

## Kalman Filter Estimation of Impedance Parameters for Medium Transmission Line

Siti Nur Aishah Mohd Amin\*, Hamzah Ahmad, Mohd Ruslim Mohamed, Mohd Mawardi Saari, Omar Aliman

Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang, Pekan Campus, 26600 Pekan, Pahang Darul Makmur, 09-4246024, Malaysia

\*Corresponding author, e-mail: aishahamin\_92@yahoo.com

### Abstract

Accurate knowledge of impedance parameters in transmission line helps to improve the system efficiency and performance. Nowadays, the estimation of impedance parameters in transmission line has become possible with the availability of computational method. This paper aims to develop Kalman filter model by using Matlab simulink to estimate accurate values of resistance (R), reactance (X), and susceptance (B) in medium transmission line. The accuracy of the parameters can be improved by reducing the unknown errors in the system. To demonstrate the effectiveness of the Kalman filter method, a case study of simulated medium transmission line is presented and comparison between Kalman Filter (KF) and Linear Least Square (LLS) method is also considered to evaluate their performances.

**Keywords:** RXB, kalman filter, matlab simulink, accuracy, transmission line

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### 1. Introduction

An electric power system consists of three principal divisions which are the generating station, transmission station and distribution station. Generally these three stations is a network of conductors between the power stations and the consumer in order to utilize the power generated. The power transmission is one of the major components in the electrical power system [1]. All transmission lines in a power system exhibit the electrical properties of resistance (R), reactance (X) and susceptance (B). It is important to consider all the R, X and B parameters in the design and operation of transmission line which these parameters will indicate the power losses and efficiency of transmission line [2]-[3]. By reducing losses and improving the efficiency, values of voltage profile and power factor in a system will be corrected, hence following the standard of IEEE an IEC system. The important consideration of R, X and B in the design and operation in transmission line are the determination of line losses, voltage drop and efficiency of transmission line. According to the previous paper, R, X and B are greatly influenced the transmission line performance in a system [5].

The frequency involve in this three constant parameter indicates the performance of a system. When considering high frequency, the magnetic field generated drive the currents to the outer edge of the conductor that carries them. As a result, the higher the frequency, the thinner the layer of metal available to carry the current and the higher the effective resistance of the cable [6]. In fact, the voltage drop in the line depends on the values of above three lines constant [1]. The resistance involved in the transmission line is the most important cause of power losses in the line and it determine the transmission line efficiency. Since there are many research has been done on R, L and C parameters value [7-9] in medium and long transmission line, so this paper will focuses on R, X an B parameters value medium transmission line. The difference of this paper to other papers is that the analysis includes the different noises conditions and it is applied to the real system which is in East Midland to analyse the performance of the system.

Power system state estimation is a main tool of energy management system. State estimation was first applied to power systems by Schweppe and Wildes in the late 1960s [4],[10]. Since then, several applications and techniques of KF method have been one of the famous methods in power system estimation. It has been used for damping and frequency [1], modal changes [11], state estimation [12], power factor detection and etc. However all of these

researches did not highlighted important indicators that give great effects in the system which are R, X and B parameters. Theoretically, values of R, X and B in long transmission line are supposedly higher than short transmission line. But due to some condition, the values of R, X and B can be opposed the actual properties such as type of cable, size of cable, length of cable, number of winding and etc. All of these properties will indicate the values of R, X and B in the transmission line.

Traditionally, all the impedance parameters involved were calculated manually by using handbook formulas based or through fault record analysis. Accounting, for any system, can be a complex undertaking. A manual accounting system requires an understanding in the accounting process in a way that may be unnecessary with a computerized accounting system. This can be an advantage or a disadvantage, depending on the person doing the bookkeeping, usually a professionally trained is needed to ensure that accounting in the system is done properly. Therefore, with the availability of computational method, it is possible to calculate and measure the impedance parameters by using computational method such as KF and LLS method. Previous paper has shown the estimation of R, X and B by using LLS method [5]. Thus, in this paper, with the same information used the estimation of R, X and B value is carried out by using KF method. Both of these methods are used for parameters estimation purposes in a system with the different in accuracy of estimation.

KF works on prediction-correction model applied for linear and time-variant or time-invariant systems [13]. The prediction model involves the actual system which is predictable with the process noise in the system [13]. While LLS method is based on weighted least squares in which past values taken in account for determining the future value. The weights are updated recursively based on memory. LLS method will only use the measurements to update the position estimate every second but, KF will provide additional information and yield a more accurate estimation. Kalman filter is a powerful tool especially when it comes to control noise in a system. Thus, in this paper the development of KF model by using Matlab simulink has been done. The mathematical model and its standard parameters are also used. The formula for the KF method estimation will be examined in the next section.

