

A Sparse Representation Image Denoising Method Based on Orthogonal Matching Pursuit

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Abstract

Image denoising is an important research aspect in the field of digital image processing, and sparse representation theory is also one of the research focuses in recent years. The sparse representation of the image can better extract the nature of the image, and use a way as concise as possible to express the image. In image denoising based on sparse representation, the useful information of the image possess certain structural features, which match the atom structure. However, noise does not possess such property, therefore, sparse representation can effectively separate the useful information from noise to achieve the purpose of denoising. Aiming at image denoising problem of low signal-to-noise ratio (SNR) image, combined with Orthogonal Matching Pursuit and sparse representation theory, this paper puts forward an image denoising method. The experiment shows that compared with the traditional image denoising based on Symlets, image denoising based on Contourlet transform, this method can delete noise in low SNR image and keep the useful information in the original image more efficiently.

Keywords: Image Denoising, OMP (Orthogonal Matching Pursuit), Sparse Representation

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1. Introduction

When people receiving outside information, 80% are visual information. Digital image is the major source of visual information. However, in the meantime, while we receiving information of the image, there are inevitably interference of both internal factors and external factors, which makes the image contain many noises and makes the received information incomplete or even incorrect [1]. Noise commonly refers to the useless information. In the process of image processing, it is required to effectively restrict noise and improve the quality and visual effect of the image, which can not only improve correct judgment for the image, but also is very meaningful for the after-processing of the image [2].

How to denoising the image is one of the research focuses in recent years. Signal processing method changes from orthogonal transform to wavelet transform, then to multi-scale transform. In recent years, along with the development of compressed sensing technology, sparse representation theory has become a new research direction in the field of image denoising. Sparse model refers to describe the exist signal only with very little linear set in basic dictionary [3]. It is well known that ordinary images can be sparse representation in certain transform domain, thus to transfer the image to this transform domain. And the fortunate thing is that noise cannot be sparse represented in transform domain. Based on this premise, sparse representation theory can effectively delete the noise in the image. The image over complete signal sparse representation theory is first put forward by Mallat in 1993, and the adopted image sparse decomposition algorithm is the Matching Pursuit (MP) algorithm put by him. Of course, until now this theory is still not completely mature, and requires further study and discussion. However, it is another new thought and direction after previous image denoising theories [4, 5].

This paper mainly based on sparse representation theory to explore the image denoising problem. It first conducts a brief introduction of noisy image and sparse representation theory, then explains the main process and idea of OMP. Based on the above mentioned research theories and technologies, it puts forward the design process of the sparse representation image denoising method based on OMP. The last part of this paper is experiment design and result analysis.

2. Noisy Image and Sparse Representation Theory

2.1. Matrix Expression of Digital Image

Digital image refers to the sampled and quantified two dimensional function, which samples through equidistant rectangular grid, and conducts equal interval quantization to the scope. The digital image discussed in this paper is limited to two dimensional grayscale static image gained through sampling and quantization (that is, digitization or A/D) of two dimensional continuous image. Usually a digital image is often expressed through two dimensional matrix.

Sample image $f(x, y)$, select $M \times N$ data, sequence these data into a matrix according to the corresponding position of the sample dot, and then quantify each array position to gain a digital matrix. Use this matrix to replace function, $f(x, y)$. In other words, digital imaged can be represented by a matrix. The elements in the matrix are called the pixels of the image, which can be described as:

$$\begin{array}{c}
 f(x, y) \xrightarrow{\text{Sampling}} \left[\begin{array}{cccc}
 f(x_0, y_0) & f(x_0, y_1) & \cdots & f(x_0, y_{N-1}) \\
 f(x_1, y_0) & f(x_1, y_1) & \cdots & f(x_1, y_{N-1}) \\
 \vdots & \vdots & \ddots & \vdots \\
 f(x_{M-1}, y_0) & f(x_{M-1}, y_1) & \cdots & f(x_{M-1}, y_{N-1})
 \end{array} \right] \\
 \longrightarrow [f(i, j)]_{M \times N} \xrightarrow{\text{Quantification}} [f_l(i, j)]_{M \times N}
 \end{array} \quad (1)$$

In which, $f_l(i, j)$ refers to the quantified pixel value.

If the sample amount is $M \times N$, quantization grade is $Q = 2^n$, then the bit needed for storing a digital image is:

$$B = M \times N \times n \quad (2)$$

Usually a two dimensional number set is used to store the digital image data. The scale of the two dimensional number set equals to that of the digital image. Each element in the number set is corresponding to each pixel in the digital image, which stores corresponding pixel grayscale value.

