Applied and Computational Mechanics 6 (2012) 185–196

Buckling analysis of thick isotropic plates by using exponential shear deformation theory

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Received 25 October 2012; received in revised form 30 December 2012

Abstract

In this paper, an exponential shear deformation theory is presented for the buckling analysis of thick isotropic plates subjected to uniaxial and biaxial in-plane forces. The theory accounts for a parabolic distribution of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. Governing equations and associated boundary conditions of the theory are obtained using the principle of virtual work. The simply supported thick isotropic square plates are considered for the detailed numerical studies. A closed form solutions for buckling analysis of square plates are obtained. Comparison studies are performed to verify the validity of the present results. The effects of aspect ratio on the critical buckling load of isotropic plates is investigated and discussed.

Keywords: shear deformation, isotropic plates, shear correction factor, buckling analysis, critical buckling load

1. Introduction

When plate is subjected to in-plane compressive forces, and if forces are sufficiently small the equilibrium of plate is stable. If the small additional disturbance result in a large response and the plate does not return to its original equilibrium configuration, the plate is said to be unstable. The onset of instability is called buckling. The magnitude of the in-plane compressive axial forces at which the plate becomes unstable is termed the critical buckling load. The magnitude of the critical buckling load depends on geometry, material properties, as well as on the buckling mode shape.

To predict the critical buckling load of plate, a number of plate theories have been proposed based on considering the transverse shear deformation effect. The well-known classical plate theory (CPT) which neglects the transverse shear deformation effect provides reasonably good results for thin plates and overpredicts the critical buckling loads for thick plates. The Reissner [7] and Mindlin [5] theories are known as the stress based and displacement based first-order shear deformation plate theory (FSDT) respectively, and account for the transverse shear effects by the way of linear variation of in-plane displacements through the thickness of plate. However, these theories do not satisfy the zero traction boundary conditions on the top and bottom surfaces of the plate, and need to use the shear correction factor to satisfy the constitutive relations for transverse shear stresses and shear strains. These shear correction factors are depends on the geometric parameters, boundary conditions and loading conditions.

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To overcome the drawbacks of the FSDT, a number of higher order shear deformation plate theories are developed. A recent reviews of such refined shear deformation theories are presented by Ghugal and Shimpi [1], Wanji and Zhen [13] and Kreja [4]. Recently Shimpi and Patel [9, 10] has developed two variable plate theory for the static and dynamic analysis of thick plate whereas Kim et al. [3] extended this theory for the buckling analysis of isotropic and orthotropic plates. Thai and Kim [12] employed Levy type solution for the buckling analysis of thick plate using two variable plate theory. Ghugal and Pawar [2] applied hyperbolic shear deformation theory for the buckling analysis of plates in which in-plane displacement field uses hyperbolic functions in terms of thickness coordinate to include the shear deformation effect.

In this paper, a displacement based an exponential shear deformation theory (ESDT) presented by Sayyad and Ghugal [8] is extended for the buckling analysis of thick isotropic square plates subjected to uniaxial and biaxial in-plane loads. Governing equations and associated boundary conditions are derived from the principle of virtual work. The Navier's solution is employed for solving the governing equations of square plates with all simply supported edges. The detail procedure of Navier's solution technique is given by Szilard [11]. Comparison studies are performed to verify the validity of the present results. The effects of aspect ratio on the critical buckling loads of isotropic plates are studied and discussed in detail.

2. Theoretical formulation

2.1. Isotropic plate under consideration

Consider a rectangular plate of sides 'a' and 'b' and a constant thickness of 'h' made up of isotropic material and subjected to in-plane compressice forces (N^0_{xx}, N^0_{yy} and N^0_{xy}) as shown in Fig. 1. The co-ordinate system (x, y, z) chosen and the coordinate parameters are such a that, the plate occupies a region given by Eq. (1)

$$0 \le x \le a, \quad 0 \le y \le b, \quad -h/2 \le z \le h/2. \tag{1}$$

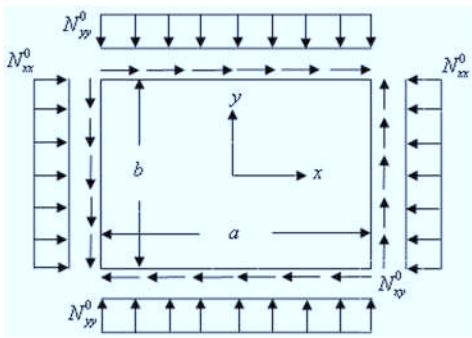


Fig. 1. Plate subjected to in-plane forces

2.2. The displacement field

The displacement field of the proposed plate theory is given by Sayyad and Ghugal [8]

