■   525

Quantized Feedback Control of Network Empowerment Ammunition With Data-Rate Limitations

Fang Jin[1], **Zhi-hua Yuan***[1], **Qing-Quan Liu**[1,2,3]**, and Zhang Xing**[4]
[1] College of Information Science and Engineering, Shenyang Ligong University
No. 6, Nan Ping Zhong Road, Hun Nan Xin District, Shenyang, 110159, China
[2] Shenyang Institute of Automation, Chinese Academy of Sciences
No. 114, Nanta Street, Shenhe District, Shenyang, 110016, China
[3] Allwin Telecommunication CO., LTD
No. 6, Gaoge Road, Hun Nan Xin District, Shenyang, 110179, China
[4] Agricultural Development Bank of China branch in Liaoning province China
*Corresponding author, e-mail: lqqneu@163.com

Abstract

This paper investigates quantized feedback control problems for network empowerment ammunition, where the sensors and the controller are connected by a digital communication network with data-rate limitations. Different from the existing ones, a new bit-allocation algorithm on the basis of the singular values of the plant matrix is proposed to encode the plant states. A lower bound on the data rate is presented to ensure stabilization of the unstable plant. It is shown in our results that, the algorithm can be employed for the more general case. An illustrative example is given to demonstrate the effectiveness of the proposed algorithm.

Keywords: network empowerment ammunition, bit-allocation algorithms, data-rate limitations, quantized control, feedback stabilization

1. Introduction

Networked control systems have attracted great interests in recent years [1-3]. In this paper, we study quantized feedback control problems for network empowerment ammunition with limited information about the plant states. This problem arises when the state measurements are to be transmitted to the controller via a limited capacity communication channel.

Issues of the type discussed are motivated by several pieces of work in the recent literature. The research on the interplay among coding, estimation, and control was initiated by [4]. A high-water mark in the study of quantized feedback using data rate limited feedback channels is known as the data rate theorem that states the larger the magnitude of the unstable poles, the larger the required data rate through the feedback loop. The intuitively appealing result was proved [5-8], indicating that it quantifies a fundamental relationship between unstable physical systems and the rate at which information must be processed in order to stably control them. When the feedback channel capacity is near the data rate limit, control designs typically exhibit chaotic instabilities. This result was generalized to different notions of stabilization and system models, and was also extended to multi-dimensional systems [9-12]. Liu and Yang investigated quantized control problems for linear time-invariant systems over a noiseless communication network [13]. Furthermore, Liu addressed coordinated motion control of autonomous and semiautonomous mobile agents in [14], and derived the condition on stabilization of unmanned air vehicles over wireless communication channels in [15].

For the multi-state case, one needs to present an optimal bit-allocation algorithm to regulate the transmission of information about each mode such that stabilization can be guaranteed for all modes. In the literature, the bit-allocation algorithms were on the basis of the eigenvalues of

---

the system matrix A. Namely, it states the larger the magnitude of the unstable eigenvalues, the larger the required data rate through the feedback loop. Thus, it needs to transform the system matrix A by a real transformation matrix $H \in \mathbb{R}^{n \times n}$ to a diagonal matrix J or a Jordan canonical form J (i.e., $J = HAH^{-1}$) in order to decouple its dynamical modes to achieve an optimal bit-allocation algorithm. However, for the more general matrix A, modal decomposition might not be possible and putting the system matrix into Jordan canonical form generally requires a transformation matrix with complex elements such that the existing bit-allocation algorithms do not work.

In this paper, we present a new bit-allocation algorithm for the more general matrix A. The algorithm proposed here is on basis of not the eigenvalues but the singular values of the system matrix A. In particular, we quantize, encode the plant states by an adaptive differential coding strategy.

The rest of the paper is organized as follows. In Section 2, the problem formulation is presented. Section 3 presents the bit-allocation algorithm for stabilization. The results of numerical simulation are presented in Section 4. Conclusions are stated in Section 5.