2.2. Noisy Model

In reality, during the process of digitization and transfer, the digital image often interfered by the image formation device and noise from outside environment, and becomes noisy image. Image denoising refers to deleting and reducing the noise in the digital image. Before image denoising we need to first build a noisy image model, which is created by adding a random noise to the original image:

$$p(x, y) = f(x, y) + q(x, y) \quad (3)$$

$f(x, y)$ refers to the image, $q(x, y)$ refers to the noise, the noisy image is described as $p(x, y)$.

2.3. Sparse Decomposition Algorithm

The sparse decomposition of the signal means that when decomposing on the over-complete dictionary, the basic functions that represent the signal can be selected flexibly according to the signal's characteristics, and it requires only a little of basic functions. In other words, the signal can be represented by the product of a set of sparse coefficients and the training dictionary. If there are only a small part of elements in a signal is non-zero, then the signal is defined as sparse. In real signal processing, in order to improve the processing effect and speed, such sparse data expressions are always needed. Replacing the original data by sparse approximation can not only reduce the signal processing cost fundamentally, but also greatly improve the compact efficiency [6, 7].

An important premise of signal sparse decomposition is to find the sparse domain of the signal, which directly relates to the reconstruction accuracy of the compact perception. Decomposing the signal on the over-complete dictionary can gain the sparse expression of the original signal. Signal sparse expression under the over-complete dictionary is more effective, and matrix combined from some types of random matrix and dictionaries with certainty has small restricted isometry constants. The sparse decomposition process of the signal can be described as: for the D dimensional Hilbert space $H = R^D$, offer a set $G = \{g_{\gamma l}, l = 1, 2, \dots, L\}$, $L \square D$, set G refers to the dictionary[8]. The redundancy of the dictionary ($L \square D$), make vector $g_{\gamma l}$ no longer linear independent. For any signal $v \in H$ with a length of D , approximate by automatically selecting the best m atoms from G , make the approximate error $\varepsilon = \|v - \tilde{v}\|^2 < \rho$, (ρ refers to a sufficiently small positive number), gain its approximation $\tilde{v} : \tilde{v} = \sum_{l=0}^{m-1} c_l g_l \rightarrow v$.

Sparse approximation refers to when the approximation error is determined, selecting the m with smallest value from the m set that meets the above equation. Signal sparse decomposition is a typical NP (Non-Deterministic Polynomial) with extremely high calculation complexity, because the involved L0 norm is non-convex. Under the over-complete condition, decomposition algorithm of polynomial time is completely unrealistic. Therefore, suboptimal approximation algorithm is required [9, 10].

3. Main Process and Concept of The OMP (Orthogonal Matching Pursuit)

The MP (Matching Pursuit) algorithm is a sparse signal representation algorithm based on a redundant dictionary. MP is a greedy signal approximation algorithm, selecting at least one atom at each iteration to best match the inner [11].

OMP possesses the property of optimal iteration, which has a smaller iteration amount. It only select one atom to update the set during each iteration. Therefore, its iteration time is closely related to sparsity K and sample amount M . The basic concept of OMP is to select sensing matrix columns through greedy iteration. The selected columns of each iteration and the current redundancy are correlated to the largest extent. Subtracting the related part from the measuring vector and iterate repeatedly until the iteration time reaches the sparsity K .

Input: over-complete dictionary $G = [g_1, g_2, \dots, g_L]$, original signal y , initialized sparsity M , redundancy $r_0 = y$, support index set $A_0 = \emptyset$, initial iteration $m = 1$.

Process: iteration, in the m circulation, operate steps (1)-(5).

(1) Finding out the corresponding footer δ of the maximum of the products of residual r and sensing matrix column g_i , calculate the support index: $\delta_m = \arg \max_{i=1, \dots, N} | \langle r_{m-1}, d_i \rangle |$.

(2) Introducing the signal support set, $\Gamma_m = \Gamma_{m-1} \cup (\delta_m)$, find the reconstructed atom set $\Psi_t = [\Psi_{t-1}, \varphi_{\delta_t}]$ from the sensing matrix.

(3) Gaining $\hat{x}_t = \arg \min_z \|y - \Psi_t \hat{x}\|_2$ through the least squares.

(4) Updating the residual: $r_m = y - G_{\delta_m} (G_{\delta_m}^T G_{\delta_m})^{-1} G_{\delta_m}^T y$.

