

# Improved lumped model for thermal analysis of high burn-up nuclear fuel rods

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## Abstract

High burn-up nuclear fuel elements have been intensively studied for prolonged lifetime of existing reactors and for next-generation advanced reactors. This paper presents an improved lumped-differential formulation for one-dimensional transient heat conduction in a heat generating cylinder with temperature-dependent thermo-physical properties typical of high burn-up nuclear fuel rods. Two-points Hermite approximations for integrals ( $H_{1,1}/H_{1,1}$ ) were used to obtain the average temperature and heat flux in the radial direction. As a testing case, transient heat conduction in a nuclear fuel rod was computed with the thermo-physical properties represented by the MATPRO correlations. The problem was formulated and solved by using the symbolic/numerical computation software system *MATHEMATICA*. The solutions of the proposed improved lumped model were validated by comparing with numerical solutions of the original distributed parameter formulation of the transient heat conduction problem, as well as by comparing with simulation results of a cold leg SBLOCA with RELAP5/MOD3.

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## 1. Introduction

High burn-up nuclear fuel elements have been intensively studied for prolonged lifetime of existing reactors and next-generation advanced reactors (see e.g., Bagger et al., 1994; Fuketa et al., 2001; Inoue et al., 2004; Suzuki et al., 2006). Inoue et al. (2004) studied fuel-to-cladding gap evolution and its impact on thermal performance of high burn-up uranium–plutonium oxide fuel pins by one-dimensional steady state thermal analysis. Recently, Suzuki et al. (2006) developed a computer code based on finite element method to analyze high burn-up fuel rod behaviors in the reactivity initiated accident (RIA) conditions.

Lumped parameter models have been widely used in the thermo-hydraulic analysis of nuclear reactors. As in the analysis of other complex thermal systems, this classical approach is extremely useful and sometimes even mandatory when a simplified formulation of the transient heat conduction is sought.

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Together with the neutron point kinetics model, the lumped parameter approach for fuel rod heat conduction is essential in the simplified models of pressurized water reactors (PWRs) and in real-time simulators of nuclear power plants (Zanobetti, 1989). It is of great technological interests to develop lumped models for thermal analysis of high burn-up nuclear fuel rods.

The classic lumped system approach has been widely used in thermal analysis of engineering problems, when simplified solutions of transient heat conduction are sought. As an inherent limitation of the lumped parameter approach, moderate to low temperature gradients within the region are assumed, which govern the accuracy of such approximate formulations through the associated problem parameters. As a rule of thumb, the classical lumped parameter approach that assumes uniform temperature within the domain is in general limited to problems with Biot numbers less than 0.1. In applications where the assumption of moderate to low temperature gradients is not reasonable, a more accurate approach should be adopted.

Cotta and Mikhailov (1998) presented a systematic formalism to provide improved lumped parameter formulations for

steady and transient heat conduction problems, based on Hermite approximations for integrals (Mennig et al., 1983) that define average temperatures and heat fluxes, as functions of the dependent variable and its derivatives at the boundaries. This formalism has been applied in a variety of heat conduction and convection problems (Aparecido and Cotta, 1989; Scofano Neto and Cotta, 1993; Corrêa and Cotta, 1998; Su, 2001, 2004; Ruperti et al., 2004; Pontedeiro et al., 2004).

Transient heat conduction in a nuclear fuel rod is a typical example where the Biot number is higher than the limit utilized in the classic lumped formulation. Regis et al. (2000) have proposed an improved lumped parameter model for the transient heat conduction in a nuclear fuel rod where the one-sided corrected trapezoidal rule was employed in the averaged temperature integral for the fuel, and the plain trapezoidal rule was used in the averaged temperature integral for the cladding and the heat fluxes ( $H_{1,0}/H_{0,0}$  approximation). Su and Cotta (2001) presented a higher order lumped parameter analysis in which the two-sided corrected trapezoidal rule was used in the averaged temperature integrals for the fuel and the cladding, and the plain trapezoidal rule was used in the averaged heat flux integrals ( $H_{1,1}/H_{0,0}$  approximation). The proposed mathematical model was applied to simulate transient neutronic and thermo-hydraulic behavior of the PWR core and the primary circuit of a nuclear power plant during reactivity insertion and partial loss-of-flow transients.