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w(x,y)}{\partial x} + z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\phi(x,y), \\ v(x,y,z) &= -z\frac{\partial w(x,y)}{\partial y} + z\exp\left[-2\left(\frac{z}{h}\right)^2\right]\psi(x,y), \\ w(x,y,z) &= w(x,y). \end{aligned} \tag{2}$$

Here u, v and w are the displacements in the x, y and z-directions respectively. The exponential function in terms of thickness coordinate in both the in-plane displacements u and v is associated with the transverse shear stress distribution through the thickness of plate. The functions ϕ and ψ are the unknown functions associated with the shear slopes. The strain field obtained by using strain-displacement relations can be given as

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + f(z)\frac{\partial \phi}{\partial x}, \\ \varepsilon_y &= \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} + f(z)\frac{\partial \psi}{\partial y}, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y} + f(z)\left(\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}\right), \\ \gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\mathrm{d}f(z)}{\mathrm{d}z}\phi, \\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\mathrm{d}f(z)}{\mathrm{d}z}\psi, \end{aligned} \tag{3}$$

where $f(z) = z\exp\left[-2\left(\frac{z}{h}\right)^2\right]$, $\frac{\mathrm{d}f(z)}{\mathrm{d}z} = \exp\left[-2\left(\frac{z}{h}\right)^2\right]\left[1 - 4\left(\frac{z}{h}\right)^2\right]$.

2.3. *Stress-strain relationships*

The stress-strain relations of an isotropic plate can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{zx} \\ \tau_{yz} \end{Bmatrix} = \frac{E}{1-\mu^2}\begin{bmatrix} 1 & \mu & 0 & 0 & 0 \\ \mu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\mu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\mu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{zx} \\ \gamma_{yz} \end{Bmatrix}, \tag{4}$$

where E is the Young's modulus and μ is the Poisson's ratio of the material.

3. Governing equations and boundary conditions

The principle of virtural work of the plate can be written as

$$\begin{aligned} &\int_{z=-h/2}^{z=h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a} \left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx} + \tau_{xy}\delta\gamma_{xy}\right]\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z - \\ &\int_{y=0}^{y=b}\int_{x=0}^{x=a} q(x,y)\delta w\,\mathrm{d}x\,\mathrm{d}y - \\ &\int_{y=0}^{y=b}\int_{x=0}^{x=a}\left[N_{xx}^0\frac{\partial^2 w}{\partial x^2} + N_{yy}^0\frac{\partial^2 w}{\partial y^2} + 2N_{xy}^0\frac{\partial^2 w}{\partial x \partial y}\right]\delta w\,\mathrm{d}x\,\mathrm{d}y = 0, \end{aligned} \tag{5}$$

where $q(x, y)$is the transverse load acting in the downword z direction on surface $z = -h/2$. Substituting Eqs. (3)–(4) into Eq. (5) and integrating through the thickness of the plate, the governing differential equations and associated boundary conditions in-terms of stress resultants are as follows:

$$\begin{aligned}\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N^0_{xx}\frac{\partial^2 w}{\partial x^2} + N^0_{yy}\frac{\partial^2 w}{\partial y^2} + 2N^0_{xy}\frac{\partial^2 w}{\partial x \partial y} + q(x,y) &= 0,\\ \frac{\partial N_{sx}}{\partial x} + \frac{\partial N_{sxy}}{\partial y} - N_{Tcx} &= 0,\\ \frac{\partial N_{sy}}{\partial y} + \frac{\partial N_{sxy}}{\partial x} - N_{Tcy} &= 0.\end{aligned} \tag{6}$$

The boundary conditions at $x = 0$ and $x = a$ obtained are of the following form:

$$\begin{aligned}&\text{Either} \quad V_x = 0 \text{ or } w \text{ is prescribed,}\\ &\text{either} \quad M_x = 0 \text{ or } \frac{\partial w}{\partial x} \text{ is prescribed,}\\ &\text{either} \quad N_{sx} = 0 \text{ or } \phi \text{ is prescribed,}\\ &\text{either} \quad N_{sxy} = 0 \text{ or } \psi \text{ is prescribed.}\end{aligned} \tag{7}$$