(5) $m = m + 1$ if $m > M$, then end the iteration, if not, return to step(1), until the iteration $y =$ termination condition: $m = M$ is met[12].

Output: support index set $\Gamma_m = \Gamma_{m-1}$, sparse coefficient $b = G_{\delta_m} (G_{\delta_m}^T G_{\delta_m})^{-1} G_{\delta_m}^T y$.

4. Sparse Representation Image De noising Algorithm Based on Orthogonal Matching Pursuit

(1) Image block classification, divide the image into three structures of smooth, texture and edge. Different structures possess different visual information. The image classification is represented by the following equations:

$$G = G_s \cup G_t \cup G_e \quad (4)$$

In equation (4), G refers to the entire image, G_s , G_t and G_e respectively refer to the smooth, texture and edge part of the image. Image block classification is the basis and premise of building the sub-content dictionary in the smooth, texture and edge parts of the image [13].

(2) Based on the block classification to the task image, gain different sub-block sets, and then conduct sparse decomposition to these sub-blocks in each corresponding training dictionary with OMP algorithm [14].

$$\arg \min \{ \|y_i - Gx_i\|_2^2 \} \quad s.t \quad \forall i, \|x_i\|_0 \leq T_0, i = 1, 2, \dots, N \quad (5)$$

Gain each type of sparse coefficient matrix Q through equation (5).

(3) After gaining the sparse matrix of the training dictionary and image to be processed, denoise the task image. Here, consider y_i as the column vector transformed from the signal of the noisy image to be processed, \hat{y}_i as the useful information of the image, $\Delta\psi$ is the \hat{y}_i frequency bandwidth, $\hat{\alpha}_{\Delta\psi}$ is the noise inside the frequency band $\Delta\psi$, $\hat{\alpha}_{\Delta\psi^c}$ is the noise outside the frequency band $\Delta\psi$. $\hat{\alpha}_{\Delta\psi^c}$ can be considered as is orthogonal with all the atoms in the dictionary, then the sparse decomposition equation of step k is [15]:

$$\|R^k y_i\|_2^2 = \|R^k (\hat{y}_i + \hat{\alpha}_{\Delta\psi} + \alpha_{\Delta\psi^c})\|_2^2 = \|R^k (\hat{y}_i + \hat{\alpha}_{\Delta\psi})\|_2^2 + \|\alpha_{\Delta\psi^c}\|_2^2 \quad (6)$$

The gained sparse representation matrix Q of the denoised image.

(4) Finally, gain the denoised image of each part of the image P_s , P_t and P_e through equation (7), reconstruct the three images to gain the denoised target image P .

$$P = \tilde{G}Q \quad (7)$$

5. Experiment Design and Result Analysis

In order to prove the efficiency and advantage of the algorithm of this paper in low SNR image denoising, two other denoising algorithms are compared in terms of noisy image denoising. These algorithms include wavelet hard threshold image denoising algorithm based on Symlets and image denoising algorithm based on Contourlet transform.

5.1. Introduction to Symlets and Contourlet Transform

Symlets is the discrete wavelet transform based on multi-resolution and multi-sampling filter theory, which is widely applied in engineering. It is a compactly supported orthogonal wavelet function improved from db wavelet. db wavelet does not possess the property of symmetry, but Symlet has better symmetry. Symlets is often represented by $\text{sym } N$ ($N = 2, 3, \dots$). The wavelet and scaling function of sym4 are shown in figure 1-3, It has properties of near symmetric, orthogonal and biorthogonal.

Contourlet transfer has the properties of multi-resolution, local positioning, multi-direction, near-critical sampling and anisotropy. Its basic functions are distributed on multi-scale and multi-direction. It can effectively capture the edge contour of the image with small amount of coefficients. Edge contour is the main characteristic of a nature image. The basic concept of Contourlet transfer is to first inspect the edge singularity through multi-scale decomposition similar to wavelet, then gather the singularities close in location into the contour according to the direction information.

Contourlet transform first use LP transform to conduct multi-scale analysis to the image. Each grade of decomposition gains a low frequency image and a high frequency image. LP decomposition first generates a low-pass approximation of the original image and a difference image between the original image and the low-pass predicted image. Further decomposition of the low-pass image can gain the next level low-pass image and difference image, like this, gaining the multi-resolution decomposition of the image through filtration step by step. Applying

the two dimensional Directional Filter Bank (DFB) in the high frequency component of each level gained through LP decomposition can gain 2^n directional subbands at any scales. Input the high-pass subband gained by each LP subband decomposition of the image to the DFB, gradually connect the singularities into a linear structure to capture the image contour. This paper selects the 4level Contourlet decomposition image denoising algorithm as the comparison algorithm.