The aim of this work is to develop an improved lumped-differential formulation for transient heat conduction in a heat generating cylinder with temperature-dependent thermo-physical properties. After presenting the mathematical formulation and the improved lumped formulation based on Hermite approximation for integrals, we consider a special case of the transient temperature behavior in a  $\text{UO}_2$  nuclear fuel rod with the temperature-dependent properties (thermal conductivity, density and specific heat) given by the correlations from MATPRO – a Library of Materials Properties for Light-Water-Reactor Accident Analysis (INEL, 1993). An improved lumped model based on two-points Hermite approximations to the average temperature and to the heat flux in the fuel rod is developed ( $H_{1,1}/H_{1,1}$  approximation). The solution of the proposed improved lumped models is then validated by a numerical solution of the original distributed parameter formulation.

## 2. Mathematical formulation

### 2.1. One-dimensional transient heat conduction equation

We consider one-dimensional transient heat conduction in a heat generating cylinder with temperature-dependent thermo-physical properties. The material is considered to be homogeneous and isotropic. The mathematical formulation governing the physical problem is written as follows:

$$\rho(T)C_p(T)\frac{\partial T(r,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk(T)\frac{\partial T(r,t)}{\partial r}\right) + g(t) \quad (1)$$

where  $T$  is the temperature,  $r$  is the radial coordinate,  $t$  is the time,  $\rho$  is the density,  $C_p$  is the specific heat,  $k$  is the thermal conductivity, and  $g$  is the spatially uniform volumetric heat generation rate which is independent of the temperature.

The heat conduction equation (1) is to be solved with appropriate boundary and initial conditions:

$$\left.\frac{\partial T(r,t)}{\partial r}\right|_{r \rightarrow 0} = 0 \quad (2)$$

$$\left.\frac{\partial T(r,t)}{\partial r}\right|_{r \rightarrow r_1} = \frac{h_{\text{ef}}(T_m(t) - T(r_1,t))}{k(T)} \quad (3)$$

$$T(r,0) = TI(r) \quad (4)$$

where  $r_1$  is the radius of the fuel rod,  $h_{\text{ef}}$  is an effective heat transfer coefficient between the heat generating cylinder and the surrounding fluid, and  $T_m(t)$  is the time-varying temperature of the surrounding fluid. The effect of the cladding and the gap in heat conduction is represented by the effective heat transfer coefficient that includes the thermal resistance of the cladding, the gap and the heat transfer between the cladding and the coolant.

### 2.2. Improved lumped formulation

The spatially averaged temperature in the fuel rod is defined as

$$T_{\text{av}}(t) = \frac{2}{r_1^2} \int_0^{r_1} rT(r,t) dr \quad (5)$$

and thus

$$\frac{dT_{\text{av}}(t)}{dt} = \frac{2 \int_0^{r_1} r(\partial T(r,t)/\partial t) dr}{r_1^2} \quad (6)$$

We operate Eq. (1) with  $2\int_0^{r_1} r dr/r_1^2$  and use the average temperature defined in Eq. (5):

$$\begin{aligned} & \frac{2}{r_1^2} \int_0^{r_1} \left( rC_p(T)\rho(T)\frac{\partial T(r,t)}{\partial t} \right) dr \\ &= \frac{2}{r_1^2} \int_0^{r_1} r \left( \frac{1}{r} \frac{\partial}{\partial r} \left( rk(T)\frac{\partial T(r,t)}{\partial r} \right) + g(t) \right) dr \end{aligned} \quad (7)$$