2. Problem Formulation

Consider the control system of network empowerment ammunition described by the state equation

$$X(k+1) = AX(k) + BU(k) \tag{1}$$

where $X(k) \in \mathbb{R}^n$ is the measurable state, and $U(k) \in \mathbb{R}^p$ is the control input. A and B are known constant matrices with appropriate dimensions. The following is assumed to hold:

Assumption-1: The pair (A, B) is controllable, and the plant states are measurable;

Assumption-2: The initial condition $X(0)$ is a random vector, satisfying $E\|X(0)\|^2 < \phi_0 < \infty$;

Assumption-3: The sensors and controllers are geographically separated and connected by errorless, bandwidth-limited, digital communication channels without time delay. The channel is also assumed to be a time-invariant, memoryless channel. Then, the encoder and decoder have access to the previous control actions.

Let $\hat{X}(k)$ denote the decoder's estimate of $X(k)$ on the basis of the channel output. Our simplified model of the channel neglects the effects of network-induced delays and data dropout, and focuses on the bit-allocation algorithm. Then, we may implement a quantized state feedback control law of the form

$$U(k) = K\hat{X}(k). \tag{2}$$

As in [9], the system (1) is said to be asymptotically MS-Stabilizable via quantized feedback if for any initial states $X(0)$, there exists a control policy relying on the quantized data such that the states of the closed-loop system are asymptotically driven to zero in the mean square sense, namely

$$\limsup_{k \to \infty} E\|X(k)\|^2 = 0. \tag{3}$$

Our main task is to present a bit-allocation algorithm for the more general matrix A, and to derive the sufficient condition on the data rate for stabilization of the system (1) in the mean square sense (3).

3. The Bit-Allocation Algorithm

This section deals with the stabilization problem under data rate constraints, and presents a bit-allocation algorithm for the more general matrix A. Since the matrix $A'A$ is a real symmetric matrix, there exists a real orthogonal matrix $H \in \mathbb{R}^{n \times n}$ that diagonalizes $A'A = H'\Lambda^2 H$. Here, we define $\Lambda := \text{diag}[\sigma_1, \cdots, \sigma_n]$ where σ_i denotes the ith singular value of A $(i = 1, \cdots, n)$.

The coding technique presented in this paper is an adaptive differential coding strategy. Define

$$\bar{X}(k) := HX(k),$$
$$\tilde{X}(k) := H\hat{X}(k).$$

Then, the system (1) may be rewritten as

$$\bar{X}(k+1) = HAH'\bar{X}(k) + HBKH'\tilde{X}(k).$$

Let $\vec{X}(k) := H(A+BK)H'\tilde{X}(k-1)$ and $Z(k) := \bar{X}(k) - \vec{X}(k)$ denote the prediction value and the prediction error of $\bar{X}(k)$, respectively. Here, we set $\vec{X}(0) = 0$. Then, $Z(0) = X(0)$. It is shown in Assumption-3 that the encoder and decoder have access to the previous control actions, update their estimator and scaling in the same manner, and obtain the same prediction value. Then, we may quantize, encode $Z(k)$, and transmit the information of $Z(k)$ via a digital channel. Let $\hat{Z}(k)$ and $V(k)$ denote the quantization value and the quantization error of $Z(k)$, respectively. Then,

$$\hat{Z}(k) = Z(k) - V(k).$$

The value of $\hat{Z}(k)$ may be computed on the basis of the channel output at the decoder. Then, the decoder's estimate is defined as

$$\hat{X}(k) := H'(\vec{X}(k) + \hat{Z}(k)).$$

Thus, $\hat{X}(k)$ may be obtained by the controller.

Let $Z(k) := [z_1(k) \ z_2(k) \cdots z_n(k)]'$. As in [16], given a positive integer M_i and a nonnegative real number $\Delta_i(k)$ $(i = 1, \cdots, n)$, define the quantizer $q : \mathbb{R} \to \mathbb{Z}$ with sensitivity $\Delta_i(k)$ and saturation value M_i by the formula

$$q(z_i(k)) = \begin{cases} M^+, & \text{if } z_i(k) > (M_i + 1/2)\Delta_i(k), \\ M^-, & \text{if } z_i(k) \leq -(M_i + 1/2)\Delta_i(k), \\ \lfloor \frac{z_i(k)}{\Delta_i(k)} + \frac{1}{2} \rfloor, & \text{if } -(M_i + 1/2)\Delta_i(k) < z_i(k) \leq (M_i + 1/2)\Delta_i(k) \end{cases} \quad (4)$$

where define $\lfloor z \rfloor := \max\{k \in \mathbb{Z} := k < z, z \in \mathbb{R}\}$. The indeces M^+ and M^- will be employed if the quantizer saturates. The algorithm to be used here is based on the hypothesis that it is possible to change the sensitivity (but not the saturation value) of the quantizer on the basis of available quantized measurements. However, it is unclear in [16] how much each M_i should at least be in order to guarantee stabilization of the system (1). A lower bound of each M_i for stabilization will be presented in our results.

