Generalized Hebbian Learning for Ellipse Fitting

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ABSTRACT

In this paper, we investigate the use of a neural network employing Genralised Hebbian Learning for the approximation of an image of a hypothetically ellipsoidal object as an ellipse. Further, we discuss how the same algorithm is used with higher dimensional data to model hyperellipsoids, with the basic aim at a specific application, namely the modelling of an object as an ellipsoid given a set of 3-dimensional points.

Keywords

Ellipse Fitting, Principal Component Analysis (PCA), Generalised Hebbian Learning(GHL)

1. INTRODUCTION

An algorithm for fitting an ellipse to the image of an ellipsoidal object using principal components was discussed in [WP04]. It suffers from the problem of not being easy to extend to multiple dimensions.

We discuss how Generalised Hebbian Learning, which is an Artificial Neural Network (ANN) approach of performing PCA on a set of data, is used in association with this algorithm to estimate the ellipse parameters in 2-dimensions. We further investigate the extension of this algorithm to multiple dimensions, with the major objective of 3-dimensional ellipsoid modelling.

2. EXISTING WORK

Most existing algorithms for ellipse fitting either fall into Hough transform based methods or least squares fitting algorithms [HF98]. In contrast, [WP04] discusses a method based on PCA. It was found that this method has an advantage over other methods with respect to speed and robustness against outliers. This makes it ideal for resource constrained environments.

This paper looks at extending the above mentioned algorithm so that it could be used for hyperellipsoid fitting in multiple dimensions.

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3. GHL FOR ELLIPSE FITTING

It was shown in [Hay94] that a network employing the learning rule known as Generalised Hebbian Learning (GHL) performs PCA for zero mean inputs. We use this principle to calculate the principal components and the variance of data along them. Then using them we determine the ellipse parameters [WP04]. The algorithm for using GHL for ellipse fitting is as follows:

- Arrange the points in a $m \times N$ matrix, x
  where, N - No. of points
  m - No. of dimensions/neurons($m = 2$ for 2-D)
- Initialize the $m \times m$ weight matrix, w with random values
- Calculate the sum of the initial length of the weight vectors
  $l = \sum w_j^2$
- Define the length convergence error, e
- Define the learning rate, η
- While the absolute value of the length of the weight vectors have not converged to 1
  - Calculate the output
    $y = W * x$
  - Calculate the weight update
    $\Delta W = \eta y(x^T - y^T W)$
  - Update the weight matrix with the weight updates
    $W = W + \Delta W$
  - Update the length of the weights
    $l = \sum w_j^2$

- Determine the eigenvectors of the input correlation matrix by the weight matrix
  $V = W^T \quad \Rightarrow \quad v_i = [V_{1,i} \cdots V_{m,i}]^T$
- Calculate the eigenvalues by the variance of the output values
  $\lambda = var(W.x)$
- Determine the axes of the ellipse using the eigenvalues and its orientation using the eigenvectors or principal components
  $x_1 + jx_2 = 2(\cos t + j \sin t)e^{j\phi} + (c_1 + jc_2)$
  where c_1, c_2 are the mean of the data,
  and $\tan\phi = v_2/v_1$
- Draw the ellipse around the points or the image

Figure 1 shows an example of an ellipse fit. In the case of an image, the method outlined in [WP04] should be used to sample the points before fitting the ellipse.

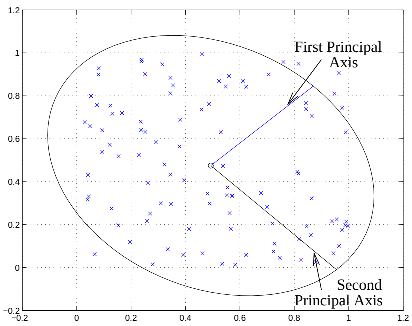

Figure 1: An example of ellipse fitting

4. ELLIPSOID MODELLING

The extension of the above algorithm to fit hyper-ellipoids in higher dimensions is done by changing the number of neurons of the neural network. This changes the input, weight and output matrices accordingly. The parameters are then determined by using the eigenvectors and eigenvalues as in the 2-D case. Figure 2 shows an example in 3-dimensions.

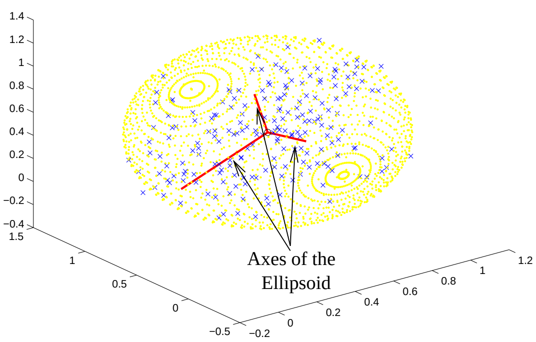

Figure 2: An example of 3-D ellipsoid modelling

5. COMPARISON OF METHODS

The neural network method discussed in this paper to calculate the ellipse parameters is compared with that used in [WP04]. The main advantage here is the simple extensibility to higher dimensions.

Further this method gives the user control over the error of fit. By changing the length convergence error, the processing and accuracy could be balanced to suit the requirements of the application.

It was also found that the above discussed method is faster, specially in higher dimensions. Figure 3 shows this comparison of average processing times, where **PCA** is the algorithm discussed in [WP04] and **GHL** is the method discussed above.

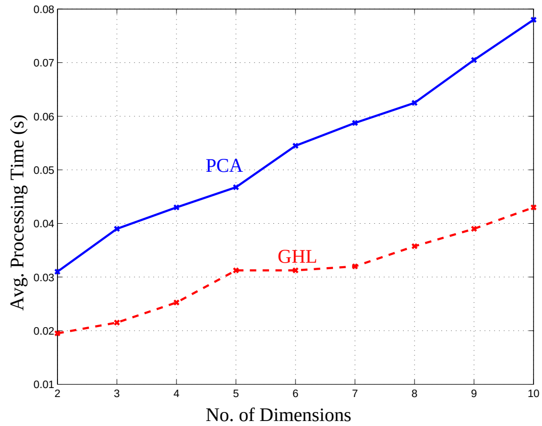

Figure 3: Processing time Vs No. of dimensions

6. CONCLUSION

In this paper we discussed how a neural network employing Generalized Hebbian Learning could be used to fit an ellipse to a set of 2-dimensional data points and how to extend this algorithm to n-dimensions.

The main strength of this algorithm lies in its extensibility to higher dimensions without substantially increasing the amount of processing.

Hence we conclude that this method is acceptable in ellipse fitting but is more efficient in higher dimensions. It could be used in applications where resource constraints require the approximation of data to minimize the processing and maximize speed.

7. REFERENCES

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