Properties of Three Shadows on a Plane

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ABSTRACT

We describe the geometrical properties of solid's shadows projected onto a flat plane by point-light sources. Even though the shape and position of each element are arbitrary and unknown, we indicate that there exists a certain constraint between three shadows on the plane; i.e., three cross points determined by pairs of tangential lines must sit on a straight line. This property is considered to be a derivation of epipolar geometry. However, the situation we treat is a special one in which an explicit feature can emerge. We also show the geometrical meaning of this property by invoking other properties.

Keywords

Epipolar Geometry, Shape from Silhouette, Shadow.

1. INTRODUCTION

Studies on the geometry of shadows have been conducted mainly in the fields of computer graphics (CG) and 3-D recognition of images.

There are several kinds of shadows, however, we deal with only the most basic one that satisfies the following conditions: (1) light source is a point, (2) the shadow is generated only by a direct light effect, i.e., not by ambient and diffuse light, (3) the shadow is a so-called cast shadow; i.e., the shadow is projected on an object such as a wall or the ground, and (4) the shape of the object onto which the shadow is projected is flat. Figure 1 illustrates an example of such shadows.

In this paper, we derive an interesting property about the geometry of these shadows. It can be summarized as follows:

Property: *Consider a flat plane (screen) on which three shadows of a single solid are projected by three different light sources. First, we select two shadows, and draw two lines so that they are common tangential lines of these two shadows. Next, we define an intersection of these two lines as* $\boldsymbol{e}_{12}$ *(Fig. 2 (a)). Since three shadows makes three different combinations of the pair, we can make three different intersection points* $\boldsymbol{e}_{12}$ *,* $\boldsymbol{e}_{23}$ *, and* $\boldsymbol{e}_{31}$ *(Figs 2 (a), (b), and (c)). These three points necessarily sit on a straight line (Fig. 2 (d)).*

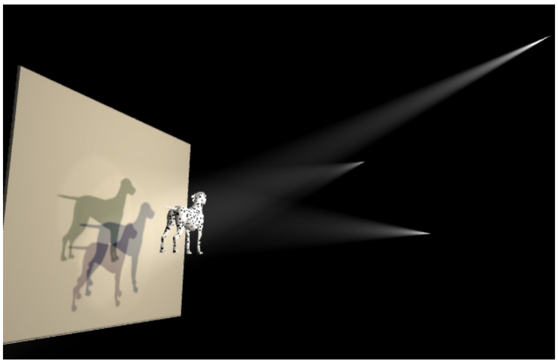
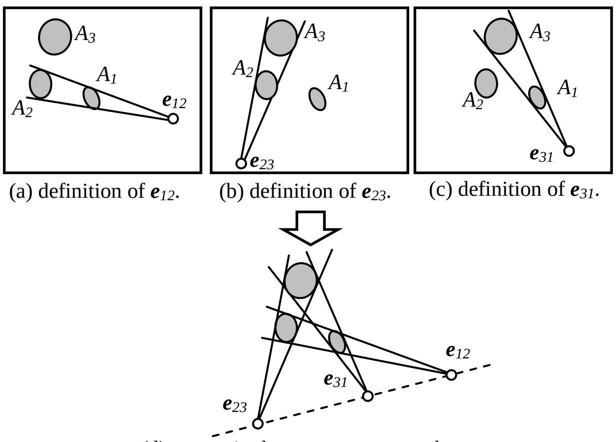


Figure 1: Shadows on a plane projected by point-light sources.



(a) definition of $\boldsymbol{e}_{12}$. (b) definition of $\boldsymbol{e}_{23}$. (c) definition of $\boldsymbol{e}_{31}$.

(d) constraint between $\boldsymbol{e}_{12}$, $\boldsymbol{e}_{23}$, and $\boldsymbol{e}_{31}$.

Figure 2: Geometrical properties of three shadows on the plane.

With this property, the shape and pose of the solid can be arbitrary and unknown; furthermore, the position of the three light sources can also be arbitrary and unknown. It must be remarked that such a specific property can hold even though any explicit correspondence between shadows is not known, i.e., each contour of the shadow generally corresponds to a different part of the solid object when the light sources are different.

1.1 Related Work

Studies of shape from silhouette have a great deal of relationship to our topic, even though most treat camera images, rather than shadows. In terms of projective geometry, the subjects of camera images and shadows on a plane basically have the same principle. We can take the camera as the point-light source, and the projective plane in perspective projection as the plane on which the shadow is projected.