The boundary conditions at $y = 0$ and $y = b$ obtained are of the following form:

$$\begin{aligned}&\text{Either} \quad V_y = 0 \text{ or } w \text{ is prescribed,}\\ &\text{either} \quad M_y = 0 \text{ or } \frac{\partial w}{\partial y} \text{ is prescribed,}\\ &\text{either} \quad N_{sxy} = 0 \text{ or } \phi \text{ is prescribed,}\\ &\text{either} \quad N_{sy} = 0 \text{ or } \psi \text{ is prescribed.}\end{aligned} \tag{8}$$

Reaction at the corners of the plate is of the following form:

$$\text{Either} \quad M_{xy} = 0 \quad \text{or } w \text{ is prescribed.} \tag{9}$$

The stress resultants appear in the governing equations and boundary conditions are given as:

$$\begin{aligned}(M_x, M_y, M_{xy}) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z \, \mathrm{d}z,\\ (N_{sx}, N_{sy}, N_{sxy}) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) f(z) \, \mathrm{d}z,\\ (N_{Tcx}, N_{Tcy}) &= \int_{-h/2}^{h/2} (\tau_{zx}, \tau_{yz}) \frac{\mathrm{d}f(z)}{\mathrm{d}z} \, \mathrm{d}z,\\ V_x &= \frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y},\\ V_y &= \frac{\partial M_y}{\partial y} + 2\frac{\partial M_{xy}}{\partial x}.\end{aligned} \tag{10}$$

The governing differential equations in-terms of unknown displacement variables used in the displacement field (w, ϕ and ψ) obtained are as follows:

$$
\begin{aligned}
D_1\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-D_2\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}+\frac{\partial^3\psi}{\partial y^3}+\frac{\partial^3\psi}{\partial x^2\partial y}\right)&=\\
q(x,y)+N^0_{xx}\frac{\partial^2 w}{\partial x^2}+N^0_{yy}\frac{\partial^2 w}{\partial y^2}+2N^0_{xy}\frac{\partial^2 w}{\partial x\partial y}&,\\
D_2\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)-D_3\left(\frac{\partial^2\phi}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2\phi}{\partial y^2}\right)+D_4\phi-D_3\left(\frac{1+\mu}{2}\right)\frac{\partial^2\psi}{\partial x\partial y}&=0,\\
D_2\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)-D_3\left(\frac{1-\mu}{2}\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+D_4\psi-D_3\left(\frac{1+\mu}{2}\right)\frac{\partial^2\phi}{\partial x\partial y}&=0.
\end{aligned}
\tag{11}
$$

The associated consistent boundary conditions in-terms of unknown displacement variables obtained along the edges $x = 0$ and $x = a$ are as below:

$$
\begin{aligned}
D_1\left[\frac{\partial^3 w}{\partial x^3}+(2-\mu)\frac{\partial^3 w}{\partial x\partial y^2}\right]-&\\
D_2\left[\frac{\partial^2\phi}{\partial x^2}+(1-\mu)\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\psi}{\partial x\partial y}\right]&=0 \qquad \text{or } w \text{ is prescribed},\\
D_1\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)-D_2\left(\frac{\partial\phi}{\partial x}+\mu\frac{\partial\psi}{\partial y}\right)&=0 \qquad \text{or } \frac{\partial w}{\partial x} \text{ is prescribed},\\
D_2\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right)-2D_3\left(\frac{\partial\phi}{\partial x}+\mu\frac{\partial\psi}{\partial y}\right)&=0 \qquad \text{or } \phi \text{ is prescribed},\\
D_3\left(\frac{\partial\psi}{\partial x}+\frac{\partial\phi}{\partial y}\right)-D_2\frac{\partial^2 w}{\partial x\partial y}&=0 \qquad \text{or } \psi \text{ is prescribed}.
\end{aligned}
\tag{12}
$$

