A QUANTITIES METHOD OF INDUCTION MOTOR UNDER UNBALANCED VOLTAGE CONDITIONS

Ayong Hiendro
Department of Electrical Engineering, Faculty of Engineering, Tanjungpura University
Jl. Jend. Ahmad Yani, Pontianak 78124
e-mail: ayongh2000@yahoo.com

Abstrak

Pendefinisian paling lengkap untuk tegangan tak seimbang adalah menggunakan complex voltage unbalance factor (CVUF) yang terdiri dari besaran amplitudo dan sudut fasa. Sayangnya, definisi ini tidak membedakan antara kondisi ketidakseimbangan tegangan kurang dan tegangan lebih. Pada makalah ini, analisis terhadap motor induksi dilakukan menggunakan metode komponen simetris dan perangkat lunak MATLAB digunakan sebagai alat bantu untuk mengevalusi kinerja motor induksi. Hasil simulasi komputer menunjukkan bahwa definisi ketidakseimbangan tegangan oleh bakuan International Electrotechnical Commission (IEC) dapat digunakan untuk mengevaluasi rugi-rugi daya dengan teliti. Akan tetapi, besaran sudut fasa dari faktor ketidakseimbangan tegangan harus dimasukkan ke dalam perhitungan untuk memperoleh hasil yang tepat dalam menentukan arus puncak dan rugi-rugi maksimum pada belitan fasa motor induksi. Selain itu, kondisi ketidakseimbangan tegangan kurang dan tegangan lebih juga harus diperhatikan untuk menghitung semua besaran tersebut.

Kata kunci*: CUVF, motor induksi, rugi-rugi daya, tegangan tak seimbang*

Abstract

The most complete definition for the voltage unbalance is using complex voltage unbalance factor (CVUF) that consists of its magnitude and angle. Unfortunately, the definition did not distinguish between undervoltage and overvoltage unbalance conditions. In this paper, the analysis of the motor is performed using the method of symmetrical component and MATLAB software is used to investigate the performance of induction motor. The simulation results show that the International Electrotechnical Commission (IEC) definition of the voltage unbalance can be applied to evaluate total copper losses precisely. However, the phase angle of unbalance factor must be included for accurate predicting of peak currents and peak copper losses of the phase windings of the motor.The unbalanced conditions which distinguish between under and over voltage unbalance must also be taken into consideration for assessing all the quatities.

Keywords*: CUVF, induction motor, losses, voltage unbalance*

1. INTRODUCTION

Voltage unbalance is a power quality problem. It is a common phenomena found in a three-phase power system. Although three-phase voltage supply is balanced in both magnitude and phase-angle at generation and transmission levels, the voltages at distribution end and utilization side can become unbalance. It is practically impossible to be obviated due to the uneven distribution of single-phase loads in the three-phase supply system and asymmetry of transmission line and transformer winding impedances.

Such condition has severe impacts on the performance of an induction motor. This motor is designed and built to be able to tolerate only a low degree of voltage unbalance and has to be derated if the unbalance is excessive.

The negative impacts of voltage unbalance on an induction motor have been studied in depth [1]-[6]. The voltage unbalance can increase stator and rotor losses of the induction motor, rise windings temperature and reduction the insulation life caused by overheating. The motor need to be derated, reducing its output horsepower so it can tolerate the extra heating imposed by the unbalanced voltage supply. According to this issue, Influence of magnitude of unbalance voltage factor on efficiency and motor losses has been investigated in [7].

In most previous studies, the method of evaluating the degree of unbalanced voltage is based on either National Electrical Manufactures Association (NEMA) standard or International Electrotechnical Commission (IEC) definition. Both the NEMA and the IEC definitions are only considering "magnitude" of voltage unbalance to describe the degree of unbalanced voltage. The three-phase voltage supply has not only magnitude but also phase-angle that giving contribution to the voltage unbalance. Hence, both definitions lead to a comparatively large error in predicting the performance of the induction motor [8].

A more precise approach in predicting the performance of an induction motor operation with unbalanced three-phase voltage is using complex quantity to specify the degree of voltage unbalance. This definition is also known as the complex unbalance voltage factor (CUVF) in some literatures. The CUVF is an extension of the IEC definition, first time evaluated by Wang [9], which consists of magnitude and angle of the voltage unbalance. However, Wang [9] did not distinguish between undervoltage and overvoltage unbalance conditions.

