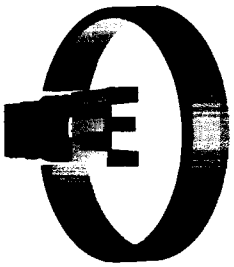

Relatório Técnico

Núcleo de Computação Eletrônica

Analyzing Problem Solving Strategies Under a Cognitive Viewpoint

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NCE -09/02

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Abstract

In this paper we investigate the human reasoning applied to the mathematical problem-resolution process. Our approach is based on two main settings: a. the investigation of mental processes involved in the human reasoning applied to problem resolution; b. the analysis of differences in the categorization and resolution of mathematical problems by beginners and specialists.

In a. we sought to contribute for the rupture of the logical-formal reasoning paradigm. In fact, we sought to contribute for the rupture of the idea that identifies the human as a completely rational entity, which invokes a thinking way that adheres the rules of an explicit form. Our results show that the human reasoning is not determined exclusively by logical-formal guidelines, but is rather determined by characteristics and aspects of the context.

In b. we sought to analyze more effectively the problem-resolution process. We concentrated our discussion on the differences in the categorization and resolution of mathematical problems by beginners and specialists. Our results indicate for a problem categorization and subsequent resolution: specialists are guided by organized logical principles, and beginners are guided by superficial elements found in its enunciation.

Keywords

human reasoning; mathematical problem resolution; cognitive science.

1. Human reasoning and the logical-formal guidelines

It was believed, during a long time, that the human reasoning was used exclusively of logical-formal guidelines. Therefore, the teaching of procedures and effective strategies in situations *prototypical* would be enough to guarantee the application of such procedures and strategies to any situations, or problems, that presented the same formal structure.

However, as indicates Gardner (1996), work about the reasoning have been questioning this point of view seriously In other words, everything indicates that the parameters of the formal logic don't serves the exclusively model for the human reasoning. In the case of the mathematics, is evident that the knowledge of procedural character, are necessary for the resolution of problems, because is through them that we took science of the tools that we dispose, however, empiric evidences have been revealing that the knowledge of such tools doesn't guarantee the success in the solution of problems of the different contexts. In this section we commented on our first empiric research where we looked for to identify the cases or situations in that the people despise the formal logic in the process of solution of problems of mathematical nature.

1.1 Methodology

This research is based in the application to people common of daily mathematical problems. We used mathematical problems to allow us to analyze how the reasoning of the subjects would base on the logical-formal parameters. We created, four groups of problems and presented for a total of 174 common people (without specialized mathematical formation) distributed in groups in a random way. In this article, from necessity of synthesis, we commented, as example, just the second group of problems for considering it fundamental in terms of empiric proof of our work hypothesis.

1.1.1. Second group of problems

In the second group, we created problems where same discounts (in money) are offered in different qualitative contexts, our objective was to discover if the different contexts *interfere* in the choices of the subjects. For example, a discount of US$ 15 can have great importance in a purchase of US$ 30; however, it can be *irrelevant* when the cost of the one that we bought is US$ 300. In this group, we created four tests and we applied each one to 18 subjects (picture 1.).

Version 1– discount of US$ 20 and of 50%

Imagine that you are at a store in the intention of acquiring a blouse. The blouse cost US$ 40. The salesperson informs that, if you is willing to wait the remittance next month, exists the possibility to acquire, the same blouse, for US$ 20. Would you be willing to wait? Justify your answer.

Version 2– discount of US$ 20 and of 1,11%

Imagine that you are at a store in the intention of acquiring a computer. The computer cost US$ 1800. The salesperson informs that, if you is willing to wait the remittance of next month, exists the possibility to acquire, the same computer, for US$ 1780. Would you be willing to wait? Justify your answer.

Picture 1

With regard to the methodological procedures, the subjects were guided to solve the problems "thinking aloud". In other words, they were well instructed so that, during the resolution of the problems, they indicated us all of the steps of their reasoning. This procedure should indicate us the underlying mental process for the use of their strategies.

1.2 work hypothesis

The pragmatic factors and the familiarity of the subject with the contents certain problems influence in the resolution of the same in way to evidence reasoning forms to permeate the formal logical reasoning.

1.3 results

1.3.1. Second group of problems

In this group of problems the subjects were guided to answer:
Yes – If disposed to look for the discount. **No** – If no disposed. **Depends** – Se they opted for conditional answers.

Results of the version 1			
Tests	**Yes**	**No**	**depends**
$\sum x$	57	8	7
$\sum x^2$	835	18	25
$\overline{X}$	14,25	2	1,75

Table 1

Results of the version 2			
	Yes	**No**	**depends**
?x	26	39	7
$\sum x^2$	194	18	17
$\overline{X}$	6,5	9,75	1,75

Table 2

We submitted these results to the test of Width of Ducan to analyze significant internal differences are existed separately among the three choice possibilities in each version.