Based on the quantization algorithm above, we can construct a code with codeword length r_i $(i = 1, \cdots, n)$. Let $(c_1 c_2 \cdots c_{r_i})$ denote the codeword corresponding to $z_i(k)$. Namely, $z_i(k)$ is quantized, encoded, and transformed into the r_i bits for transmission. Then we may compute $c_j \in \{0,1\}$ $(j = 1, \cdots, r_i)$

$$(c_1 c_2 \cdots c_{r_i-1}) = \arg\max_{(c_1 c_2 \cdots c_{r_i-1})} \sum_{j=1}^{r_i-1} c_j 2^{j-1} \quad (5)$$

subject to the condition:

$$\sum_{j=1}^{r_i-1} c_j 2^{j-1} \leq |\lfloor \frac{z_i(k)}{\Delta_i(k)} + \frac{1}{2} \rfloor|.$$

Furthermore, we set $c_{r_i} = 0$ when $z_i(k) \geq 0$ and set $c_{r_i} = 1$ when $z_i(k) < 0$. This implies that

$$r_i = \log_2 M_i + 1.$$

Then the data rate of the channel is given by

$$R = \sum_{i=1}^{n} r_i \ \text{(bits/sample)}.$$

Our main result is the following theorem:

Theorem 1: Consider the system (1). Suppose that all the eigenvalues of $A + BK$ lie inside the unit circle. Then, for any given $\varepsilon \in (0,1)$, there exist a control law of the form (2), a quantization algorithm of the form (4), and a coding algorithm of the form (5) to stabilize the system (1) in the mean square sense (3) if the data rate of the channel satisfies the following condition:

$$R > \sum_{i \in \Xi} \frac{1}{2} \log_2 \frac{\sigma_i^2}{\varepsilon} \text{ (bits/sample)}$$

where $\Xi := \{i \in \{1, 2, \cdots, n\} : \frac{\sigma_i^2}{\varepsilon} > 1\}$.

Proof: Consider the closed-loop system

$$X(k+1) = AX(k) + BK\hat{X}(k)$$

which is equivalent to

$$\bar{X}(k+1) = HAH'\bar{X}(k) + HBKH'\tilde{X}(k).$$

Furthermore, notice that

$$\vec{X}(k+1) = H(A + BK)H'\tilde{X}(k).$$

By the definitions above, we have $\hat{X}(k) := H'(\vec{X}(k) + \hat{Z}(k))$, $Z(k) := \bar{X}(k) - \vec{X}(k)$, $\hat{Z}(k) = Z(k) - V(k)$, and $\tilde{X}(k) := H\hat{X}(k)$. Thus, it follows that

$$
\begin{aligned}
Z(k+1) &= \bar{X}(k+1) - \vec{X}(k+1) \\
&= HAH'(\bar{X}(k) - \tilde{X}(k)) \\
&= HAH'[(\vec{X}(k) + Z(k)) - (\vec{X}(k) + \hat{Z}(k))] \\
&= HAH'V(k).
\end{aligned}
\tag{6}
$$

Clearly, the prediction error $Z(k+1)$ is determined by the previous quantization error $V(k)$, and is independent of the control signals applied.