In this paper, the stator and rotor losses of an induction motor and its peak currents are studied precisely. The angle of the CUVF is taken account to estimate the peak currents and losses of the induction motor operating with both under-voltages unbalance and over-voltage unbalance conditions.The symmetrical component theory approach is used to analyze the operation of induction motor under such conditions. MATLAB program is also used for computer simulation. Finally, recommendations are suggested for accurate calculating the quantities of the induction motor operation under unbalanced voltage conditions.

2. RESEARCH METHOD

The motor parameters used are: 3-phase, Y-Connected, 380V (line-to-line), 5.5kW, 50Hz, 4-poles. The impedances of its equivalent circuit in Ohm/phase referred to the stator are: R_s=0.34, R_r=0.25, X_s=0.73, X_r=0.47, X_m=15.12.

Analysis of an induction motor operating with unbalanced voltage supply using symmetrical component approach [10] requires positive and negative sequence equivalent circuit of a three phase induction motor representing in Figure 1.

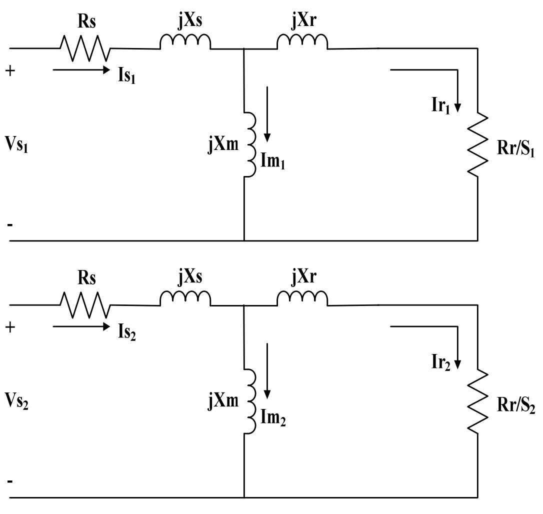

Figure 1. Sequence equivalent circuit

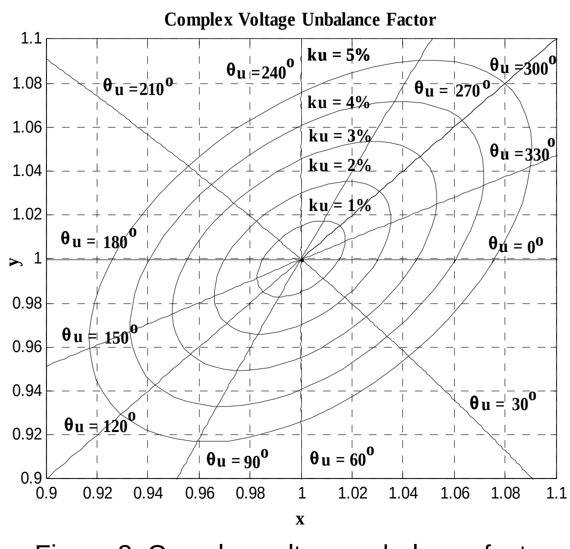

Figure 2. Complex voltage unbalance factor (CVUF) diagram

Each circuit performs to both positive and negative sequence circuits. The only difference between the circuits is the load resistance defined by positive and negative sequence slips. Positive sequence slip is s_1=s, negative sequence slip is s_2=2–s and slip s is as shown equation (1), where n_s is synchronous speed and n_r is rotor speed.

$$s = \frac{n_s - n_r}{n_s} \tag{1}$$

Input sequence impedances for the positive and negative sequence circuit are shown in equation (2), where i=1 for positive sequence and i=2 for negative sequence.

$$\mathbf{Z}_i = R_s + jX_s + \frac{(jX_m)*\left(\frac{R_r}{s_i} + jX_r\right)}{\frac{R_r}{s_i} + j(X_m + X_r)} \tag{2}$$

Let $\mathbf{U}_{\mathbf{s}ab}$, $\mathbf{U}_{\mathbf{s}bc}$ and $\mathbf{U}_{\mathbf{s}ca}$ be the line voltages of the stator. The corresponding zero, positive and negative sequence voltages ($\mathbf{U}_{\mathbf{s}0}$, $\mathbf{U}_{\mathbf{s}1}$,$\mathbf{U}_{\mathbf{s}2}$) are given by equation (3), where $\mathbf{a}$=1∗exp(j2π/3) is Fortescue operator, and $\mathbf{a}^2$=1∗exp(-j2π/3).