Problems of the version 1		
Table of the test of Ducan (significance level = 1%)		
Number of the groups	2	3
Rp values (df = 69)	3,76	3,92
RP values	3,83	3,99
difference among the averages		
	Minimum value	**Found value**
Yes / depends	3,99	12,5
Yes / not	3,83	12,25
No / depends	3,83	0,25

Table 3

The analysis of the results of the problems of the version 1, through the test of Width of Ducan, reveals, as display the table 3 that there is a significant difference for 1% between the option yes and the other options. In short, in the problems of the version 1, the option yes it prevails from a significant way to 1%.

problems of the version 2		
Table of the test of Ducan (significance level = 1%)		
Number of the groups	2	3
Rp values (df = 69)	3,76	3,92
RP values	4,17	4,35
difference among the averages		
	Minimum value	**Found value**
no / it depends	4,35	8
no / yes	4,17	4,75
yes / it depends	4,17	3,25

Table 4

Already the analysis of the results of the problems of the version 2, through the test of Width of Ducan, reveals that there is a significant difference for 1% among the option no and the other options. In short, in the problems of the version 2, it is the option no that it prevails from a significant way to 1%.

1.4 Discussions of the results

As we imagined, in the problems of the version 1 (Picture 1), the option *yes* it prevailed in a significant way. While in the problems of the version 2 (Picture 1) the option ***no*** that it prevails in a significant way.

In our evaluation these results challenge seriously the idea that the human reasoning follows logical-formal guidelines, because, in objective terms, the problems of the version 1 and the problems of the version 2 are identical and, therefore, they should generate similar solutions on the part of the subjects. However, our results demonstrate opposite reactions. In the version 1, the predominant choice is to look for the discount, while in the version 2, the predominant choice is to despise the discount.

The choices presented by the subjects involve, probably, an analysis of the context of the situation. In other words, the subjects didn't decide following objective criterions. In a general way, discounts of the same value in money presented different weights in the decisions of the subjects. The weight that the discount acted in the decisions of the subjects was certain, mainly, for the relationships between the value of the discount and the cost of the product. This relationship was revealed fundamental to determine "it was worthwhile" or no the efforts proposed in exchange for the obtaining of the discounts.

2. Problem solving categorizations

2.1 Introduction

In this section, we commented on our second researches empiric that approached the differences, in the categorization and resolution of mathematical problems, between beginners and specialists. In general lines, this research had the intention of verifying the relationships between the categorization and specialized resolution of mathematical problems and the acquisition and use of the logical-formal principles.

2.2 Methodology

2.2.1. Materials

To reach our research objective, we selected 20 problems of medium level divided in five basic categories: Percentage, numeric Groups, Functions, Trigonometry, and Probability. We selected two problem types, which we called problems of direct categorization (DC) and problems of indirect categorization (IC). The problems of direct categorization count with more evident statements than they appear directly for the category, which they refer. Already the problems of indirect categorizing count with statements that don't lead to the category which they refer, but they appear, most of the time, in the direction of other categories of problems. This way, for they are classified appropriately, they demand a mathematical knowledge more deepened and organized.

Below, we presented examples of problems used in the category percentage.

Problem of direct categorization (DC):
When buying an appliance that cost US$ 210 I obtained a reduction of US$ 17,85. Which the percentile corresponding to the discount that I obtained?

Problem of indirect categorization (IC):
In a factory, 64% of the workers are women and the remaining 108 are men. How many workers do have the factory?

Picture 2

The problems were introduced to twenty beginners and six specialists. We called beginners university students of the mathematics. The twenty beginners that participated in our tests are students in mathematics of UENF. We called specialists subject with formation in mathematics, graduate and postgraduates. Of the six specialists that participated in the tests, 3 are doctors and 3 are graduated.

The twenty problems were distributed in two tests: test A and test B. Each test containing two problems of each one of the categories presented above totaling ten problems. The test A counts seven problems of IC and three problems of DC. The test B, to the opposite, counts seven problems of DC and three problems of IC. Therefore, they were used, in the total, ten problems of DC and ten problems of IC.

Each test was introduced to ten beginners and three specialists. The subjects were guided, firstly, to classify the ten problems starting from the reading of the statement without you solve them indeed. After the conclusion of this stage, they were guided to solve the problems.

2.3 Hypotheses

The specialized resolution of mathematical problems requests a great amount of knowledge and it depends on the agreement of enough logical principles. This way, the specialists base the categorization and consequent resolution of mathematical problems on the mathematical logical principles fundamental while the beginners, besides using superficial strategies, will be more easily influenced by principles that don't follow the logical-formal parameters of the problems in subject.