Silhouettes are images of contour generators on a surface of the solid object [1]. One of the remarkable aspects about silhouette images is the existence of the so-called frontier point [2][3], which is the intersection of two contour generators and lies on an epipolar plane tangent to the surface (Fig. 3). The discussion described in this paper can be regarded as one of the singular conditions of this property.

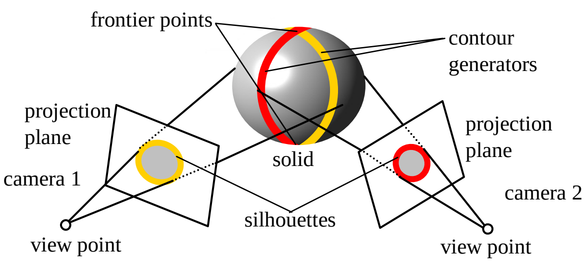


Figure 3: Silhouette of two camera images.

2. CONSTRAINT FOR SHADOWS OF POINTS

2.1 Single Point and Two Light Sources

Consider a situation in which a single point is projected by two light sources (Fig. 4).

Since the point P_A and the two light sources $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ make a single plane Ω_{12A}, we can derive the following property of the shadows as shown in Fig. 5:

***Property 1**: Any 3-D point's shadows $\boldsymbol{c}_{1K}$, $\boldsymbol{c}_{2K}$, projected by two arbitrary light sources must be on a line C_{12K} that goes through a single point $\boldsymbol{e}_{12}$ determined by a line segment from one light source $\boldsymbol{b}_1$ to the other $\boldsymbol{b}_2$.*

The geometrical condition shown in Fig. 5 can be considered to be a particular case of the epipolar constraint [4][5] in the camera image. Therefore, $\boldsymbol{e}_{12}$ is substantially equivalent to the epipole, and C_{12K} is also substantially equivalent to the epipolar line.

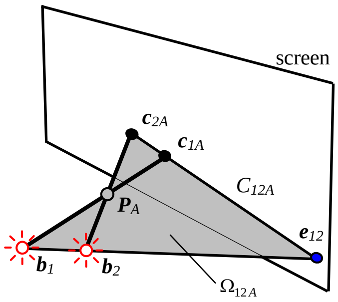


Figure 4: Intersectional line C_{12A} determined by two planes (screen and Ω_{12A}).

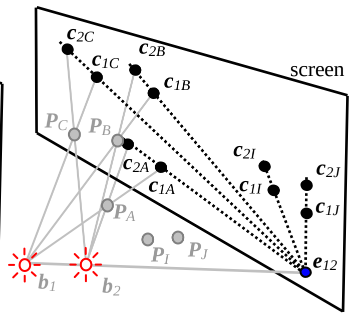


Figure 5: Properties of shadows projected by two light sources.

2.2 Three Light Sources

Next, consider the geometrical condition of $\boldsymbol{e}_{ij}$ cited in subsection 2.1, when the number of light sources is three.

Since $\boldsymbol{e}_{ij}$ is the intersection of line $\boldsymbol{b}_i$ - $\boldsymbol{b}_j$ and the screen, there are three $\boldsymbol{e}_{ij}$ s ($\boldsymbol{e}_{12}$, $\boldsymbol{e}_{23}$, $\boldsymbol{e}_{31}$), in combinations of $\boldsymbol{b}_1 - \boldsymbol{b}_2$, $\boldsymbol{b}_2 - \boldsymbol{b}_3$, and $\boldsymbol{b}_3 - \boldsymbol{b}_1$ (Fig. 6). In addition, since three points $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, and $\boldsymbol{b}_3$ necessarily define a plane Ω_{123}, the following property must hold:

***Property 2**: Three epipoles $\boldsymbol{e}_{12}$, $\boldsymbol{e}_{23}$, and $\boldsymbol{e}_{31}$ on the screen must be on a straight line E_{123}, where E_{123} is the intersection of the screen and the plane Ω_{123} determined by three light sources $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, and $\boldsymbol{b}_3$.*

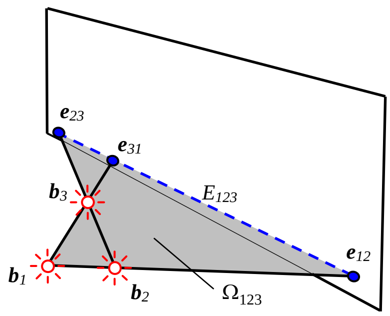


Figure 6: Intersectional line E_{123} determined by two planes (screen and Ω_{123}).