### 2.3. Heat conduction in a nuclear fuel rod

Now we proceed to analyze transient heat conduction in a nuclear fuel rod made of  $\text{UO}_2$  ceramic material. The MATPRO correlations (INEL, 1993) for the thermo-physical properties of  $\text{UO}_2$  ceramic material are as follows:

$$k(T) = \frac{1}{a + bT(r,t)} + c \frac{e^{-(d/T(r,t))}}{T(r,t)^2} \quad (8)$$

$$C_p(T) = \frac{k_1 \theta^2 e^{(\theta/T(r,t))}}{T^2(r,t)(e^{(\theta/T(r,t))} - 1)^2} + k_2 T(r,t) + \frac{k_3 EE_{dd}}{RT^2(r,t)} e^{-(EE_{dd}/RT(r,t))} \quad (9)$$

$$\rho(T) = \rho_0(1 - 3(K_1 T(r,t) - K_2 + K_3 e^{-(E_1/K_B T(r,t))})) \quad (10)$$

where  $a = 0.0375$ ,  $b = 2.165$ ,  $c = 4.75$ , and  $d = 16,361$ ;  $k_1 = 296.7$ ,  $\theta = 535.285$ ,  $k_2 = 2.43 \times 10^{-2}$ ,  $k_3 = 8.745 \times 10^7$ ,  $EE_{dd} = 1.577 \times 10^5$  and  $R = 8.3143$ ;  $K_1 = 10^{-5}$ ,  $K_2 = 3.0 \times 10^{-3}$ ,  $K_3 = 4.0 \times 10^{-2}$ ,  $K_B = 1.38 \times 10^{-23}$ ,  $E_1 = 6.9 \times 10^{-20}$  and  $\rho_0 = 10980.0$ .

Fig. 1 shows the thermal conductivity of UO<sub>2</sub> ceramic material as a function of temperature, given by Eq. (8).

As the product  $\rho C_p$  has a rather small variation in the temperature range between 500 and 2000 K, as shown in Fig. 2, it is possible to write Eq. (7) using  $\rho(T_{av})$  and  $C_p(T_{av})$ , as

$$\frac{2C_p(T_{av})\rho(T_{av})}{r_1^2} \int_0^{r_1} r \frac{\partial T(r,t)}{\partial t} dr = \frac{2}{r_1^2} \int_0^{r_1} r \left( \frac{1}{r} \frac{\partial}{\partial r} \left( rk(T) \frac{\partial T(r,t)}{\partial r} \right) + g(t) \right) dr \quad (11)$$

From Eqs. (6) and (11), and using the boundary condition, Eq. (3), we have

$$\frac{dT_{av}(t)}{dt} = \frac{g(t) + (2h_{ef}/r_1)(T_m(t) - T(r_1,t))}{\rho(T_{av})C_p(T_{av})} \quad (12)$$

Eq. (12) is an equivalent integro-differential formulation of the original distributed parameter formulation, Eqs. (1)–(4). Assuming that the temperature gradient is sufficiently smooth over the whole spatial solution domain, the classical lumped system analysis (CLSA) is based on the assumption that the boundary temperatures can be reasonably well approximated by the average temperature.

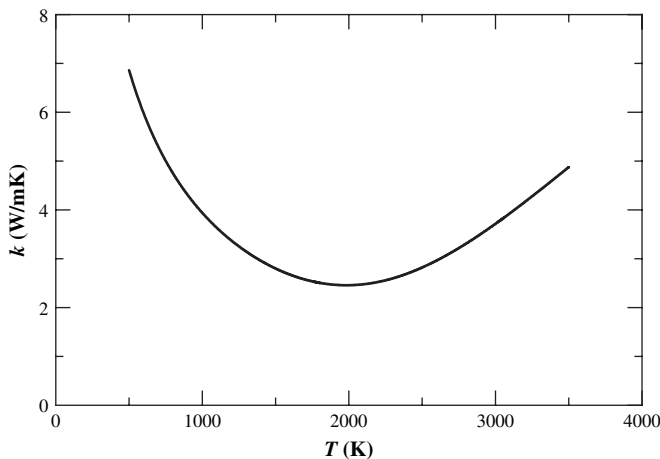


Fig. 1. The temperature dependence of the thermal conductivity of the UO<sub>2</sub> ceramic material.