The associated consistent boundary conditions in-terms of unknown displacement variables obtained along the edges $y = 0$ and $y = b$ are as below:

$$
\begin{aligned}
D_1\left[\frac{\partial^3 w}{\partial y^3}+(2-\mu)\frac{\partial^3 w}{\partial x^2\partial y}\right]-&\\
D_2\left[\frac{\partial^2\psi}{\partial y^2}+(1-\mu)\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\phi}{\partial x\partial y}\right]&=0 \qquad \text{or } w \text{ is prescribed},\\
D_1\left(\mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-D_2\left(\mu\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}\right)&=0 \qquad \text{or } \frac{\partial w}{\partial y} \text{ is prescribed},\\
D_3\left(\frac{\partial\psi}{\partial x}+\frac{\partial\phi}{\partial y}\right)-D_2\frac{\partial^2 w}{\partial x\partial y}&=0 \qquad \text{or } \phi \text{ is prescribed},\\
D_2\left(\mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-2D_3\left(\mu\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y}\right)&=0 \qquad \text{or } \psi \text{ is prescribed}.
\end{aligned}
\tag{13}
$$

The boundary condition in-terms of unknown displacement variables (w, ϕ and ψ) obtained along the corners of plate is:

$$
2D_1\frac{\partial^2 w}{\partial x\partial y}-D_2\left(\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}\right)=0 \quad \text{or } w \text{ is prescribed}, \tag{14}
$$

where constants appeared in governing equations and boundary conditions are as follows:

$$
D_1=\frac{Eh^3}{12(1-\mu^2)}, \qquad D_2=\frac{A_0E}{(1-\mu^2)}, \qquad D_3=\frac{B_0E}{(1-\mu^2)}, \qquad D_4=\frac{C_0E}{2(1+\mu)} \tag{15}
$$

and

$$A_0 = \int_{-h/2}^{h/2} z f(z)\,\mathrm{d}z, \qquad B_0 = \int_{-h/2}^{h/2} f^2(z)\,\mathrm{d}z, \qquad C_0 = \int_{-h/2}^{h/2} \left[\frac{\mathrm{d}f(z)}{\mathrm{d}z}\right]^2 \mathrm{d}z. \tag{16}$$

4. Buckling analysis of isotropic plates subjected to in-plane forces

The critical buckling loads of simply supported, isotropic, square plate will be determined in this paper by using the Navier solution. For the buckling analysis, we assume that the only applied loads are the in-plane forces and all other forces are zero, i.e. $q(x,y) = 0$. The governing equations of plate in case of static buckling when, $N_{xx}^0 = -N_0$, $N_{yy}^0 = kN_{xx}^0$ and $N_{xy}^0 = 0$ are given by

$$\begin{aligned}
&D_1\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) - D_2\left(\frac{\partial^3 \phi}{\partial x^3} + \frac{\partial^3 \phi}{\partial x \partial y^2} + \frac{\partial^3 \psi}{\partial y^3} + \frac{\partial^3 \psi}{\partial x^2 \partial y}\right) = \\
&\qquad\qquad -N_0\left(\frac{\partial^2 w}{\partial x^2} + k\frac{\partial^2 w}{\partial y^2}\right), \\
&D_2\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) - D_3\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 \phi}{\partial y^2}\right) + D_4\phi - D_3\left(\frac{1+\mu}{2}\right)\frac{\partial^2 \psi}{\partial x \partial y} = 0, \\
&D_2\left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right) - D_3\left(\frac{1-\mu}{2}\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) + D_4\psi - D_3\left(\frac{1+\mu}{2}\right)\frac{\partial^2 \phi}{\partial x \partial y} = 0.
\end{aligned} \tag{17}$$

The following are the boundary conditions of the simply supported isotropic plate

$$w = \psi = M_x = N_{sx} = 0 \quad \text{at } x = 0 \text{ and } x = a, \tag{18}$$

$$w = \phi = M_y = N_{sy} = 0 \quad \text{at } y = 0 \text{ and } y = b. \tag{19}$$