The rest of this paper is structured as follows. In section 2, the transmission line model is presented. In section 3, KF method based impedance estimation is discussed while LLS method based impedance estimation in section 4. Simulink model of KF method is shown in section 5. System model analysis and discussion are made in section 6. Section 7 concludes this paper.

## 2. Transmission Line Model

A transmission line can be defined as a medium length line if its length is more than 50 km and the voltage is comparatively higher than 20kV. In this case, the resistance, inductive reactance and shunt capacitor are taken into account. Balanced conditions of the line can be represented by the equivalent circuit of a single phase with resistance (R), and inductance (L) in series (series impedance) as well as shunt capacitor [2]. The nominal pi circuit of standard model for medium transmission line is shown in Figure 1 [8]. Based on this figure, phasor values  $V_s$ ,  $V_r$  and  $I_s$ ,  $I_r$  represents the sending and receiving end voltages and current respectively.

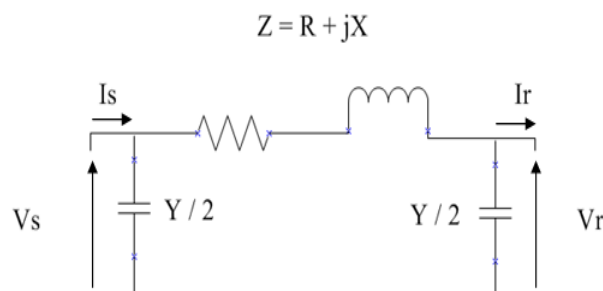


Figure 1. Nominal pi circuit diagram for medium length of transmission line [8]

For this circuit, the equations are derived based on Kirchhoff's voltage and current law [14]. The following equations are:

a. Kirchhoff's voltage law

$$V_s = Z \left( I_s - \frac{Y}{2} V_s \right) + V_r \quad (1)$$

b. Kirchhoff's current law

$$I_s = (V_s + V_r) \frac{Y}{2} + I_r \quad (2)$$

Where  $Z = R + jX$ ,  $X = 2\pi fL$ ,  $Y = G + jB$  and  $B = 2\pi fC$ .  $X$  is the inductive reactance and  $B$  is the capacitive susceptance. Formula for impedance,  $Z$  and admittance,  $Y$  can be written as below by substituting equation (2) into (1)

$$Z = \frac{V_s^2 - V_r^2}{V_s I_r + V_r I_s} \quad (3)$$

$$Y = 2 \frac{I_s - I_r}{V_s + V_r} \quad (4)$$

The values of  $Z$  and  $Y$  can be calculated.  $R$ ,  $X$ ,  $G$  and  $B$  are obtained from real and imaginary parts of  $Z$  and  $Y$  respectively.

### 3. Kalman filter based impedance estimation

a. R, X and B model

For a linear system, the process model of Kalman filter is described as

$$X_k = AX_{k-1} + BU_k + W_k \quad (5)$$

$$Z_k = HX_k + V_k \quad (6)$$

Where  $X_k$  is the estimation value of  $X$ ,  $X_{k-1}$  is the estimate value on the previous state,  $U_k$  is the control input,  $W_k$  and  $V_k$  is noise covariance matrix,  $A$ ,  $B$  and  $C$  will be numerical constant. The estimation value of  $X_k$  is represented by  $R$ ,  $X$  and  $B$  as

$$X_k = \begin{bmatrix} R \\ X \\ B \end{bmatrix} \quad (7)$$

Where  $R$  is resistance,  $X$  is reactance and  $B$  is susceptance. In this study, current,  $A$  is used as a control input,  $U_k$ .

$$U_k = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix} \quad (8)$$

The estimated value on the previous state,  $X_{k-1}$  is represented by

$$X_{k-1} = \begin{bmatrix} R_{k-1} \\ X_{k-1} \\ B_{k-1} \end{bmatrix} \quad (9)$$

b. Kalman filter algorithm

Kalman filter is used to provide an estimation of  $R$ ,  $X$  and  $B$  values in the system. Kalman filter recursively computes estimation for a state  $X_k$  according to the process and observation model. The stages of Kalman filter algorithm are as follows:

1) Prediction (time update) to estimate priori estimation of state and its error covariance matrix:

$$X_k = AX_{k-1} + BU_k \quad (10)$$

$$P_k = AP_{k-1}A^T + Q \quad (11)$$

Where  $P_k$  is the prior error covariance. This prior value is used in the measurement update equations.