$$\begin{bmatrix} \mathbf{U}_{\mathbf{s}0} \\ \mathbf{U}_{\mathbf{s}1} \\ \mathbf{U}_{\mathbf{s}2} \end{bmatrix} = \frac{1}{3} * \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} * \begin{bmatrix} \mathbf{U}_{\mathbf{s}ab} \\ \mathbf{U}_{\mathbf{s}bc} \\ \mathbf{U}_{\mathbf{s}ca} \end{bmatrix} \tag{3}$$

The sequence line voltages can be transformed directly to the sequence phase voltages by equation (4), where t=$\frac{1}{\sqrt{3}}$∗exp(-jπ/6), and its conjugate is t*=$\frac{1}{\sqrt{3}}$∗exp(jπ/6).

$$\begin{bmatrix} \mathbf{V}_{\mathbf{s}0} \\ \mathbf{V}_{\mathbf{s}1} \\ \mathbf{V}_{\mathbf{s}2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathbf{t} & 0 \\ 0 & 0 & \mathbf{t}^* \end{bmatrix} * \begin{bmatrix} \mathbf{U}_{\mathbf{s}ab} \\ \mathbf{U}_{\mathbf{s}bc} \\ \mathbf{U}_{\mathbf{s}ca} \end{bmatrix} \tag{4}$$

If the sequence phase voltages are applied to the sequence equivalent circuits (Figure 1), then the sequence currents can be computed. Positive sequence stator and rotor currents are shown in equation (5) and (6), and negative sequence stator and rotor currents are shown in equation (7) and (8).

.

$$\mathbf{I}_{\mathbf{s}1} = \frac{\mathbf{V}_{\mathbf{s}1}}{\mathbf{Z}_1} \tag{5}$$

$$\mathbf{I}_{\mathbf{r}1} = \mathbf{I}_{\mathbf{s}1} * \frac{(jX_m)}{\frac{R_r}{s_1} + j(X_m + X_r)} \tag{6}$$

$$\mathbf{I}_{\mathbf{s}2} = \frac{\mathbf{V}_{\mathbf{s}2}}{\mathbf{Z}_2} \tag{7}$$

$$\mathbf{I}_{\mathbf{r}2} = \mathbf{I}_{\mathbf{s}2} * \frac{(jX_m)}{\frac{R_r}{s_2} + j(X_m + X_r)} \quad (8)$$

Calculating stator and rotor losses requires phase stator and rotor currents. The phase currents are determined by performing the transformation back. Transforming the stator currents using Fortescue matrix as shown in equation (9), and similarly the rotor currents of the induction motor can be calculated by transformation as shown in equation (10). In (10) and (11), there are no zero sequence currents ($\mathbf{I}_{s0}$=$\mathbf{I}_{r0}$=0) for the motor connecting in delta or ungrounded wye.

$$\begin{bmatrix} \mathbf{I}_{\mathbf{s}a} \\ \mathbf{I}_{\mathbf{s}b} \\ \mathbf{I}_{\mathbf{s}c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} * \begin{bmatrix} \mathbf{I}_{\mathbf{s}0} \\ \mathbf{I}_{\mathbf{s}1} \\ \mathbf{I}_{\mathbf{s}2} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} \mathbf{I}_{\mathbf{r}a} \\ \mathbf{I}_{\mathbf{r}b} \\ \mathbf{I}_{\mathbf{r}c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} * \begin{bmatrix} \mathbf{I}_{\mathbf{r}0} \\ \mathbf{I}_{\mathbf{r}1} \\ \mathbf{I}_{\mathbf{r}2} \end{bmatrix} \quad (10)$$

