■ 648

Control Synthesis for Marine Vessels in Case of Limited Disturbances

Mikhail N. Smirnov*, Maria A. Smirnova
St. Petersburg State University, 7/9 Universitetskaya nab. St. Petersburg, 199034, Russia
*Corresponding author, e-mail: mikhail.smirnov@spbu.ru

Abstract

In view of rapid development of computer technology digital systems of automatic control are installed on the modern marine vessels for performance of various manoeuvres at optimal trajectories taking into account features of the ship and active disturbances. In this connection, great number of problems that deal with construction of automatic control systems, such as minimizing the fulfillment time of the maneuver, searching for the optimal trajectory, the suppression of various types of exogenous disturbances like wind and rough sea, arises. In the work, the problem of suppression of exogenous disturbances acting on a marine vessel, about which we have no information except its boundedness, is considered. The problem of searching of the controller as a static state feedback is the basis of offered approach. The system MATLAB-Simulink is accepted as the basic tool of the computer support. An example of modeling control system for the carrier is presented.

Keywords: controller, disturbance, marine vessel

1. Introduction

Modern marine vessels are the integral part of transport system for every country. It perform great number of different tasks, including the most difficult research efforts and rescue expeditions. Thereby the control problem of marine vessel in real-life environment, that is under the influence of some exogenous disturbances, arises [1]. In view of rapid development of computer technology digital systems of automatic control are installed on the modern marine vessels for performance of various manoeuvres at optimal trajectories taking into account features of the ship and active disturbances [2]. In this connection, great number of problems that deal with construction of automatic control systems, such as minimizing the fulfillment time of the maneuver, searching for the optimal trajectory, the suppression of various types of exogenous disturbances like wind and rough sea, arises [3-8]. Earlier in literature different scientists consider systems without disturbances or systems with disturbances of a specified form or decreasing ones with the yaw of time. In the work, the problem of suppression of exogenous disturbance, about which we have no information except its boundedness, is considered. In this situation, it requires to choose the parameters of the controller, which will give the best possible result under the worst bounded disturbance.

In case of successful solution of this problem with use of modern computer technologies obtained control system is easily integrated into on-board computer complex structure and provides the desired dynamics of control processes. Particularly, in the work much prominence is given to questions, associated with computer synthesis and modeling of control laws that suppress bounded exogenous disturbances. An example of modeling control system for the vessel with displacement ton 6000 is performed.

2. Research Method

Let us consider the mathematical model of marine vessel:

$$
\begin{aligned}
\dot{\beta} &= a_{11}\beta + a_{12}\omega + b_1\delta + h_1 d(t), \\
\dot{\omega} &= a_{21}\beta + a_{22}\omega + b_2\delta + h_2 d(t), \\
\dot{\varphi} &= \omega, \\
\dot{\delta} &= u.
\end{aligned}
\tag{1}
$$

Here ω is an angular velocity relative to the vertical axis, φ is a yaw (the turn to port side is considered positive), δ is a deviation angle of the vertical rudders, β is a drift angle (angle between the velocity vector and longitudinal axis of the ship), u is a control, $d(t)$ is a bounded exogenous disturbance as shown in Figure 1:

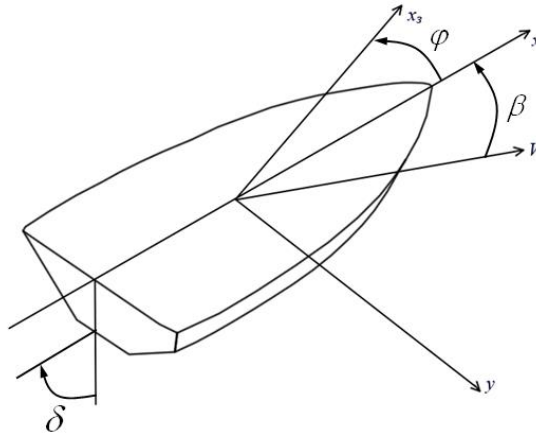$$d^T(t)d(t) \le 1, \quad 0 \le t < \infty. \tag{2}$$

Figure 1. Parameters of the vessel

It is offered to search for the control as a static state feedback with mathematical model:

$$u = k_1\beta + k_2\omega + k_3\varphi + k_4\delta, \tag{3}$$

where k_1, k_2, k_3, k_4 are parameters need to be found and provide the desired dynamics of the closed loop system. Deviation of the rudders and the velocity turn (that is control) are constrained:

$$|\delta| \le 30^o, |u| \le 3\,^o/c.$$

Let us designate $x = (\beta, \omega, \varphi, \delta)^T$ as a state vector, $y(t)$ as a output vector.
Then the system (1) may be rewritten as:

$$\begin{aligned}
\dot{x} &= Ax + Bu + Hd(t), \\
y &= Cx,
\end{aligned} \tag{4}$$

where matrices A, B, C, D are:

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 & b_1 \\ a_{21} & a_{22} & 0 & b_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad H = \begin{pmatrix} h_1 \\ h_2 \\ 0 \\ 0 \end{pmatrix}.$$

Then closed-loop system is modified to:

$$\dot{x} = A_c x + H_c d(t),$$
$$y = Cx,$$

(5)

where $A_c = A + BKC = A + BK$, $H_c = H$.