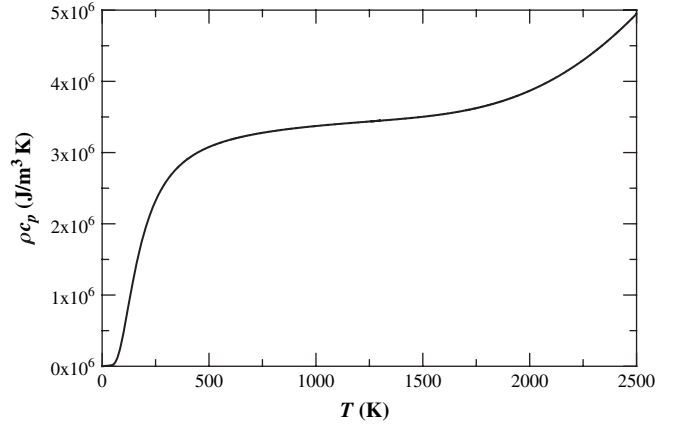


Fig. 2. The temperature dependence of the specific heat capacity ( $\rho C_p$ ) of the UO<sub>2</sub> ceramic material.

$$\frac{dT_{av}(t)}{dt} = \frac{g(t) + (2h_{ef}/r_1)(T_m(t) - T_{av}(t))}{\rho(T_{av}(t))c_p(T_{av}(t))} \quad (13)$$

We now seek an improved lumped model, in an attempt to offer enhancement to the approximation approach of the classical lumped model. The basic idea of the improved lumped-differential approach is to provide reasonably accurate relations between the boundary fluxes and the averaged temperatures, which are to be developed from two point Hermite approximations for the integrals that define the average temperatures and heat fluxes (Cotta and Mikhailov, 1998; Mennig et al., 1983).

Applying the  $H_{1,1}/H_{1,1}$  Hermite formulation both to the average temperature and the heat flux, we have

$$T_{av}(t) = \frac{k(T(r_1,t))(T(0,t) + 5T(r_1,t)) + r_1 h_{ef}(T(r_1,t) - T_m(t))}{6k(T(r_1,t))} \quad (14)$$

$$T(r_1,t) - T(0,t) = \frac{r_1}{2} \frac{\partial T(0,t)}{\partial r} + \frac{r_1}{2} \frac{\partial T(r_1,t)}{\partial r} + \frac{r_1^2}{12} \frac{\partial^2 T(0,t)}{\partial r^2} - \frac{r_1^2}{12} \frac{\partial^2 T(r_1,t)}{\partial r^2} \quad (15)$$

where

$$\frac{\partial^2 T(0,t)}{\partial r^2} = \frac{-g(t) + \rho(T_{av})C_p(T_{av})(\partial T(0,t)/\partial t)}{2k(T(0,t))} \quad (16)$$

$$\begin{aligned} \frac{\partial^2 T(r_1,t)}{\partial r^2} = & \frac{1}{r_1 k(T(r_1,t))^3} \left( h_{ef}(T(r_1,t) - r_1 g(t))k(T(r_1,t))^2 \right. \\ & - T_m(t) (k(T(r_1,t))^2 + r_1 h_{ef}(-T(r_1,t))) \\ & \left. + T_m(t) \right) \frac{dk(T(r_1,t))}{dT} \\ & + r_1 k(T(r_1,t))^2 \rho(T_{av})C_p(T_{av}) \frac{\partial T(r_1,t)}{\partial t} \end{aligned} \quad (17)$$

$$C_p(T_{av}) = \frac{k_1 \theta^2 e^{(\theta/T_{av}(t))}}{T_{av}^2(t)(e^{(\theta/T_{av}(t))} - 1)^2} + k_2 T_{av}(t) + \frac{k_3 EE_{dd}}{RT_{av}^2(t)} e^{-(EE_{dd}/RT_{av}(t))} \quad (18)$$

$$\rho(T_{av}) = \rho_0(1 - 3(K_1 T_{av}(t) - K_2 + K_3 e^{-(E_1/K_B T_{av}(t))})) \quad (19)$$

$$Y_1 = \frac{r_1^2(-g(t) + \rho(T_{av})C_p(T_{av}))(\partial T(0,t)/\partial t)}{24\left(\left(\frac{c e^{-(d/T(0,t))}}{T(0,t)^2}\right) + (1/(a + bT(0,t)))\right)}$$