Furthermore, stator copper losses of the motor can be calculated by equation (11) and its rotor copper losses is given by equation (12). Input active power of the motor is as shown equation (13) and and its input reactive power is defined by equation (14)

$$P_{slosses} = \left(|\mathbf{I}_{\mathbf{s}a}|^2 + |\mathbf{I}_{\mathbf{s}b}|^2 + |\mathbf{I}_{\mathbf{s}c}|^2 \right) * R_s \quad (11)$$

$$P_{rlosses} = \left(|\mathbf{I}_{\mathbf{r}a}|^2 + |\mathbf{I}_{\mathbf{r}b}|^2 + |\mathbf{I}_{\mathbf{r}c}|^2 \right) * R_r \quad (12)$$

$$P_{in} = \mathrm{Re}\left[3 * \left(\mathbf{V}_{\mathbf{s}1} * (\mathbf{I}_{\mathbf{s}1})^* + \mathbf{V}_{\mathbf{s}2} * (\mathbf{I}_{\mathbf{s}2})^* \right) \right] \quad (13)$$

$$Q_{in} = \mathrm{Im}\left[3 * \left(\mathbf{Vs}_1 * (\mathbf{Is}_1)^* + \mathbf{Vs}_2 * (\mathbf{Is}_2)^* \right) \right] \quad (14)$$

As the motor has no core and mechanical losses, output power is

$$P_{out} = P_1 + P_2 \quad (15)$$

where $P_1 = 3 * |\mathbf{I}_{\mathbf{r}1}|^2 * \left(\frac{1 - s_1}{s_1} \right) * R_r$ and $P_2 = 3 * |\mathbf{I}_{\mathbf{r}2}|^2 * \left(\frac{1 - s_2}{s_2} \right) * R_r$

As no core and mechanical losses, output power is

$$P_{out} = P_1 + P_2 \quad (16)$$

where, $P_1 = 3 * |\mathbf{Ir}_1|^2 * \left(\frac{1 - s_1}{s_1} \right) * Rr$ and $P_2 = 3 * |\mathbf{Ir}_2|^2 * \left(\frac{1 - s_2}{s_2} \right) * Rr$

The motor output torque is

$$T_{out} = T_1 + T_2 \quad (17)$$

Each positive and negative sequence of the output torques are $T_1 = \frac{P_1}{\omega_m} = \frac{3 * Ir_1^{\ 2} * Rr}{s_1 * \omega_s}$ and $T_2 = \frac{P_2}{\omega_m} = -\frac{3 * Ir_2^{\ 2} * Rr}{s_2 * \omega_s}$, where angular speed $\omega_m = \omega_s(1 - s)$ and ω_s is synchronous speed (in rad/s).

The CVUF is defined as shown equation (20), where k_v is the magnitude and θ_v is the angle of the CVUF.

$$\mathbf{k}_v = \frac{\mathbf{V}_{\mathbf{s}2}}{\mathbf{V}_{\mathbf{s}1}} = k_v \angle \theta_v \tag{18}$$

Let $\mathbf{U}_{\mathbf{s}ab}$, $\mathbf{U}_{\mathbf{s}bc}$ and $\mathbf{U}_{\mathbf{s}ca}$ be the line voltages of stators. Their symmetrical components are given by (3) mentioned above. In contrary, the line voltages can be obtained from their positive and negative sequences, but minus the zero sequence component ($\mathbf{U}_{\mathbf{s}0}$=0).

$$\mathbf{U}_{\mathbf{s}ab} = \mathbf{U}_{\mathbf{s}1} + \mathbf{U}_{\mathbf{s}2};\ \mathbf{U}_{\mathbf{s}bc} = \mathbf{a}^2 * \mathbf{U}_{\mathbf{s}1} + \mathbf{a} * \mathbf{U}_{\mathbf{s}2};\ \mathbf{U}_{\mathbf{s}ca} = \mathbf{a} * \mathbf{U}_{\mathbf{s}1} + \mathbf{a}^2 * \mathbf{U}_{\mathbf{s}2} \tag{19}$$

By the CVUF definition,

$$\mathbf{k}_u = \frac{\mathbf{U}_{\mathbf{s}2}}{\mathbf{U}_{\mathbf{s}1}} = k_u \angle \theta_u \tag{20}$$

As described in [8], if the ratio of the line voltages is defined by

$$\mathbf{U}_{\mathbf{s}ab} : \mathbf{U}_{\mathbf{s}bc} : \mathbf{U}_{\mathbf{s}ca} = 1 : x : y \tag{21}$$

Then the relations between (k_u , θ_u) and (x,y) according to (19), (20) and (21) are

$$\mathbf{x} = \frac{1 + \mathbf{k}_u}{\mathbf{a}^2 + \mathbf{a} * \mathbf{k}_u} \tag{22}$$

$$\mathbf{y} = \frac{\mathbf{a} + \mathbf{a}^2 * \mathbf{k}_u}{\mathbf{a}^2 + \mathbf{a} * \mathbf{k}_u} \tag{23}$$

Using (22) and (23) the relations between (k_u ,θ_u) and (x,y) can be shown in x-y plane (Figure 2). The loci of constant k_u looks like ellipses while the loci of constant θ_u looks like lines.