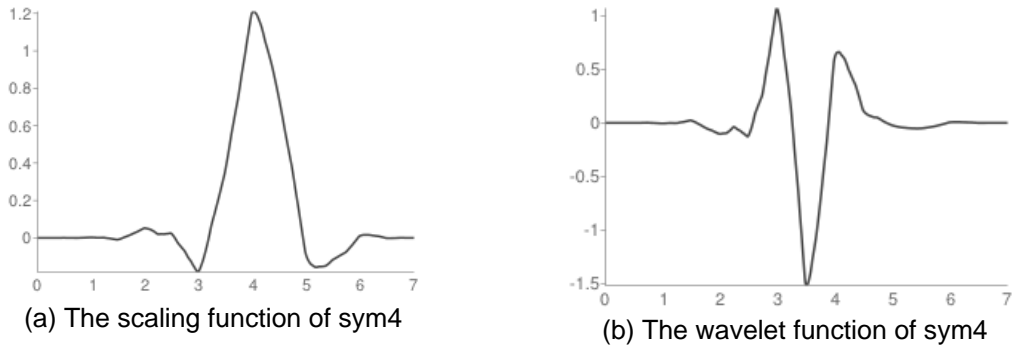


Figure 1. The scaling function and wavelet function of sym4

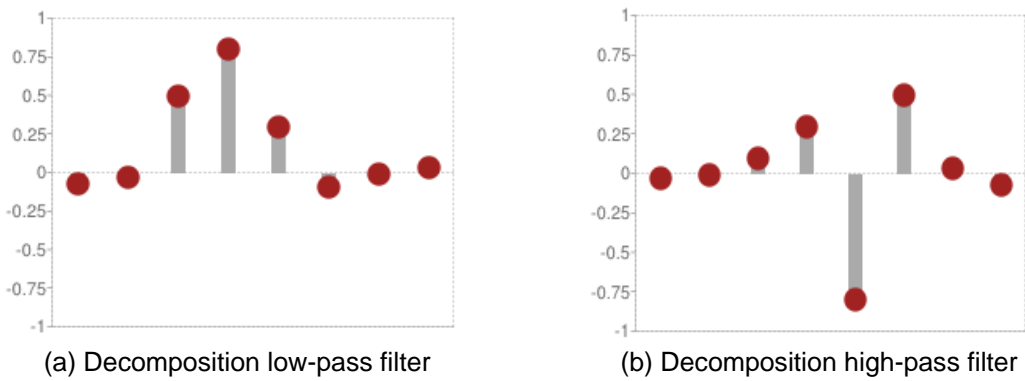


Figure 2. The decomposition low-pass and high-pass filters of sym4

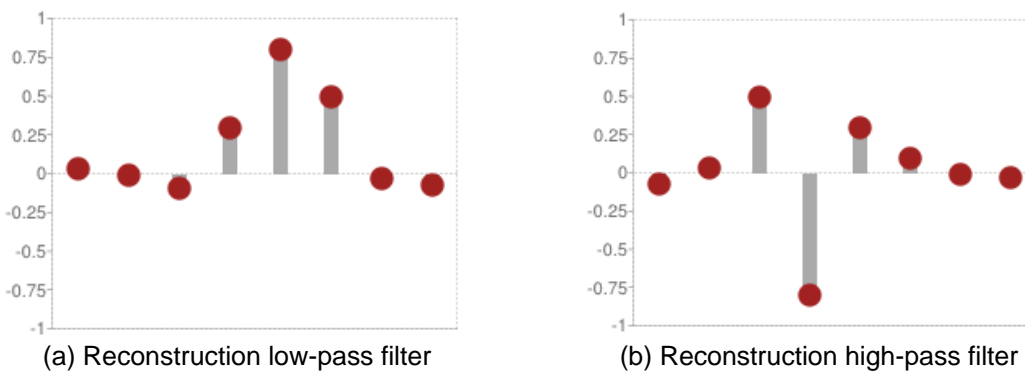


Figure 3. The reconstruction low-pass and high-pass filters of sym4

5.2. Experiment Result and Analysis

In order to test the validity of the algorithm, the denoising comparison of the noisy image is conducted. Figure 4 shows the denoised images by the above mentioned two different denoising methods.