3. CONSTRAINT FOR SHADOWS OF A SOLID

In this section, we evolve properties described in Section 2 into shadows of a single solid.

3.1 Two Light Sources

Consider a screen image on which two shadows are projected by two light sources as in Fig. 7. We denote the shadow area projected by light source 1 as A_1, and by light source 2 as A_2. From property 1, there must be an epipole $\boldsymbol{e}_{12}$ on this screen. Let us hypothesize that this epipole is at the position $\tilde{\boldsymbol{e}}_{12}$ shown in Fig. 7. However, this situation is geometrically impossible, because some line C_{1XX} that crosses A_1 but does not cross A_2 exists. According to property 1, two shadows corresponding to the same 3-D point must be on the line C_{12K}, but in this situation (Fig. 7), there is no corresponding shadow of light source 2 on A_2, e.g., there is no $\boldsymbol{c}_{2K}$ that corresponds to $\boldsymbol{c}_{1K}$ on the screen.

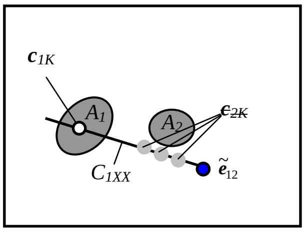


Figure 7: False position of $\boldsymbol{e}_{12}$.

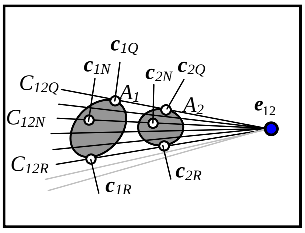


Figure 8: True position of $\boldsymbol{e}_{12}$.

If we exclude such impossible situations, only the condition represented by Fig. 8 remains. In this situation, any line from the point $\boldsymbol{e}_{12}$ must either: (1) go through both A_1 and A_2, or (2) go through neither A_1 nor A_2.

We can determine such point $\boldsymbol{e}_{12}$ as follows:

***Property 3**: If we make two outermost common tangential lines (bi-tangent lines) of the two shadows, the intersection of these two lines is an epipole $\boldsymbol{e}_{12}$. In addition, the pair of two tangent points {$\boldsymbol{c}_{1Q}$, $\boldsymbol{c}_{2Q}$}, {$\boldsymbol{c}_{1R}$, $\boldsymbol{c}_{2R}$} are the shadows of two common points on the surface of the solid.*

These two points are the same as "frontier points" described in Section 1.

We can mathematically express the condition described above as follows:

$$\forall \boldsymbol{x}_1 \in A_1,\ \exists \boldsymbol{x}_2 \in A_2,\ \exists\ \alpha_1 \in \Re,\ \boldsymbol{x}_2 = \alpha_1 \boldsymbol{x}_1 + (1-\alpha_1)\boldsymbol{e}_{12}, \quad (1)$$

$$\forall \boldsymbol{x}_2 \in A_2,\ \exists \boldsymbol{x}_1 \in A_1,\ \exists\ \alpha_2 \in \Re,\ \boldsymbol{x}_1 = \alpha_2 \boldsymbol{x}_2 + (1-\alpha_2)\boldsymbol{e}_{12}, \quad (2)$$

and the point $\boldsymbol{e}_{12}$ can be defined so that it simultaneously satisfies both Eqs (1) and (2).

The similar property has already been shown in Ref. [1], however, they used two camera images and two individual epipoles. We newly indicated that it can be simply explained by using Fig. 8 and Eqs (1) and (2), only if we can assume that screen is single and epople is common in our situation.

3.2 Consideration of Exceptions

3.2.1 Combinations of bi-tangent lines

There are generally two combinations of tangential lines that make the point $\boldsymbol{e}_{12}$ satisfying Eqs (1) and (2), as is shown in Figs. 9 (a) and (b). However, the situation shown in Fig. 9 (b) is one in which light source is in between the 3-D point and its shadow. This situation is only theoretically possible; therefore, we can exclude it from our discussion.

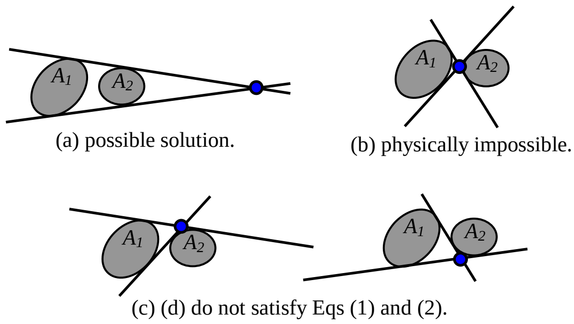


Figure 9: Combinations of bi-tangent lines.