$$Z_1 = r_1 \rho(T_{av}) C_p(T_{av}) \frac{\partial T(r_1,t)}{\partial t}$$

$$X_2 = \frac{r_1 h_{ef}(T(r_1,t) - T_m(t))\left(\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^4}\right) - (2c e^{-(d/T(r_1,t))}/T(r_1,t)^3) - (1/(a + bT(r_1,t))^2)\right)}{\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2}\right) + (1/(a + bT(r_1,t)))}$$

$$k(T(0,t)) = \frac{c e^{-(d/T(0,t))}}{T(0,t)^2} + \frac{1}{a + bT(0,t)} \quad (20)$$

$$k(T(r_1,t)) = \frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2} + \frac{1}{a + bT(r_1,t)} \quad (21)$$

The thermal conductivity derivative for  $T(r_1,t)$  is

$$\frac{dk(T(r_1,t))}{dT} = \frac{cd e^{-(d/T(r_1,t))}}{T(r_1,t)^4} - \frac{2c e^{-(d/T(r_1,t))}}{T(r_1,t)^3} - \frac{b}{(a + bT(r_1,t))^2} \quad (22)$$

In order to construct the system of equations to be solved, we need to define two ordinary differential equations and two initial conditions. The first equation is obtained by substituting Eqs. (14), (16)–(22) and the boundary conditions, Eqs. (2) and (3), into Eq. (15). By this way we can eliminate  $T_{av}$ , getting Eq. (23), as a function of  $T(0,t)$ ,  $T(r_1,t)$ ,  $(\partial T(0,t)/\partial t)$ ,  $(\partial T(r_1,t)/\partial t)$ :

$$T(r_1,t) - T(0,t) = X_1 + Y_1 - \frac{r_1(h_{ef}(T(r_1,t)) - r_1 g(t) - T_m(t)) + X_2 + Z_1}{X_3} \quad (23)$$

where

$$X_1 = \frac{r_1 h_{ef}(T(r_1,t) - T_m(t))}{2\left(\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2}\right) + (1/(a + bT(r_1,t)))\right)}$$

$$M = \frac{e^{-(2d/T(r_1,t))} (ac + bcT(r_1,t) + e^{(d/T(r_1,t))} T(r_1,t)^2)^2}{T(r_1,t)^4 (a + bT(r_1,t))^2 \left[\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2}\right) + (1/(a + bT(r_1,t)))\right]^2}$$

$$S = \frac{r_1 h_{ef}(a + 2bT(r_1,t) - bT_m(t))}{4(a + bT(r_1,t))^2 \left[\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2}\right) + (1/(a + bT(r_1,t)))\right]^2}$$

$$X_3 = 12 \left( \frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2} + \frac{1}{a + bT(r_1,t)} \right)$$

The second differential equation can be obtained by operating both sides of Eq. (14), using differentiation in time, and combining it with Eq. (12), in order to eliminate  $T_{av}$ .

By this way, we can eliminate  $T_{av}$ , substituting Eq. (14) into Eq. (24), as a function of  $T(0,t)$ ,  $T(r_1,t)$ ,  $(\partial T(r_1,t)/\partial t)$ :

$$\frac{\partial T(r_1,t)}{\partial t} = \frac{P + Q}{M + S + V} \quad (24)$$

where

$$P = \frac{g(t) + (2h_{ef}(-T(r_1,t) + T_m(t))/r_1)}{\rho_0(P_1 + k_2 T_{av}(t)) (1 - 3(-K_2 + e^{-(E_1/K_B T_{av}(t))} K_3 + K_1 T_{av}(t)))}$$

$$P_1 = \frac{e^{(\theta/T_{av}(t))} \theta^2 k_1}{(-1 + e^{(\theta/T_{av}(t))})^2 T_{av}(t)^2} + \frac{e^{-(EE_{dd}/RT_{av}(t))} EE_{dd} k_3}{RT_{av}(t)^2}$$