2) Correction (measurement update) to provide correction based on the measurement  $Z_k$  to yield the posteriori state estimate and its error covariance:

$$K_k = P_k H^T (HP_k H^T + R)^{-1} \quad (12)$$

$$X_k = X_k + K_k (Z_k - HX_k) \quad (13)$$

$$P_k = (1 - K_k H)P_k \quad (14)$$

where  $K_k$  is Kalman gain.

#### 4. Linear Least Square Method Based Impedance Estimation

The proposed method was applied to improve the accuracy of the calculated  $Z$  and  $Y$  values. For comparison, an existing method which is linear squares method was applied to obtain the parameter estimation [8]. The transmission line was modelled as a general two port network, in which voltages and currents are related to.

$$V_s = A * V_r + B * I_r \quad (15)$$

$$I_s = H * V_r + D * I_r \quad (16)$$

Where  $A$ ,  $B$ ,  $C$  and  $D$  are constant. The real and imaginary parts of  $A$  and  $B$  were computed through unbiased linear LS estimation [5]. By assuming a pi circuit in Figure 1, the constant  $A$ ,  $B$ ,  $C$  and  $D$  inside the two port network can be expressed in terms of impedance,  $Z$  and admittance,  $Y$  as shown below.

$$A = 1 + \frac{YZ}{2} \quad (17)$$

$$B = Z \quad (18)$$

$$C = Y \left( 1 + \frac{YZ}{4} \right) \quad (19)$$

$$D = 1 + \frac{YZ}{2} \quad (20)$$

$Z$  and  $Y$  are calculated from LS estimation of  $A$  and  $B$  by using (3) and (4).

$$Z = B \quad (21)$$

$$Y = \frac{2(A-1)}{B} \quad (22)$$

#### 5. Simulink Model of Kalman Filter

Matlab simulink is a practical tool for modelling the real scenario in computational method. Figure 2 shows the general development of Kalman filter model by using Matlab simulink.

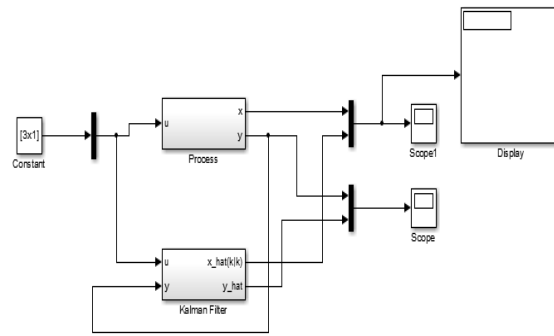


Figure 2. Kalman filter model

**6. System Model Analysis and Discussion**

This subsection investigates the results of having different noise conditions to the estimation of medium transmission line. This paper takes into account system noises of 0.1 and 50 to simulate the unknown parameters which can effects the estimation results such as size, length and type of cables. The results obtained are described as follows.

A single phase of the 400 kV, 102 km long transmission line data located between substations Grendon and Staythorpe, East Midlands, England was used and simulated by using Matlab simulink [5],[15]. This section analyse how Kalman Filter works to estimate those value efficiently.

Simulation was conducted for the equivalent circuit of medium transmission line and tuning process also take place in order to gain converge graph of each parameters of the circuit. Figure 3, Figure 4 and Figure 5 below shows the real and estimation values of R, X and B over a period. The real values of R, X and B are 2.96 Ω, 32.40 Ω and 3.69 x 10<sup>-4</sup> S while the values of estimated are 2.94 Ω, 30.92 Ω and 3.50 x 10<sup>-4</sup> respectively with noise 0.1. There are small differences between the real and new estimation values of R, X and B. Both of the real and estimated parameters are seen overlapping with each other due to the approximate parameter values. The result shows the higher consistency involved in R, X and B value due to the approximate results between real and estimated values.