Notice that

$$X(k) = H'\bar{X}(k) = H'(\vec{X}(k) + Z(k)).$$

Thus, it holds that

$$E\|X(k)\|^2 = E\|\bar{X}(k)\|^2 = E\|\vec{X}(k)\|^2 + E\|Z(k)\|^2 + 2E[\vec{X}'(k)Z(k)]. \tag{7}$$

By the definition above, we see that

$$\vec{X}(k) = H(A + BK)H'\tilde{X}(k-1).$$

Furthermore, it follows from (6) that

$$Z(k) = HAH'V(k-1).$$

Thus, we have

$$
\begin{aligned}
E[\vec{X}'(k)Z(k)] &= E[\tilde{X}'(k-1)H(A + BK)'AH'V(k-1)] \\
&= E[(\bar{X}(k-1) - V(k-1))'H(A + BK)'AH'V(k-1)].
\end{aligned}
\tag{8}
$$

Here, $\bar{X}(k-1)$ and $V(k-1)$ are mutually independent random variables. This implies

$$E[\bar{X}'(k-1)V(k-1)] = 0. \tag{9}$$

Substitute (9) into (8), and obtain

$$E[\vec{X}'(k)Z(k)] = -E[V'(k-1)H(A + BK)'AH'V(k-1)]. \tag{10}$$

Thus, substituting (10) into (7), we may obtain

$$E\|X(k)\|^2 = E\|\vec{X}(k)\|^2 + E\|Z(k)\|^2 - 2E[V'(k-1)H(A + BK)'AH'V(k-1)]. \tag{11}$$

For any given vector X, we define $\Phi_X := E[XX']$. It follows from (6) that

$$
\begin{aligned}
E\|Z(k+1)\|^2 &= \mathrm{tr}[\Phi_{Z(k+1)}] \\
&= \mathrm{tr}[(HAH')\Phi_{V(k)}(HAH')'] \\
&= \mathrm{tr}[(HAH')'(HAH')\Phi_{V(k)}] \\
&= \mathrm{tr}[(HA'AH')\Phi_{V(k)}] \\
&= \mathrm{tr}[\Lambda^2 \Phi_{V(k)}]
\end{aligned}
\tag{12}
$$

Let $Z(k) := [z_1(k)\ z_2(k)\cdots z_n(k)]'$ and $V(k) := [v_1(k)\ v_2(k)\cdots v_n(k)]'$. If there exists the quantization algorithm of the form (4) such that the following condition holds:

$$
\varepsilon E[z_i^2(k)] > \sigma_i^2 E[v_i^2(k)], \ \ (i = 1, 2, \cdots, n)
\tag{13}
$$

then we have

$$
E[z_i^2(k+1)] < \varepsilon E[z_i^2(k)] \ \ (i = 1, 2, \cdots, n).
$$

This means that

$$
E[z_i^2(k)] < \varepsilon^k E[z_i^2(0)] = \varepsilon^k E[x_i^2(0)] \ \ (i = 1, 2, \cdots, n).
$$

Thus, it follows that

$$
\limsup_{k\to\infty} E[z_i^2(k)] = 0 \ \ (i = 1, 2, \cdots, n).
$$

Namely,

$$
\limsup_{k\to\infty} E\|Z(k)\|^2 = 0.
\tag{14}
$$

Clearly, it follows from (6) and (14) that

$$
\limsup_{k\to\infty} E\|V(k)\|^2 = 0.
\tag{15}
$$

This implies

$$
\limsup_{k\to\infty} E[V'(k-1)H(A+BK)'AH'V(k-1)] = 0.
\tag{16}
$$

Notice that

$$
\begin{aligned}
\vec{X}(k+1) &= H(A+BK)H'\tilde{X}(k) \\
&= H(A+BK)H'\vec{X}(k) + H(A+BK)H'\hat{Z}(k).
\end{aligned}
$$

It follows from (14) and (15) that

$$
\limsup_{k\to\infty} E\|\hat{Z}(k)\|^2 = 0.
$$

Furthermore, we know that all eigenvalues of $A+BK$ lie inside the unit circle. Thus, it follows that

$$
\limsup_{k\to\infty} E\|\vec{X}(k)\|^2 = 0.
\tag{17}
$$

Thus, substitute (14), (16), and (17) into (11), and obtain

$$
\limsup_{k\to\infty} E\|X(k)\|^2 = 0.
$$

This means that, the system (1) is asymptotically stabilizable in the mean square sense (3) if there exists the quantization algorithm of the form (4) such that the condition (13) holds.