For the construction of this controller let use the method, associated with evaluations of the reachability set, that is the range of output's values of the system that we can obtain. We have to determine the minimum ellipsoid, in a sense, comprising the optimized output y under any bounded disturbances $d(t)$. Let's use the Theorem [9]:

Let the matrix A is stable, the pair (A, B) is controllable, C is the matrix of a full range. Then the set of reachable outputs $y(t)$ in the system:

$$\dot{x} = Ax + Hd(t), \qquad x(0) = 0,$$
$$y = Cx, \qquad d^T(t)d(t) \le 1, \quad 0 \le t < \infty$$

(6)

contains in the ellipsoid $\varepsilon = \{y: y^T (CPC^T)^{-1} y \le 1\}$, where $P = P(\alpha)$, $\alpha > 0$ is the solution of Lyapunov's equation:

$$AP + PA^T + \alpha P + \alpha^{-1} HH^T = 0, \quad P > 0.$$

(7)

Moreover, when solving one-parameter minimization problem

$$\min_{\alpha > 0} tr(CP(\alpha)C^T)$$

(8)

we obtain the ellipsoid with minimum trace among all ellipsoids, containing the reachability set of outputs.

Let's apply this theorem to closed-loop system (5). When substitute matrix A for matrix A_c into formula (7) we obtain:

$$AP + PA^T + PK^T B^T + BKP + \alpha P + \alpha^{-1} HH^T + Q \le 0$$

There are two matrix variables P and K, that are included in non-linear type, Q is a positive definite matrix. Let $Y = KP$ is a new matrix variable. Then we obtain linear matrix inequality

$$AP + PA^T + Y^T B^T + BY + \alpha P + \alpha^{-1} HH^T + Q \le 0$$

Thereby to find required minimum ellipsoid we need to solve the following minimization problem:

$$\min_{\alpha, \gamma > 0} tr(CP(\alpha, \gamma)C^T),$$
$$AP + PA^T + Y^T B^T + BY + \alpha P + \alpha^{-1} HH^T + Q \le 0, \quad \alpha > 0, \quad P > 0, Q > 0.$$

(9)

The solution in the problem (9) is reached at $Y = -\gamma B^T$ and with replacing the sense of inequality to equals sign. Thereby $P = P(\alpha, \gamma)$ is the solution of Lyapunov's equation

$$AP + PA^T - \gamma BB^T + \alpha P + \alpha^{-1} HH^T + Q = 0, \quad P > 0$$

The final solution is reached by solving the minimization problem

$$\min_{\alpha,\gamma>0} tr(CP(\alpha,\gamma)C^T), \qquad (10)$$

at the same time such values α,γ that $P(\alpha,\gamma)>0$ are considered. If we solve the problem (10), then we obtain the controller:

$$u = YP^{-1}x,$$

that provide inequality $y^T(CPC^T)^{-1}y \le 1$ for the output $y(t)$ of the system (5) with this controller under any disturbance $\|d\|_\infty \le 1$.

3. Results and Analysis

One of the most effective tools permitting to form and to use in researches computer models of dynamic system is the package MATLAB with subsystem Simulink. Let us consider mathematical model (1) of the vessel with displacement ton 6000. Its coefficients for a fixed velocity possess the following values:

$$a_{11} = -0.03408, a_{12} = 0.56, a_{21} = 0.015, a_{22} = -0.306,$$

$$b_1 = -0.0099, b_2 = -0.00417, h_1 = -0.0648, h_2 = -0.0046.$$

$$Q = \begin{pmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{pmatrix}.$$

Let's form the Simulink-model of the control system for the marine vessel, which scheme is represented on the Figure2. Figure 3 represents the graph of the bounded exogenous disturbance that was used for the computer modeling.
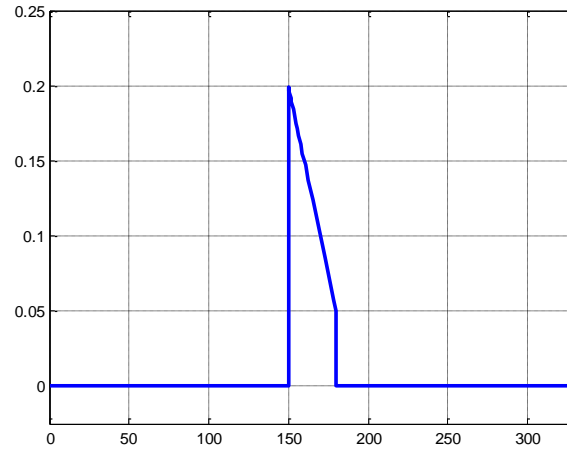