4.1. The Navier's solution

The following displacement functions $w(x,y)$, $\phi(x,y)$, and $\psi(x,y)$ are chosen to automatically satisfy the boundary conditions in Eqs. (18)–(19)

$$\begin{aligned}
w(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} \sin\alpha x \sin\beta y, \\
\phi(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi_{mn} \cos\alpha x \sin\beta y, \\
\psi(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_{mn} \sin\alpha x \cos\beta y,
\end{aligned} \tag{20}$$

where $\alpha = m\pi/a$, $\beta = n\pi/b$ and w_{mn}, ϕ_{mn} and ψ_{mn} are unknown coefficients. Substitute Eq. (20) in the set of three governing differential Eq. (17) resulting the following matrix form

$$\left\{\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - N_0 \begin{bmatrix} N_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\} \begin{Bmatrix} w_{mn} \\ \phi_{mn} \\ \psi_{mn} \end{Bmatrix} = 0, \tag{21}$$

where

$$\begin{aligned}
K_{11} &= D_1(\alpha^4 + \beta^4 + 2\alpha^2\beta^2),\\
K_{12} &= -D_2(\alpha^3 + \alpha\beta^2),\\
K_{13} &= -D_2(\beta^3 + \alpha^2\beta),\\
K_{22} &= D_3\left[\frac{1-\mu}{2}\beta^2 + \alpha^2\right] + D_4,\\
K_{23} &= D_3\left(\frac{1+\mu}{2}\right)\alpha\beta,\\
K_{33} &= D_3\left[\frac{1-\mu}{2}\alpha^2 + \beta^2\right] + D_4,\\
N_{11} &= \alpha^2 + k\beta^2.
\end{aligned} \tag{22}$$

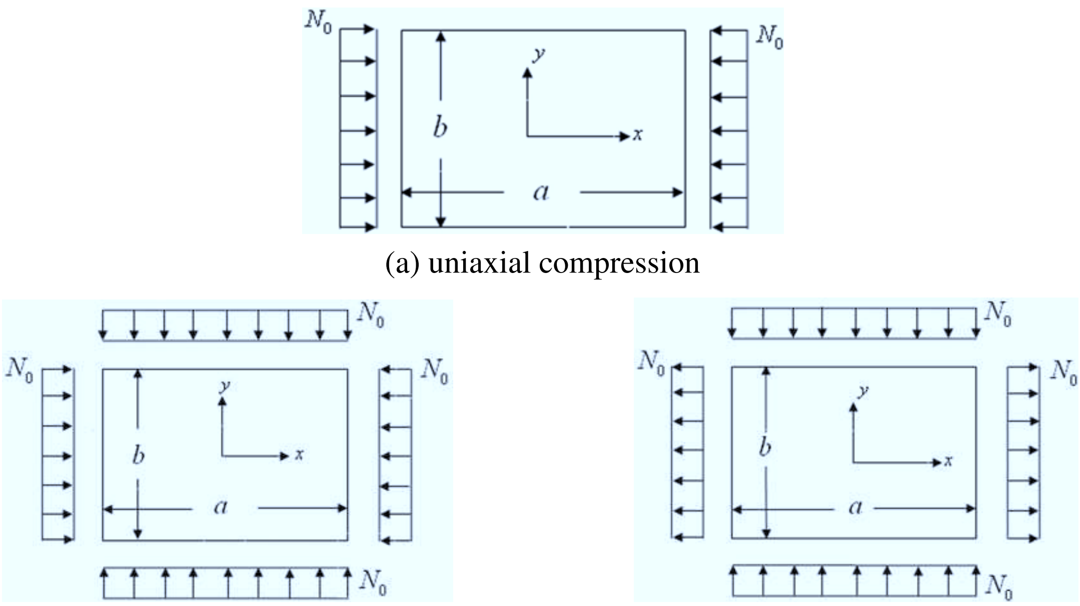
For nontrivial solution, the determinant of the coefficient matrix in Eq. (21) must be zero. This gives the following expression for buckling load:

$$N_0 = \frac{K_{11}\begin{vmatrix} K_{22} & K_{23}\\ K_{32} & K_{33}\end{vmatrix} - K_{12}\begin{vmatrix} K_{21} & K_{23}\\ K_{31} & K_{33}\end{vmatrix} + K_{13}\begin{vmatrix} K_{21} & K_{22}\\ K_{31} & K_{32}\end{vmatrix}}{N_{11}\begin{vmatrix} K_{22} & K_{23}\\ K_{32} & K_{33}\end{vmatrix}}. \tag{23}$$

For each choice of m and n, there is a corresponsive unique value of N_0. The critical buckling load is the smallest value of $N_0(m, n)$.