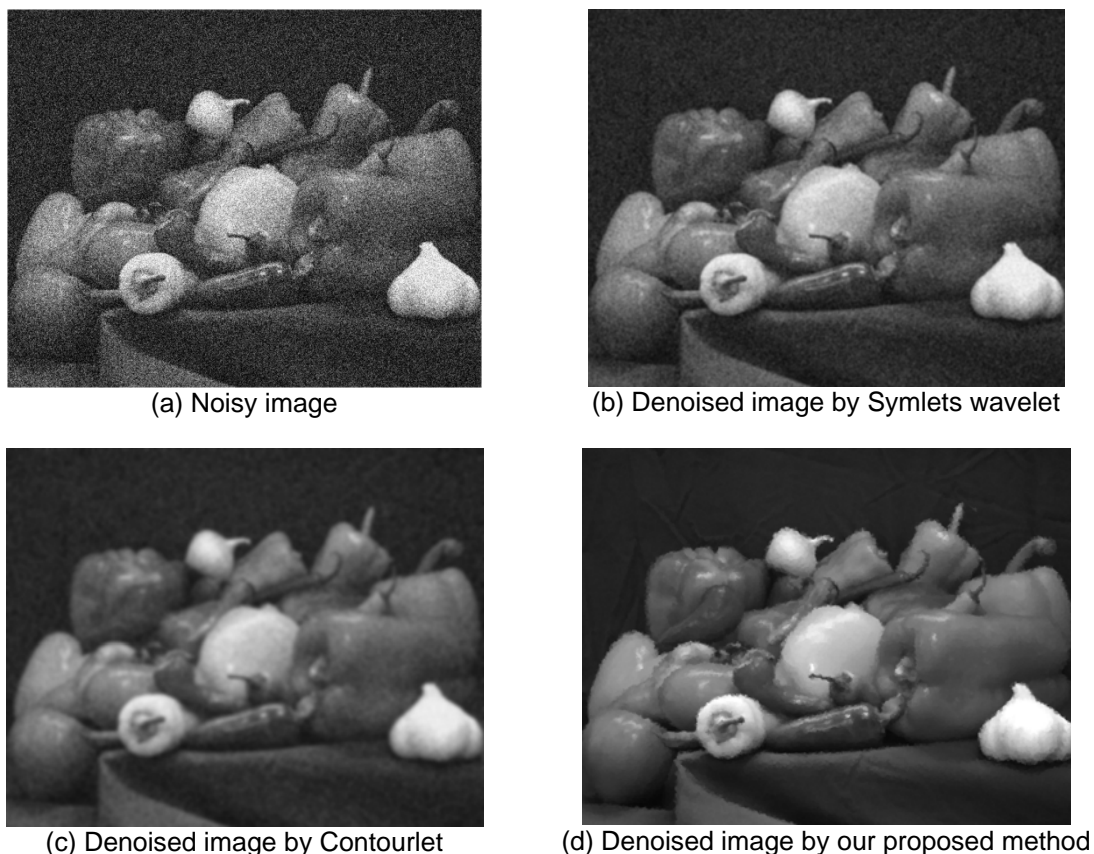


Figure 4. Denoised peppers images by different denoising methods

It can be seen from figure 4 that the signal of the reconstructed image based on Symlets transform and Contourlet transform will be attenuated and distorted. The reconstruction accuracy is poor and the denoising result is unsatisfying, which is not only caused by multi-scale decomposition and the reconstruction tool themselves, but also because these two algorithms possess the deficiency of huge redundancy, and will consider the edge of the image and other high frequency information as the noise and filter them, therefore, cannot maintain the basic coefficient distribution rule. On the contrary, the denoising algorithm put forward by this paper can denoise effectively, and gain better visual effect. It is because the two major tasks of sparse representation are dictionary building and sparse decomposition. When conducting sparse representation of the signal with analytic dictionary, the expression form of the signal is automatic and is more adaptive to different image data. Moreover, it separated the useful information from noise, which is only slightly affected by noise intensity and bandwidth, therefore, is still effective in denoising high noise and low SNR images, and can extract reconstruction information from the noisy image more thoroughly. Its image reconstruction effect and accuracy are quite satisfying.

6. Conclusion

Taking image denoising as the starting point, this paper has analyzed the traditional image denoising algorithm and image denoising algorithm based on sparse representation. It

have focused on orthogonal matching pursuit and sparse representation theory, with certain improvement, it put forward an efficient low SNR image denoising method. The experiment simulation has proved that this method can effectively denoise and maintain more detailed image texture information.

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