By the quantization algorithm of the form (4), we know that the quantization error $V(k)$ may be small enough to make the condition (13) hold if the quantization levels are large enough. Notice that for the case with $\frac{\sigma_i^2}{\varepsilon} < 1$, the condition (13) must hold though we do not quantize the corresponding $z_i(k)$, and do not transmit its information to the controller. Thus, in order to make the condition (13) hold, we set

$$
M_i > \frac{\sigma_i}{2\sqrt{\varepsilon}},
$$
$$
r_i > \frac{1}{2}\log_2 \frac{\sigma_i^2}{\varepsilon}
$$

when $\frac{\sigma_i^2}{\varepsilon} > 1$ holds. Thus, it follows that

$$R = \sum_{i \in \Xi} r_i > \sum_{i \in \Xi} \frac{1}{2} \log_2 \frac{\sigma_i^2}{\varepsilon} \text{ (bits/sample)}$$

where $\Xi := \{i \in \{1, 2, \cdots, n\} : \frac{\sigma_i^2}{\varepsilon} > 1\}$.   □

Remark 2:

- Theorem 1 states that, the bit-allocation algorithm on the basis of not the eigenvalues but the singular values of the system matrix A can still guarantee stabilization of the system (1).

- The parameter ε has an important effect on the rate of convergence. For the case with $\varepsilon = 1$, we may also present a similar argument. However, we stress that, the system (1) may be not asymptotically stabilizable, but be boundable if the data rate satisfies the condition:

$$R > \sum_{i \in \Xi} \log_2 |\sigma_i| \text{ (bits/sample)}$$

  where $\Xi := \{i \in \{1, 2, \cdots, n\} : |\sigma_i| > 1\}$.

- If we set $\varepsilon = 1$ and assume that all the singular values of the system matrix A are larger than 1, it follows from Theorem 1 that

$$R > \sum_{i=1}^{n} \log_2 |\sigma_i| = \log_2 |\det(A)| \text{ (bits/sample)}.$$

Namely, our result may reduce to the well know data rate theorem [9,11].

4. Numerical Example

In this section, we present a numerical example for network empowerment ammunition to illustrate the effectiveness of the bit-allocation algorithm given in our results. Let us consider a networked control system which evolves in discrete-time according to

$$X(t+1) = \begin{bmatrix} -3.4331 & 17.3913 & -37.9582 \\ -12.4331 & 32.3913 & -33.9582 \\ -7.4331 & 17.3913 & -7.9582 \end{bmatrix} X(t) + \begin{bmatrix} 1.4 \\ 1.7 \\ 2.1 \end{bmatrix} U(t).$$

A control law is given by $K = [11.32830 \ -24.3046 \ 21.9182]'$. The initial plant state is given by $X_0 = [2000 \ 1000 \ -200]'$. The eigenvalues of the system matrix A are 7, $7 + 6i$, $7 - 6i$, and have magnitudes of 7, 9.2195, 9.2195. To achieve the minimum bit-rate needed to stabilize the unstable plant, one may find a matrix

$$\bar{H} = \begin{bmatrix} 0.9464 & 0.8546 & 0.8546 \\ 0.2983 & 0.4037 - 0.2202i & 0.4037 + 0.2202i \\ -0.1234 & -0.0499 - 0.236i & -0.0499 + 0.236i \end{bmatrix}$$

which diagonalizes A or transforms A to a Jordan canonical form. Then, by employing coordinate transform, we define

$$\bar{X}(k) := \bar{H} X(k),$$

and quantize $\bar{X}(k)$, encode $\bar{X}(k)$, and transmit the information of $\bar{X}(k)$ to the controller over a digital communication channel. Let $\bar{X}(k) := [\bar{x}_1(k) \ \bar{x}_2(k) \ \bar{x}_3(k)]'$. Clearly, $\bar{x}_2(k)$ and $\bar{x}_3(k)$ are complex values, but are not complex conjugates of each other. Thus, each $\bar{x}_2(k)$ and $\bar{x}_3(k)$ could not be reconstructed from only its real part or its complex part. Then, one has to quantize the real and imaginary parts of each $\bar{x}_2(k)$ and $\bar{x}_3(k)$ in order to guarantee boundedness of each $\bar{x}_i(k)$. Thus, the minimum bit-rate needed to stabilize the unstable plant is 19 bits/sample (since the bit rate must be an integer).