5. Numerical results and discussion

A simply supported square ($a = b$) plate subjected to the loading conditions, as shown in Fig. 2, is considered to illustrate the accuracy of the present theory in predicting the buckling behavior of the isotropic plate.



(a) uniaxial compression

(b) biaxial compression

(c) tension in the x direction and compression in the y direction

Fig. 2. The loading conditions of square plate for (a) uniaxial compression, (b) biaxial compression and (c) tension in the x direction and compression in the y direction

Results obtained for critical buckling load and the effects of aspect ratio on the critical buckling load of isotropic plates is investigated and discussed in detail. For verification purpose, corresponding results are also generated by higher order shear deformation theory (HSDT) of Reddy [6], classical plate theory (CPT) and first order shear deformation theory (FSDT) of Mindlin [5]. The exact elasticity solution for buckling analysis of plate is not available in the literature. Following material properties of isotropic plates are used

$$\begin{aligned} E &= 210 \text{ GPa and } \mu = 0.3, \\ E &= 70 \text{ GPa and } \mu = 0.33. \end{aligned} \tag{24}$$

For convenience, the following nondimensional buckling load is used:

$$\bar{N}_{cr} = \frac{N_0 a^2}{E h^3}. \tag{25}$$

5.1. Discussion of results

The results of critical buckling load of simply supported square plates are presented in Tables 1–6 and Figs. 3–5. Tables 1–3 shows the comparison of critical buckling load for the steel plates whereas Tables 4–6 shows the comparison of critical buckling load for the aluminum plates subjected to in-plane forces. In case of plate subjected to uniaxial compression (Fig. 2a) and biaxial compression (Fig. 2b), buckling load is critical when mode for the plate is $(1, 1)$ whereas in case of plate subjected to tension in x direction and compression in y direction (Fig. 2c), buckling load is critical when mode for the plate is $(1, 2)$.

Table 1. Comparison of non-dimensional critical buckling load ($\bar{N}_{cr}$) of square plates subjected to uniaxial compression ($k = 0$, $E = 210$ GPa and $\mu = 0.3$)

Mode for the plate		Aspect Ratio ($S = a/h$)				
(m, n)	Theory	5	10	20	50	100
$(1, 1)$	Present (ESDT)	2.960 3	3.424 2	3.565 4	3.607 2	3.613 2
	Reddy (HSDT)	2.951 2	3.422 4	3.564 9	3.606 8	3.613 0
	Mindlin (FSDT)	2.949 8	3.422 2	3.564 9	3.607 1	3.613 0
	CPT	3.615 2	3.615 2	3.615 2	3.615 2	3.615 2

Table 2. Comparison of non-dimensional buckling load ($\bar{N}_{cr}$) of square plates subjected to biaxial compression ($k = 1$, $E = 210$ GPa and $\mu = 0.3$)

Mode for the plate		Aspect Ratio ($S = a/h$)				
(m, n)	Theory	5	10	20	50	100
$(1, 1)$	Present (ESDT)	1.480 2	1.712 1	1.782 7	1.803 8	1.806 5
	Reddy (HSDT)	1.475 6	1.711 2	1.782 5	1.803 4	1.806 5
	Mindlin (FSDT)	1.474 9	1.711 1	1.782 5	1.803 5	1.806 5
	CPT	1.807 6	1.807 6	1.807 6	1.807 6	1.807 6

Table 3. Comparison of non-dimensional critical buckling load ($\bar{N}_{cr}$) of square plates subjected tension in the x direction and compression in the y direction ($k = 1$, $E = 210$ GPa and $\mu = 0.3$)

Mode for the plate (m, n)	Theory	Aspect Ratio ($S = a/h$) 5	10	20	50	100
$(1, 2)$	Present (ESDT)	4.879 8	6.613 3	7.277 7	7.489 8	7.521 2
	Reddy (HSDT)	4.827 4	6.602 4	7.275 4	7.489 3	7.520 1
	Mindlin (FSDT)	4.815 8	6.601 0	7.275 3	7.489 5	7.521 1
	CPT	7.531 7	7.531 7	7.531 7	7.531 7	7.531 7

Table 4. Comparison of non-dimensional critical buckling load ($\bar{N}_{cr}$) of square plates subjected to uniaxial compression ($k = 0$, $E = 70$ GPa and $\mu = 0.33$)