Further more, the definition of the CVUF can be viewed more closely as a cylinder in three-dimensional. As example, the variations of terminal voltages (in per-unit value) for k_u=5% with phase angle in 0-360° range is shown in Figure 3.

There are infinite combinations of terminal voltages of the induction motor for k_u=5%. Every point of the outer surface of cylinder will give the same level of voltage unbalance. The large range of variation of the terminal voltages for a given value of voltage unbalance can be reduced by considering the phase angle θ_u and the definition of under and over voltage conditions. For instant, the variations of terminal voltages under a 0% voltage unbalance is shown in Figure4. Considering to the definition above, there are two sections of the graphic. The first, k_u=0% is for undervoltage if U_{ab}, U_{bc}, U_{ca} < 1 and the other, k_u=0% is for overvoltage if U_{ab}, U_{bc}, U_{ca} > 1.

In this work, magnitude of the voltage unbalance factor of 0%, 1%, 2%, 3%, 4%, 5% and 6% are used for computation. Undervoltage condition is defined by 0.7≤ f < 1.0 and overvoltage condition is 1.0 < f ≤ 1.4. Torque of the machine is kept constant during computation process, T_{out}=40.3578 N/m^2 for calculating peak current and peak losses to obtain rate current and T_{out}=33.8046 N/m^2 for calculating total copper losses. Matlab programming is employed to calculate the currents and losses by using equations (1) – (29).

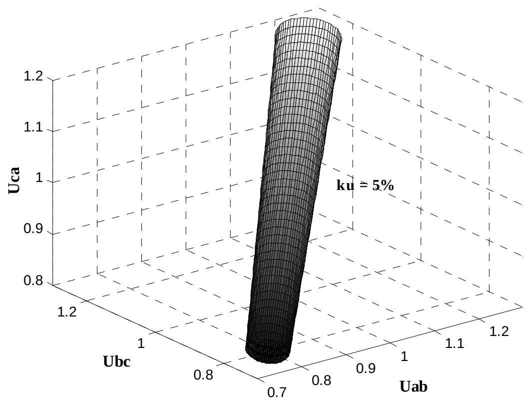

Figure 3. Terminal Voltage of a Three-Phase Induction Motor with k_u=5%

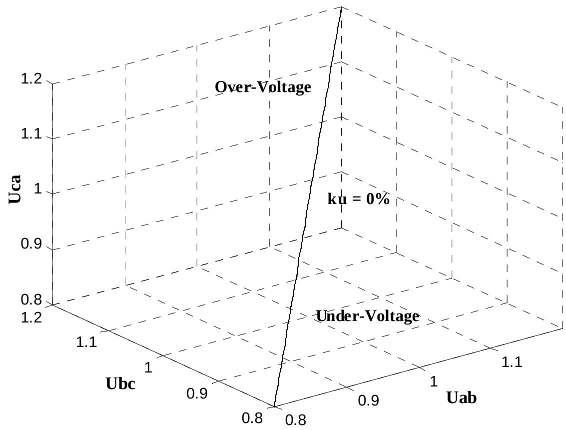

Figure 4. Terminal Voltage of a Three-Phase Induction Motor with k_u=0%

3. RESULTS AND ANALYSIS

Faiz et.al in [7] reported that copper losses of the induction motor increases related to magnitude of voltage unbalance factor. They did not investigate the influence of phase angle of the unbalance factor. Here, Figs. 5 and 6 describe the effects of phase angle of the voltage unbalance factor on stator and rotor copper losses.