Figure 1. Trajectory of plant states.

To reduce the conservatism, we find a real matrix

$$H = \begin{bmatrix} 0.8802 & -0.4288 & 0.2034 \\ 0.4580 & 0.6549 & -0.6011 \\ 0.1246 & 0.6223 & 0.7728 \end{bmatrix}$$

which can diagonalize $A'A = H'\Lambda^2 H$ with $\Lambda = $ diag[0.5477, 15.4175, 65.5362]. It follows from Theorem 1 that, the minimum bit-rate needed to stabilize the unstable plant is 11 bits/sample which is same as that in [9], [11], etc. The corresponding simulation is obtained and shown in Fig.1. Clearly, the system is stabilizable.

5. Conclusion

In this paper, we presented a new bit-allocation algorithm for the more general system matrix, on which no assumption that there exists a real transformation matrix such that the system matrix can be transformed to a diagonal matrix or a Jordan canonical form was made. Different from the existing bit-allocation algorithms, the algorithm proposed here was on the basis of not the eigenvalues but the singular values of the system matrix. It was derived that the bit-allocation algorithm can guarantee stabilization of the system. The simulation results on network empowerment ammunition have illustrated the effectiveness of the proposed algorithm.

References

[1] Kotta H Z, Rantelobo K, Tena S, Klau G. Wireless Sensor Network For Landslide Monitoring in Nusa Tenggara Timur. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2011; 9(1):9-18.

[2] Tumiran C, Nandar S. Power Oscillation Damping Control Using Robust Coordinated Smart Devices. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2011; 9(1):65-72.

[3] Atia D M, Fahmy F H, Ahmed N M, Dorrah H T. A New Control and Design of PEM Fuel Cell System Powered Diffused Air Aeration System. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2012; 10(2):291-302.

[4] Wong W S, Brockett R W. Systems With Finite Communication Bandwidth Constraints II: Stabilization With Limited Information Feedback. *IEEE Trans. Automat. Control*. 1999; 44(5):1049-1053.

[5] Baillieul J. Feedback Designs For Controlling Device Arrays With Communication Channel Bandwidth Constraints. in *ARO Workshop on Smart Structures*, Pennsylvania State Univ, Aug. 1999.

[6] Baillieul J. Feedback Designs in Information Based Control. in *Stochastic Theory and Control Proceedings of a Workshop Held in Lawrence, Kansas*. B. Pasik-Duncan, Ed. New York: Springer-Verlag, 2001; 35-57.

[7] Baillieul J. Feedback Coding for Information-Based Control: Operating Near The Data-Rate Limit. in *Proc. 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada USA, 2002; 3229-3236.

[8] Li K, Baillieul J. Robust Quantization For Digital Finite Communication Bandwidth (DFCB) Control. *IEEE Trans. Automat. Control*. 2004; 49(9):1573-1584.

[9] Nair G N, Evans R J. Stabilizability of Stochastic Linear Systems With Finite Feedback Data Rates. *SIAM J. Control Optim.*. 2004; 43(2):413-436.

[10] Elia N. When Bode Meets Shannon: Control-Oriented Feedback Communication Schemes. *IEEE Trans. Automat. Control*. 2004; 49(9):1477-1488.

[11] Tatikonda S, Mitter S K. Control Under Communication Constraints. *IEEE Trans. Automat. Control*. 2004; 49(7):1056-1068.

[12] Tatikonda S, Sahai A, Mitter S K. Stochastic Linear Control Over a Communication Channel. *IEEE Trans. Automat. Control*. 2004; 49(9):1549-1561.

[13] Liu Q, Yang G H. Quantized Feedback Control For Networked Control Systems Under Information Limitation. *Information Technology And Control*. 2011; 40(3):218-226.

[14] Liu Q. Stabilization of Unmanned Air Vehicles Over Wireless Communication Channels. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 11(7):1701-1708.

[15] Liu Q. Coordinated Motion Control of Autonomous and Semiautonomous Mobile Agents. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 10(8):1929-1935.

[16] Liberzon D, Nešić D. Input-To-State Stabilization of Linear Systems With Quantized State Measurements. *IEEE Transactions on Automatic Control*. 2007; 52(5):767-781.