Mode for the plate (m, n)	Theory	Aspect Ratio ($S = a/h$) 5	10	20	50	100
(1, 1)	Present (ESDT)	2.999 1	3.488 6	3.638 8	3.683 3	3.689 8
	Reddy (HSDT)	2.989 3	3.486 6	3.638 3	3.683 3	3.689 6
	Mindlin (FSDT)	2.987 7	3.486 5	3.638 3	3.683 2	3.690 0
	CPT	3.691 9	3.691 9	3.691 9	3.691 9	3.691 9

Table 5. Comparison of non-dimensional critical buckling load ($\bar{N}_{cr}$) of square plates subjected to biaxial compression ($k = 1$, $E = 70$ GPa and $\mu = 0.33$)

Mode for the plate (m, n)	Theory	Aspect Ratio ($S = a/h$) 5	10	20	50	100
$(1, 1)$	Present (ESDT)	1.499 5	1.744 3	1.819 4	1.841 6	1.844 9
	Reddy (HSDT)	1.494 7	1.743 3	1.819 2	1.841 6	1.844 8
	Mindlin (FSDT)	1.493 9	1.743 3	1.819 2	1.841 5	1.845 0
	CPT	1.845 9	1.845 9	1.845 9	1.845 9	1.845 9

Table 6. Comparison of non-dimensional critical buckling load ($\bar{N}_{cr}$) of square plates subjected tension in the x direction and compression in the y direction ($k = 1$, $E = 70$ GPa and $\mu = 0.33$)

Mode for the plate (m, n)	Theory	Aspect Ratio ($S = a/h$) 5	10	20	50	100
$(1, 2)$	Present (ESDT)	4.908 3	6.717 2	7.420 8	7.646 8	7.680 3
	Reddy (HSDT)	4.852 3	6.705 5	7.418 4	7.646 5	7.680 4
	Mindlin (FSDT)	4.839 8	6.704 0	7.418 3	7.646 5	7.681 0
	CPT	7.691 5	7.691 5	7.691 5	7.691 5	7.691 5

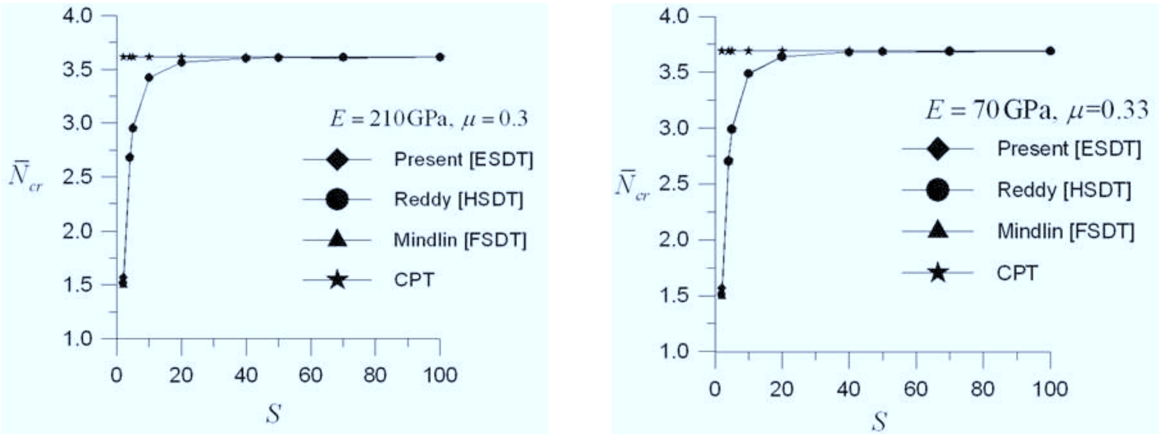


Fig. 3. The effect of aspect ratios on the critical buckling load of square plate subjected to uniaxial compression