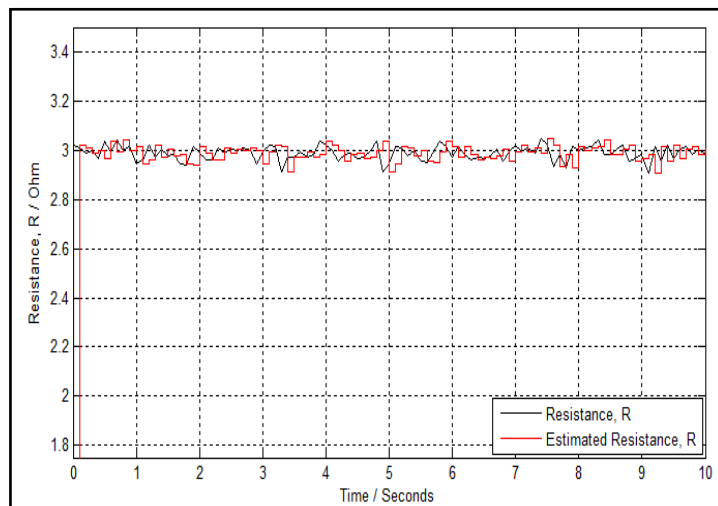


Figure 3. Graph of real and estimated value of resistance, R

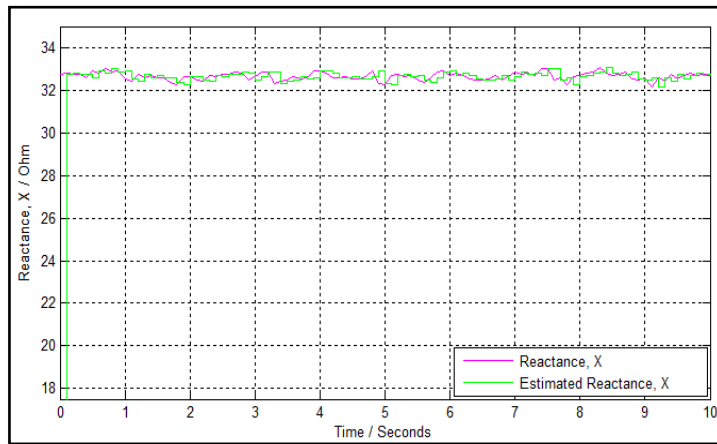


Figure 4. Graph of real and estimated value of reactance, X

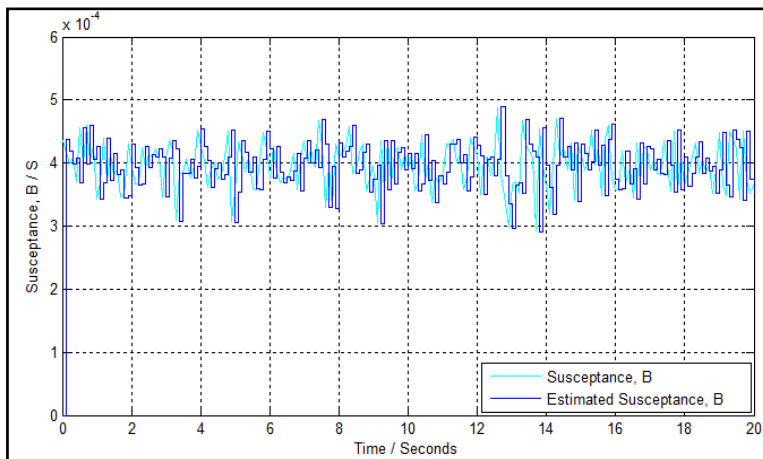


Figure 5. Graph of real and estimated value of susceptance, B

Table 1 defines the system parameters of the simulink model with the small value of noise which is 0.1. It shows the accuracy of R, X and B in the system more precise with the small value of noise which are 99.32 %, 95.43 % and 94.85 % respectively. Thus, it is shown that higher estimation accuracy will be achieved with the decreasing values of the noise.

	Real values	Estimation values	Accuracy
Resistance, R	2.96 Ω	2.94 Ω	99.32
Reactance, X	32.4 Ω	30.92 Ω	95.43
Susceptance, B	3.69 x 10 <sup>-4</sup> S	3.50 x 10 <sup>-4</sup> S	94.85

Figure 6, Figure 7 and Figure 8 shows the real and estimated values of R, X and B in the system. The real values of R, X and B are 2.96 Ω, 32.40 Ω and 3.69 x 10<sup>-4</sup> S while the estimated values R, X and B are 2.66 Ω, 33.02 Ω and 2.84 x 10<sup>-4</sup> S respectively with higher system noise of 50. Both of the calculated and estimated parameters are seen less consistency and the results are slightly different due to the greater noise in the system.