$$Q = \frac{r_1 h_{ef}(dT_m(t)/dt)}{4\left(\left(\frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2}\right) + (1/(a + bT(r_1,t)))\right)^2} \times \left[ \frac{c e^{-(d/T(r_1,t))}}{T(r_1,t)^2} + \frac{1}{(a + bT(r_1,t))} \right]$$

$$V = \frac{ce^{-d/T(r_1,t)}r_1h_{ef}\left(3T(r_1,t)^2+dT_m(t)-T(r_1,t)(d+2T_m(t))\right)}{4T(r_1,t)^4\left[\left(ce^{-d/T(r_1,t)}/T(r_1,t)^2\right)+\left(1/(a+bT(r_1,t))\right)\right]^2}$$

The above system allows us to obtain the transient temperature at the center and at the outer surface of the fuel rod ( $T(0,t)$  and  $T(r_1,t)$ ). Then, if required,  $T_{av}(t)$  can be evaluated from Eq. (14).

### 3. Results and discussion

In order to evaluate the accuracy of the improved lumped model proposed in this work, we have carried out a comparative study of transient heat conduction in a cylindrical nuclear fuel rod using the classical lumped model, Eq. (13), the improved lumped model and the original distributed parameter model, Eqs. (1)–(4). The numerical results obtained by the classical lumped model and the improved lumped model  $H_{11}-H_{11}$  are compared with the results obtained by a numerical solution of the original distributed parameter formulation, using the built-in function of MATHEMATICA NDSolve (Wolfram, 1999). The MATPRO correlations (INEL, 1993) for the thermo-physical properties of UO<sub>2</sub> ceramic material given in Eqs. (8)–(10) are used in all test examples.

#### 3.1. Example 1 ( $0.09 < Bi < 0.29$ )

The first numerical example, chosen for validation purposes, utilizes values for the fuel material properties according to Eqs. (8)–(10) and the system formed by Eqs. (22), (23) and (13) solved with the following conditions:  $h_{ef} = 130.0 \text{ W/m}^2 \text{ K}$ ;  $r_1 = 0.0045 \text{ m}$ ;  $T_m = 613.15 \text{ K}$ ;  $T(0,0) = T(r_1,0) = 613.15 \text{ K}$ ;  $g(t) = 0.0$  if  $t < 20 \text{ s}$ ;  $g(t) = 628.7603 \text{ W/m}^3$ . The Biot number  $Bi (=h_{ef}r_1/k)$  varies between 0.09 and 0.29 due to the variation of the heat conductivity with the temperature.

Table 1 shows the relative errors of the average, center and surface temperatures calculated by the classical and improved lumped models when compared with the reference numerical

Table 1  
Relative errors of the classical and improved lumped models

Time (s)	Temperature	CLSA	$H_{1,1}-H_{1,1}$
4	$T_{av}$	1.9	0.3
	$T_c$		0.7
	$T_w$		0.5
50	$T_{av}$	1.5	0.0
	$T_c$		0.0
	$T_w$		0.0
100	$T_{av}$	1.1	0.0
	$T_c$		0.1
	$T_w$		0.0
300	$T_{av}$	0.8	0.0
	$T_c$		0.0
	$T_w$		0.0

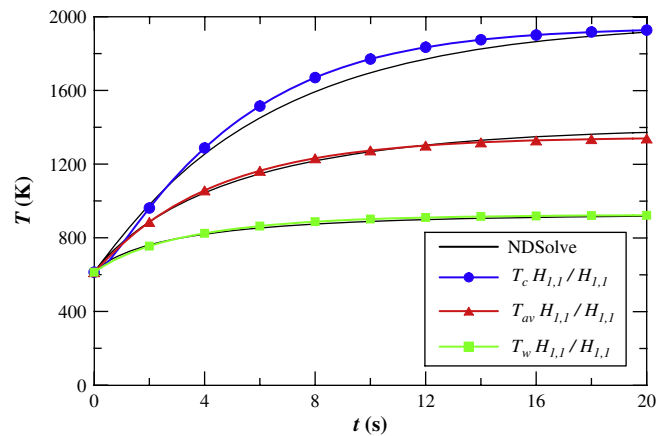


Fig. 3. Comparison of the improved lumped models and numerical solution of the distributed parameter model for constant heat generation rate.

solution obtained by NDSolve, for the times of 4, 50, 100, and 300 s. It can be seen that all calculated relative errors are less than 2%, showing the good performance of both lumped models for Biot numbers between 0.09 and 0.29.