3.2.2 No bi-tangent line exists

If two shadows are related as shown in Fig. 10, i.e., one shadow is entirely inside the other shadow, we can not find any common tangential line. This situation is possible when one light source is inside the cone determined by the other light source and solid. This condition is equivalent to the vision problem condition that there is no frontier point; therefore, we can not apply our theory to this situation.

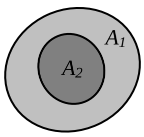


Figure 10: Situation in which $\boldsymbol{e}_{12}$ cannot be determined.

3.3 Three Light Sources

By using properties 2, and 3, we can easily derive the following remarkable property:

***Property 4**: Given three shadows of an arbitrary solid projected by three arbitrary light sources, three points determined by two outermost common tangential lines of any pair of two shadows are necessarily on a straight line.*

Additionally,

***4.1**: These three points represent line segments between each pair of light sources.*

***4.2**: The tangent points of each tangential line are the shadows of the common point on the solid surface.*

We show this property in the examples in Fig. 11. These image were created by using a well-established CG software.

4. CONCLUSION

We described the geometrical properties of shadow images projected by point-light sources onto a flat screen. Related properties might have been used implicitly in some prior art concerning camera image recognition. However, we believe that only our problem definition can explain the properties simply and explicitly. Since the result is universal and intuitively understandable, it can be conveniently used as a hint and basis for developing algorithms related to the multiple shadows.

This paper focused only on theory. Further study appropriate to the individual application is future work.

Acknowledgment

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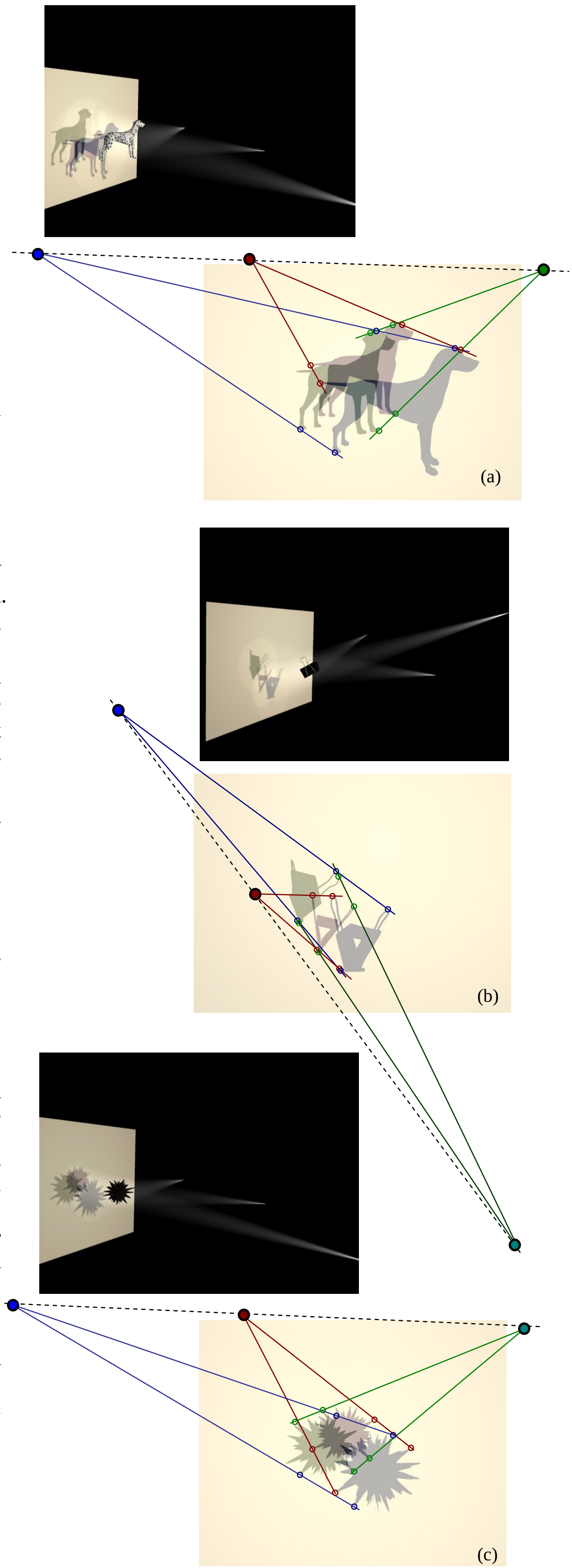

Figure 11: Examples of property 4.