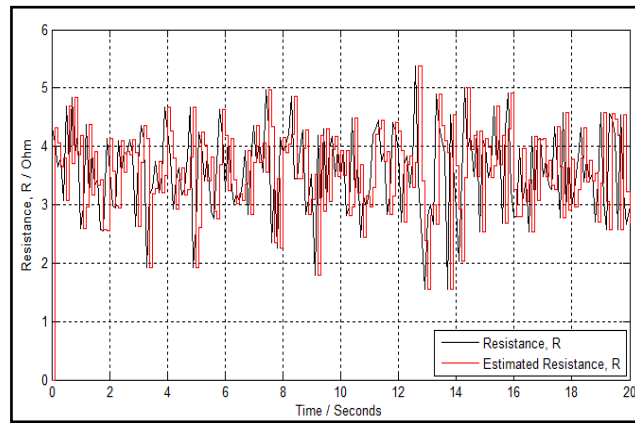


Figure 6. Graph of real and estimated value of resistance, R

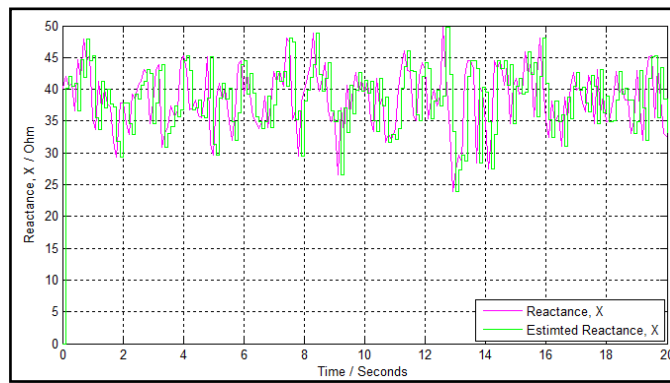


Figure 7. Graph of real and estimated value of reactance, X

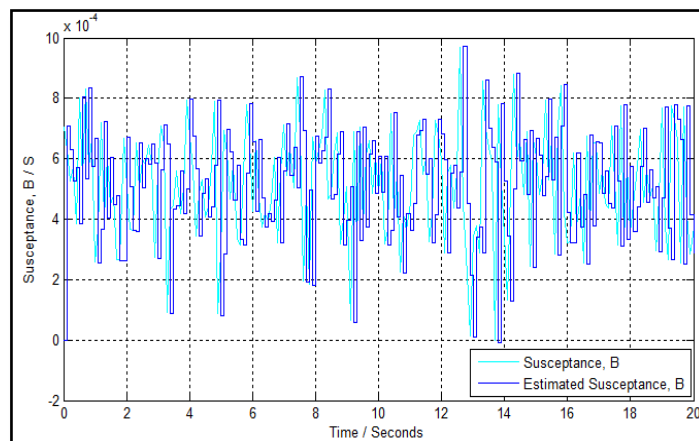


Figure 8. Graph of real and estimated value of susceptance, B

Table 2 defines the system parameters of the simulink model with the greater value of noise which is 50. It shows the accuracy of R, X and B in the system are inaccurate with the

greater value of noise which are 89.86 %, 98.08 % and 76.96 % respectively. Thus, it is proved that the accuracy in the system will decrease with the increasing noise.

Table 2. Parameter values

	Real values	Estimation values	Accuracy
Resistance, R	2.96 $\Omega$	2.66 $\Omega$	89.86
Reactance, X	32.4 $\Omega$	33.02 $\Omega$	98.08
Susceptance, B	3.69 x $10^{-4}$ S	2.84 x $10^{-4}$ S	76.96

According to these table and graphs presented previously, the value of noise affected the accuracy of R, X and B estimation in the system. The higher the value of noises or disturbances in the system, the performance will degrades. These results of R, X and B estimation indicate the possibility for parameters estimation even in different places. Noted that, it is important to consider that the values of R, X and B affected the determination of system efficiency in the transmission line.

Comparison of parameter errors between Kalman filter and Least Square method also has been done [5]. The line parameter errors of R, X and B by using Kalman filter are 0.02  $\Omega$ , 1.48  $\Omega$  and 0.16 S while the line parameter errors of R, X and B by using Linear Square method are 11.9  $\Omega$ , 0.212  $\Omega$  and 38.7 S. Generally, from the results presented in Table 3 below, Kalman filter give a higher accuracy compared to the Least Square method. From these two results of accuracy, it can be proven that Kalman filter is a better method that can be used for estimation.

Table 3. Parameter errors

	R	X	B
<b>KF</b>	0.02	1.48	0.16
<b>LS</b>	11.9	0.212	38.7

**KF:** Kalma filter method

**LS:** Least Squares Method

Remark that, it is important to consider of R, X and B as they affected the determination of power transfer, voltage drop and efficiency in the transmission line. The voltage drop in the line depends upon the values of above three lines constant. Similarly, the resistance of transmission line is the most important cause of power losses in the line and determine the transmission efficiency.

## 7. Conclusion

This paper presented the analysis of R, X and B estimation in a system. The analysis has proven that Kalman filter is one of the optimal methods that can be used for parameter estimation in transmission line. From this research, it has been revealed that noise in the system indicates the performance of medium transmission line.

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