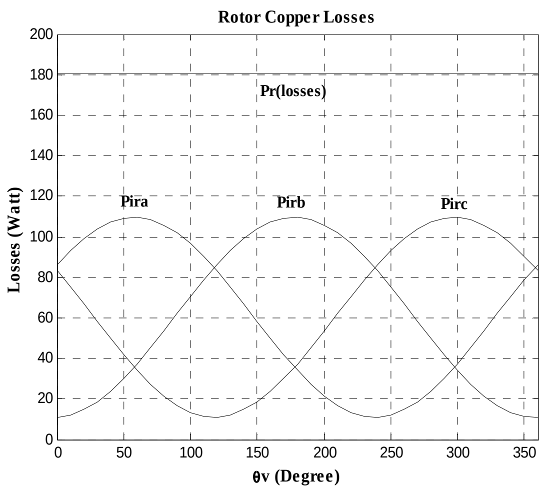

Figure 5. Variations of phase a, b and c Rotor Losses with θ_v

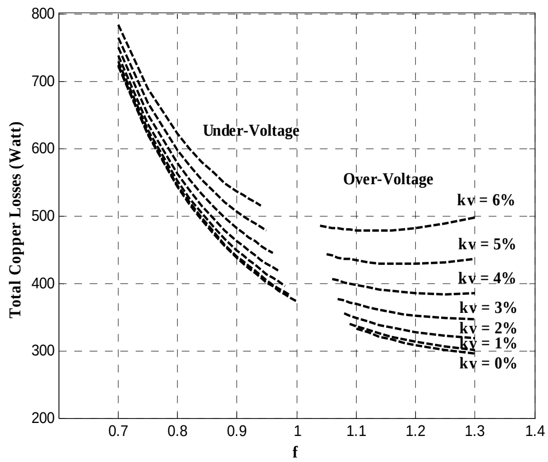

Figure 6. Variations of total copper losses with unbalanced conditions at torque=33.8046 N/m^2

The phases a, b and c of the stator losses (P_{isa}, P_{isb}, P_{isc}) and the rotor losses (P_{ira}, P_{irb}, P_{irc}) vary from θ_v=0° to 360° for a constant k_v=6% and output torque=40.3578 N/m^2 at slip s=0.0224. The losses in the three phases are extremely non-uniform under the unbalance voltage condition and vary significantly with θ_v , but total of stator and rotor losses remain constant and are not dependent on the phase angle of unbalanced factor. The peak losses of each phase refer to peak currents.

The peak current of stator occurs when the phase angle of unbalanced factor is

$$\theta_v = \text{angle}(\mathbf{Z}_2) - \text{angle}(\mathbf{Z}_1) + 2n\pi/3,\ n=0,1,2 \quad (24)$$

The phase angle of the unbalanced factor for the rotor peak current is

$$\theta_v = (\text{angle}(\mathbf{Z}_2) - \text{angle}(\mathbf{Z}_1)) - (\text{angle}(\mathbf{CD}_2) - \text{angle}(\mathbf{CD}_1)) + 2n\pi/3,\ n=0,1,2 \quad (25)$$

where $\mathbf{CD}_1$ and $\mathbf{CD}_2$ are positive and negative sequence current dividers and defined by

$$\mathbf{CD}_1 = \frac{(jX_m)}{\frac{R_r}{s_1} + j(X_m + X_r)} \text{ and } \mathbf{CD}_2 = \frac{(jX_m)}{\frac{R_r}{s_2} + j(X_m + X_r)}.$$

If at an angle of θ_v the current in phase-a has a peak value, then in the next 120° interval (= θ_v + 120°), phase-b would have a peak and phase-c would have its peak current in the subsequent 120° interval (= θv + 240°). Once a phase has the value of peak current above the rate current, it will lead to excessive heating in the motor's windings. The motor is not allowed to operate at above the rated current or it will harm to the stator and rotor windings.

Table 1 shows variations of peak losses and peak currents occurred in the rotor and stator phases of the induction motor operation under k_v=6% and output torque=40.3578 N/m^2 with considering over or under voltage unbalance conditions. Comparing Tables 1 and 2, it can be found that peak losses and peak current in one of stator and rotor phase windings become very high if the induction motor is operated with either over voltage unbalance or under-voltage unbalance conditions. Under the same value of k_v, the under-voltage unbalance gives higher peak losses and peak currents than the three-phase over voltage unbalance does. A large value of unbalanced may create the current flowing in one of the phase windings exceeds the rated value. This condition will damage the insulation of the motor's windings and may shorten the motor's life for long-term operation. Precise derating for induction motors with unbalanced voltage investigated by Faiz et.al. in [8]. They found that both the NEMA and IEC definitions lead to large error in predicting the performance of the machine when operation with unbalanced voltage conditions. It is true for calculating the motor derating factor. Moreover, they [8],[9] did not define unbalanced condition to distinguish between over and under voltage unbalance to obtain more accurate results. Figure 6 shows that the IEC definition of voltage unbalance combined with the unbalanced condition can be applied to evaluate total copper losses precisely.