#### 3.2. Example 2 ( $3 < Bi < 10$ )

The second numerical example also utilizes values for the fuel material properties according to Eqs. (8)–(10). A representative heat transfer coefficient for a PWR fuel rod is used:  $h_{ef} = 4540.0 \text{ W/m}^2 \text{ K}$ , which corresponds to Biot numbers between 3 and 10, due to the variation of the heat conductivity with the temperature.

Again, the numerical results obtained by the improved lumped system  $H_{1,1}/H_{1,1}$  are compared with the results obtained by a numerical solution of the original distributed parameter formulation, using NDSolve, as shown in Figs. 3 and 4. Fig. 3 shows the temperature evolution predicted by the improved lumped model  $H_{1,1}/H_{1,1}$  for the first 20 s of the transient compared with numerical solution by NDSolve. Fig. 4 shows typical results of a slow transient in which the

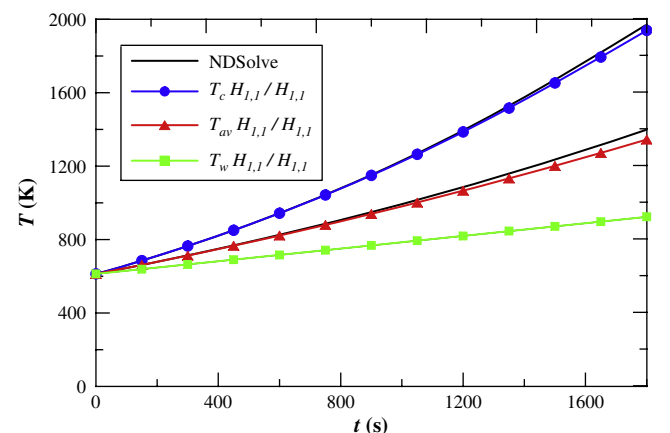


Fig. 4. Comparison of the improved lumped models and numerical solution of the distributed parameter model for a slow transient of linear power rise.

Table 2  
The event sequence of a cold leg SBLOCA

Time (s)	Main events
20	Start of leakage
~ 32	Reactor shutdown
~ 595	Cladding dryout

reactor power rises linearly during the first 30 min. It can be seen that the solution of the improved lumped model agrees very well with the numerical solution.

### 3.3. Example 3: SBLOCA in a cold leg of NPP Angra 2

In the following example, we simulated a Small Break Loss of Coolant Accident (SBLOCA) in a cold leg of the Brazilian Nuclear Power Plant Angra 2, based on simulation data obtained by Madeira et al. (2007) using RELAP5/MOD3. The sequence of events until the occurrence of dryout is given in Table 2. Beyond this point, the present model is no longer applicable as the fuel rod cladding must be modeled separately.

Fig. 5 shows the curve of the normalized reactor power during 800 s and Fig. 6 shows the average coolant temperature in the reactor core during the same period. The heat generation rate in the fuel rod was first adjusted to obtain the core power output level of the RELAP simulation. Second, a period of 20 s was used for the model to reach the steady state from which the sequence of events starts. As shown in Figs. 5 and 6, the leakage starts at 20 s with the coolant temperature rises until 32 s when the reactor is shutdown. From this instant forward the reactor power begins to drop, as well as the coolant and fuel temperatures. It can be seen that few minutes after the reactor shutdown the residual power is already less than 1% of the full power level. Correspondingly, the coolant temperature drops very quickly. Fig. 7 shows the results of the temperature at the center of the fuel rod compared with the simulation results by RELAP5 which uses a finite difference method. The same correlations of thermo-physical properties have been used in both simulations. An excellent agreement between