Figure 2. Simulink-model

Figure 3. Bounded disturbance

One of the effective approaches to the increasing of the digital system's performance consists in the search for such values of tuned parameters with fixed structure of the feedback, those provide the desired dynamics of the closed-loop system.

As the result of this algorithm for the given ship the controller with the coefficients $k_1 = 2.44$, $k_2 = 66.6$, $k_3 = -0.038$, $k_4 = 0.00133$, which provides the desired dynamics, is obtained. At the same time all technical constraints are taking into account. For the performance testing of the found controller let use it for automatic control of a sea-going ship under bounded exogenous disturbance. Figures 4 and 5 are showing the graphs of yaw and deviation angle of the vertical rudders.
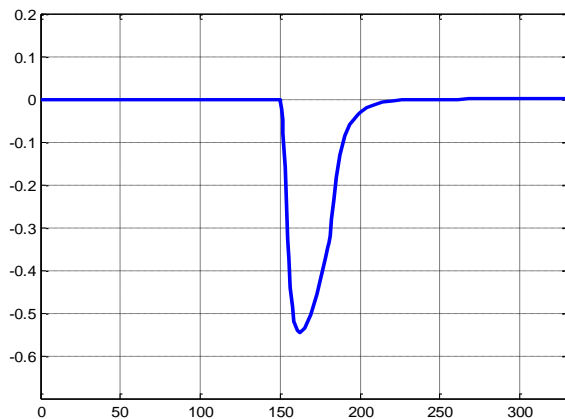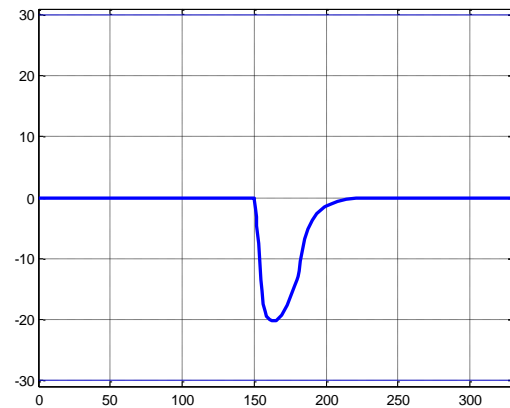

Figure 4. Yaw

figure 5. vertical rudders

4. Conclusion

In the work the method of suppression of bounded disturbances based on using of invariant ellipsoids is stated. For the specific model of a sea-going ship was found the controller, provided the restriction of exogenous disturbances, taking into account all technical requirements.

References
[1] Fossen TI. Guidance and Control of Ocean Vehicles. New York: John Wiley & Sons. 1999.

[2]   Lukomskiy UA, Korchanov VM. The control of the marine movable objects. St.Petersburg: Elmor. 1996.

[3]   Veremey EI, Pogozhev SV, Korchanov VM. Computer modeling of control systems for marine movable objects. St.Petersburg: SPbSU. 2002.

[4]   Veremey EI. The synthesis of the laws of multipurpose control for marine movable objects. *Gyroscopy and Navigation.* 2009; 4: 3-14.

[5]   Yang Qing, Gao De-xin. Optimal disturbance rejection control of underactuated autonomous underwater in vertical plane. *TELKOMNIKA (Indonesian Journal of Electrical Engineering).* 2015; 13(1): 91-100.

[6]   Saruchi SA, Zamzuri H, Zulkarnain N, Ariff MHM, Wahid N. Composite nonlinear feedback with disturbance observer for active front steering. *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS).* 2017; 7(2):434-441.

[7]   Wang Liyun, Xiao Jianmei, Wang Xihuai. Ship Dynamic Positioning Systems based on Fuzzy Predictive Control. *TELKOMNIKA (Telecommunication, Computing, Electronics and Control).* 2013; 11(11): 6769-6779.

[8]   Smirnov NV, Smirnova MA, Smirnova TE, Smirnov MN. Optimization of the size of minimal invariant ellipsoid with providing the desired modal properties. *Lecture Notes in Engineering and Computer Science.* 2016; 1: 238-241.

[9]   Scherbakov PS, Polyak BT. Robust stability and control. Moscow: Nauka. 2002.