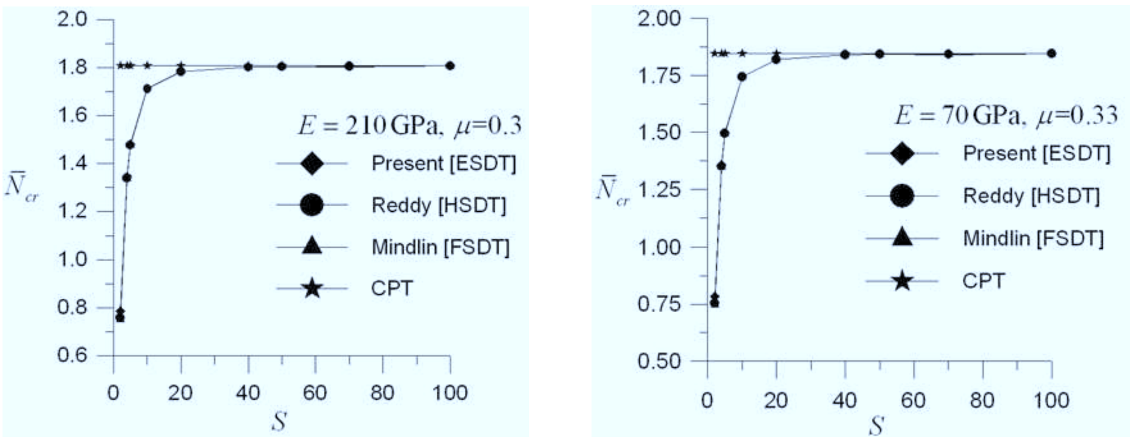


Fig. 4. The effect of aspect ratios on the critical buckling load of square plate subjected to biaxial compression

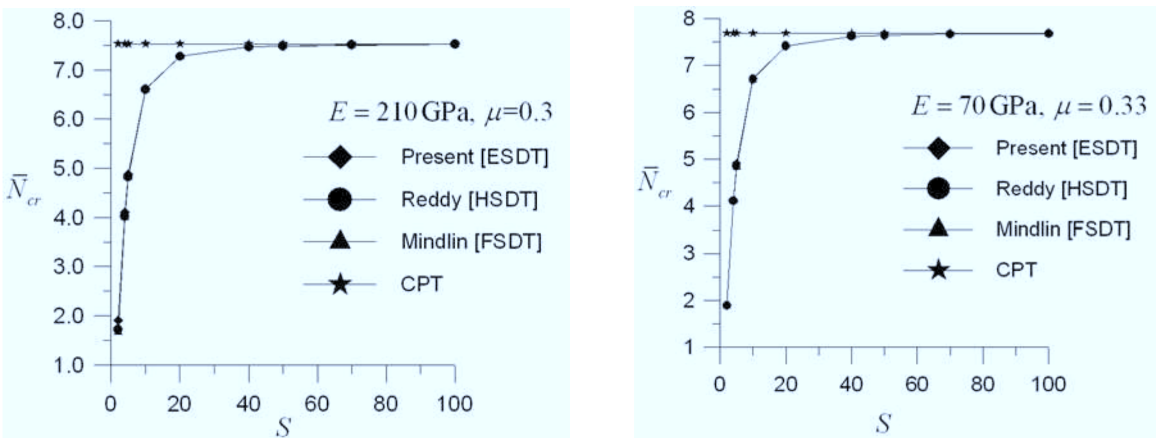


Fig. 5. The effect of aspect ratios on the critical buckling load of square plate subjected to tension in the x direction and compression in the y direction

From the examination of Tables 1–6, it is observed that, the critical buckling load obtained by present theory (ESDT) and Reddy's theory (HSDT) is in excellent agreement with each other even though the plate is very thick due to inclusion of effect of transverse shear deformation. It is also observed that, the value of critical buckling load is increased with increase in aspect (a/h) ratio. As compared to ESDT and HSDT, FSDT underestimates the values of critical buckling load in all cases due to use of shear correction factor (5/6) whereas CPT overestimates the same due to neglect of transverse shear deformation. In case of CPT, critical buckling load is independent of aspect ratio. Figs. 3–5 shows that, for the higher value of aspect (a/h) ratio, the results obtained by ESDT, HSDT, FSDT and CPT are more or less same.

6. Conclusions

An exponential shear deformation theory (ESDT) presented by Sayyad and Ghugal [8] has been applied in this paper for buckling behavior of thick isotropic plates. From the numerical results and discussions following conclusions are drawn.

1. The theory takes account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate, hence it is unnecessary to use shear correction factors.

2. It can be concluded that the presented theory can accurately predict the critical buckling loads of the isotropic plates.

3. Critical buckling load is increased with increase in aspect ratio.

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