Table 1. Comparison of peak losses and peak current under k_v=6% and T_{out}=40.3578 N/m^2

Conditions	f	is(max) A	ir(max) A	Ps(max) W	Pr(max) W
Under Voltage Unbalance	0.8	32.4699	31.0314	309.9621	138.6649
	0.9	30.7941	28.9150	278.7937	120.3949
Over Voltage Unbalance	1.1	29.7999	26.7127	261.0816	102.7537
	1.2	30.0384	26.1997	265.2776	98.8454

Table 2. Peak losses and peak current under balanced voltage and T_{out}=40.3578 N/m^2

Conditions	f	is(max) A	ir(max) A	Ps(max) W	Pr(max) W
Balanced Voltage	1.0	20.3875	18.0996	122.2020	47.1738

The total copper losses are summation of the stator and rotor copper losses and do not depend on the phase angle of voltage unbalance. At the same degree of voltage unbalance, the total losses depend upon unbalance conditions. As seen in Figure 6, the motor operates in undervoltage unbalance condition gives greater losses compared to it is in over voltage unbalance condition at k_v=0%, 1%, 2%, 3%, 4%, 5% and 6%.

4. CONCLUSIONS

The degree of voltage unbalance is a quantity required in studying the performance of a three phase induction motor operating with an unbalanced voltage supply. It has been asserted that the prevailing operation conditions of the motor are not accurately assessed without knowing the phase angle of unbalanced factor and the voltage unbalance conditions (under or over voltage unbalance).

The IEC definition of voltage unbalance combined with the unbalanced condition can be applied to evaluate total copper losses precisely. However, the phase angle of unbalanced factor must be included for accurate predicting of peak current and peak copper losses of the phase windings.

REFERENCES

[1]. Woll RF. Effect of Unbalanced Voltage on the Operation of Poly-Phase Induction Motor. *IEEE Transactions on Industrial Appliactions*. 1975; IA-11(1): 38-42.
[2]. Cummings PB, Dunki-Jacobs JR, Kerr RH. Protection of Induction Motors Against Unbalanced Voltage Operation. *IEEE Transactions on Industrial Appliactions*. 1985; 21(4): 778-792.
[3]. Kersting WH, Philips WH. Phase Frame Analysis of the Effects of Voltage Unbalance on Induction Machine. *IEEE Transactions on Industrial Applications*. 1997; 33(2): 415-420.
[4]. Pillay P, Hoftman P, Manyage M. Derating of Induction Motors Operating with the Combination of Unbalanced Voltages and Over or Under Voltages. *IEEE Transactions on Energy Conversion*. 2002; 17: 485-491.
[5]. Refoufi L, Bentarzi H, Dekhandji FZ. *Voltage Unbalance Effects on induction Motor Performance*. Proceeding of the 6th WSEAS Int. Conf. on Simulation, Modelling and Ottimization. Portugal. 2006; 112-117.
[6]. Sandhu KS, Chaudhary V. Steady State Modelling of Induction Motor Operating with Unbalanced Supply System. *WSEAS Transactions on Circuit and Systems*. 2009; 8(2): 197-206.
[7]. Faiz J, Ebrahimpour H, Pillay P. Influence of Unbalanced Voltage Supply on Efficiency of Three Phase Squirrel Cage Induction Motor and Economic Analysis. *Energy Conversion and Management*. 2006; 47: 289 – 302.
[8]. Faiz J, Ebrahimpour H. Precise Derating of Three-Phase Induction Motors with Unbalanced Voltages. *Energy Conversion and Management*. 2007; 48: 2579-2586.
[9]. Wang YJ. Analysis of Effects of Three Phase Voltage Unbalance on Induction Motors with Emphasis on the Angle of the Complex Voltage Unbalance Factor. *IEEE Transactions on Energy Conversion*. 2001; 16(3): 270-275.
[10]. Williams JE. Operation of 3-Phase Induction Motors on Unbalanced Voltages. A*IEE Transactions* .1954; Pt. III-A: 125-133.