the number of crossovers, regardless of the total population. Mutation was performed on a specified number of randomly selected individuals at each step of evolution. The quality of each individual surface was the sum distance from each point of cloud to the surface. Thus, we can obtain a set of populations at each step of evolution by standard GA operations such as mutation and crossover. In other words this set is a set of NURBS-surfaces that already meet the requirements of smoothness since they are based on the point cloud that is obtained in turns from the main shipbuilding curves. Finally, the GA allows us to get the surface closer and closer to the initial point cloud that is actually an equivalent to the "growing" of the required surface similar to a living organism.



**Figure 9. The initial point cloud to build NURBS-surface**

The best representatives of the hull surface in some generations are shown in Table 2. The 6th method of ship surface quality control mentioned above is used, that is the automatic control of the surface deviation from the original data. The time of obtaining of each generation representative and the value of the sum distance from the initial cloud to the surface are presented in Table 3. This distance is used as a criterion for the quality of the surface retrieval. The improvement of the last individuals of subsequent generations did not occur.

From Table 3 it is clear that to obtain a ship hull surface represented by a single NURBS-surface with acceptable quality about 3000 generations are needed. This process takes approximately 1 minute of CPU (AMD Phenom™ II N930 Quad-Core Processor 2.00 GHz).

**The best representatives of some generations**

**Table 2**

generation #	Best individual
100	
500	
1500	
3000	

In this case, the overall deviation of the final NURBS-surfaces from the original point cloud does not exceed 83 mm. This corresponds to the shipbuilding accuracy of individual point deviation



from the surface up to  $0.0001B$ , where  $B$  is the width of the hull.

### The quality factor

Table 3

generation #	Quality of the best individual (total distance), mm	Compute time, s
100	17254.612	4.34
500	12703.928	20.53
1500	9870.289	32.92
3000	82.731	48.93

Thus, on the basis of the mathematical NURBS approximation apparatus and the approach based on genetic algorithm for the retrieval of smooth complex ship hull surface is developed. The developed approach can be successfully used for the retrieval of this type of surfaces for which smoothness is a major requirement.

The final NURBS-surface of the trawler hull without plane part of the board is shown in Figure 10. The whole final NURBS-surface of the trawler hull is presented in Figure 11.

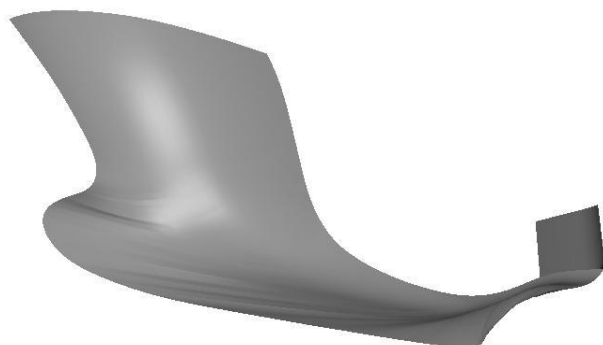


Figure 10. The final NURBS-surface of the hull without plane part of the board

## 4. CONCLUSION

In conclusion it should be stated that the NURBS-surface can be obtained by GA on the controlling points which may be solved on the given scattered point cloud. The genetic gene and evaluating function are given and the controlling points and

their corresponding weight genes are computed. By numerical simulation this approach is verified for the validity of the simplified representation of the fitting surface.

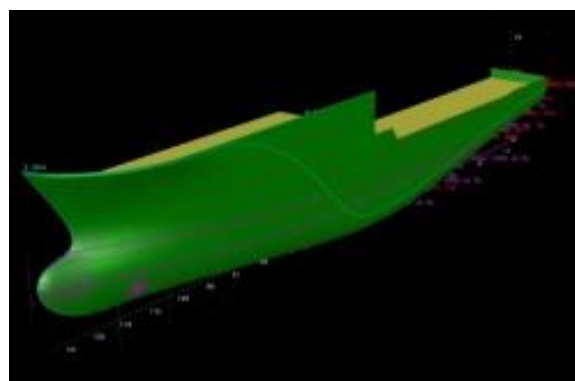


Figure 11. The whole final NURBS-surface

## 5. REFERENCES

- [1] Tomassini M. A survey of genetic algorithms. In D. Stauffer, editor, *Annual Reviews of Computational Physics*, volume III, pages 87-118. World Scientific, 1995.
- [2] Kirkpatrick S., Toulouse G. Configuration space analysis of traveling salesman problems. *J. Phys. (Paris)* v46, 1985
- [3] Holland J. H. *Adaptation in natural and artificial systems*. Ann Arbor: University of Michigan Press. 1975.
- [4] Boyer, D. O., Martnez, C. H. & Pedrajas, N. G. Crossover Operator for Evolutionary Algorithms Based on Population Features, *Applied Soft Computing J.*, 39(3), 2007.
- [5] Aggarwal C. C., Orlin J. B., Tai R. P. Optimized crossover for maximum independent set. *Oper. Res.* v45, 1997.
- [6] E. Ulker, NURBS curve fitting using artificial immune system, *Int. J of Innovative Computing, Information and Control*, vol.8, no.4, April 2012.
- [7] A. Limaiem, A. Nassef and H. A. Elmaghraby, Data fitting using dual krigging and genetic algorithms, *CIRP Annals*, vol.45, pp.129-134, 1996.
- [8] F. Yoshimoto, M. Moriyama and T. Harada, Automatic knot placement by a genetic algorithm for data fitting with a spline, *Proc. of the International Conference on Shape Modeling and Applications*, pp.162-169, 1999.



- [9] M. Sarfraz, Representing shapes by fitting data using an evolutionary approach, *Int. J. of Computer-Aided Design & Applications*, vol.1, no.1-4, pp.179-186, 2004.
- [10] A. Swee Wen, S. M. H. Shamsuddin, Y. Samian., Ship Hull Fairing Using Nurbs, *Proceedings of the Postgraduate Annual Research Seminar*, pp.225-228, 2005.
- [11] A. Swee Wen, Optimization of ship hull NURBS surface fitting using simulated annealing, University of Malaysia, 2007.
- [12] K.G. Pigounakis, P.D. Kaklis and A.D. Papanikolaou: Ship Hull Fairing under Shape and Integral Constraints, *Proceedings of IMAM '95 / 5th Congress of the International Maritime Association of Mediterranean*, Dubrovnik, 1995.
- [13] Popov, E. V., Rekshinsky, A. V. A ship hull surface design using genetic algorithms, "Vestnik IzhGTU", *Periodic scientific and theoretical J.*, Izhevsk, # 3, 2007.