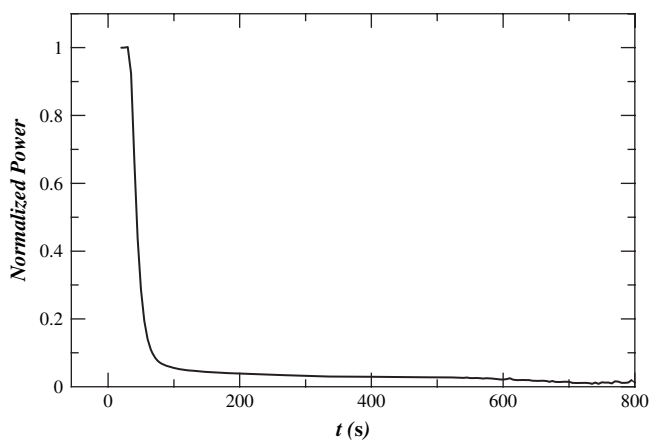


Fig. 5. Normalized reactor power of NPP Angra 2 during a cold leg SBLOCA given by RELAP5 simulation.

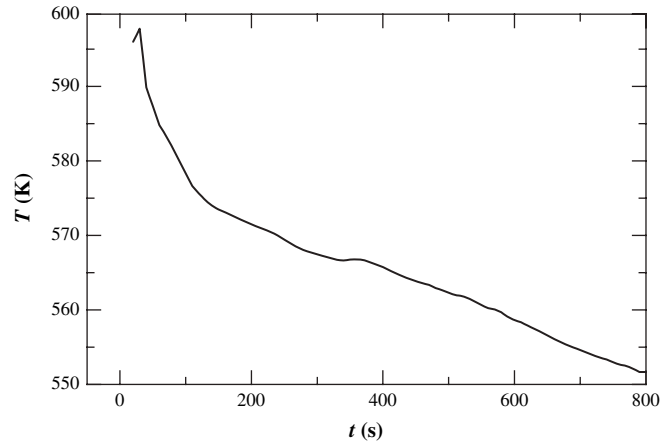


Fig. 6. Average coolant temperature in the reactor core of NPP Angra 2 during a cold leg SBLOCA given by RELAP5 simulation.

the solutions by improved lumped model and the RELAP5 simulation can be observed until the occurrence of dryout after which the RELAP5 simulation predicts a rapid rise of the fuel temperature.

## 4. Conclusions

It was shown in this paper that the improved lumped parameter formulation could be an interesting alternative approximate methodology to simulate the temperature transients in a nuclear fuel rod. It was useful to analyze the behavior of the improved lumped approach in predicting nuclear fuel rod temperature transient, especially with respect to the fuel centerline temperature, which is an important criterion for nuclear reactors design. By using the simplified model with density and specific heat as a function of the average temperature the results provided by the improved lumped model were quite good to UO<sub>2</sub> ceramic material. The possibility of applying this approach to a situation with temperature-dependent thermal properties, still with reasonably accurate results, confirms that the improved lumped parameter formulation might be

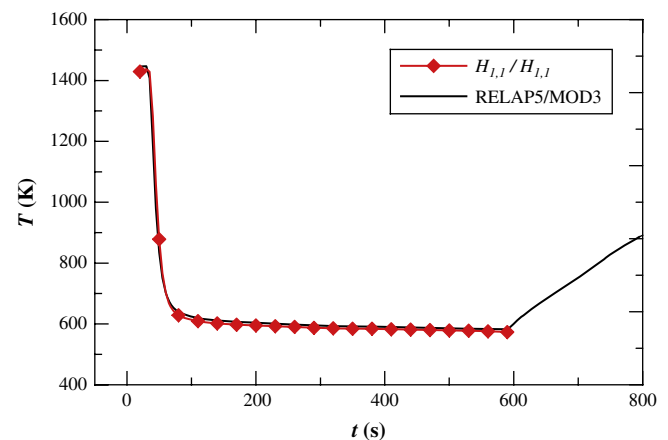


Fig. 7. Comparison of the temperature in the fuel rod center during a cold leg SBLOCA given by ILSA  $H_{1,1}$ – $H_{1,1}$  model and RELAP5 simulation.

very useful for design, safety analysis and plant simulators of nuclear reactors.

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