2.4 Results

2.4.1. Results of the categorization

As we commented on in the section methodology, problems of DC and IC were created. We compared the beginners' acting in the categorization of these two problem types. Soon afterwards, we make the same with the specialists.

To evaluate the acting of the subjects, we divided the problems classified correctly of the total of problems, in each category (DC and IC) and we multiplied for ten.[1].

The beginners' acting in the categorization		
Subject	**DC**	**IC**
$\sum x$	169	42
$\sum x^2$	1473	142
$\overline{X}$	8,45	2,09

Table 5

These results were submitted to the "test t" of Student for we are verified the differences found healthy really significant or she should be attributed to casual variations.

Found results	
df*	18
t	12,47
P**	0,01

* degree of freedom
* * probability of a casual result

Table 6

In agreement with the "test t" of Student, these results can be expected less than 1 time in 100 by chance. The beginners' best acting in the categorization of the problems of DC, in relation to the categorization of the problems of IC, is very significant. In practical terms, we can say that this result should not be considered, in any hypothesis, a casual result.

The specialists' acting in the categorization		
Subject	**DC**	**IC**
$\sum x$	53,7	42,3
$\sum x^2$	489	369
$\overline{X}$	8,95	7,05

Table 7

These resulted also were submitted to the "test t" of Student for we have verified the differences found healthy really significant or she should be attributed to casual variations.

Found results	
df	4
t	0,532
P	0,6

Table 8

1This multiplier went utility just to facilitate the calculations and it didn't have any influence in the results.

In agreement with the "test t" of Student, these results can be expected 60 times in 100 by chance. This means that the specialists' best acting in the categorization of the problems of DC, in relation to categorization of the problems of IC, is not significant. In practical terms, we can say that this was a casual result.

2.4.2 Discussion of the results

Regarding the categorization, the obtained results are in agreement with our initial suppositions. In the beginners' case, it happened a very significant difference among the acting in the categorization of the problems of DC and the acting in the categorization of the problems of IC. In the specialists' case, the difference also exists, but it is not significant. Comparing the beginners' results and of the specialists, we noticed an advantage a little significant, in favor of the specialists, in the problems of DC. However, in the problems of IC, this advantage is very significant.

In our opinion, this happens because, in the problems of DC, the categories which belong the problems are very evident in the terms used in the statement, already, in the problems of IC, the beginners don't get to perceive, starting from the statement, the internal structure of the problem. For instance, when we used the term "percentile" and percentile values in problems of the category numeric groups they have a strong inclination for classify them as percentage problems. The specialists, for their time, get to perceive the principles that are behind the terms used in the statement. In short, the obtained results appear for a categorization guided by formal logical-mathematical principles very organized in the specialists' case and a categorization guided by superficial elements found in the statement in the beginners' case.

2.4.3. Results of the resolution

In this section, we presented the resolution of a problems of indirect categorization (IC) made by a beginner and a specialist. Before the resolution, we presented the categorization accomplished initially by the beginner and for the specialist. We selected this problem for believing that the same evidences the importance of a correct categorization clearly for the remaining of the resolution process.

A sixth of the life of John was your childhood. And a fourth took until that if it married. One year later Steve, your only and dear son was born. When reaching half of your father's life, Steve took him to live with itself. It is lived like this John their last 5 years. With that age John died?

BEGINNER 11:		*SPECIALIST 6:*	
Categorization:	**Resolution:**	**Categorization:**	**Resolução:**
Fracion problem	$\frac{1}{6} + \frac{1}{4} + 1 + \frac{1}{2} + 5 = \frac{11}{12} + 6$	*Equation*	*X = John'age to the died.* $\frac{X}{6} + \frac{X}{4} + 1 + \frac{X}{2} + 5 = X$ *X = 72 years old.*

Picture 3

In what concerns the resolution of the problems, as we had foreseen, the most exacts categorization of the specialist's favors the access, in the memory, to the knowledge and necessary information for resolution of the problems. On the other hand, the beginners, taken by the employed terms in the statements, classify in a mistaken way the problems and they access, inadequate knowledge to make your resolution.

3. Final considerations

With regard to the first group of experiments, our results appear for the breaking of the paradigm of the formal logical reasoning. We perceived clearly, in the obtained results, the intervention, decisive, of aspects no inherent to the formal logic, as representative and context. In short, in what concerns the results of the first group of experiments, everything takes to believe that the human reasoning doesn't base exclusively on logical-formal criteria. In other words, the human reasoning cannot totally be understood through formal principles.

Although the human reasoning is, in certain circumstances, in agreement with the principles of the formal logical, seems us a reduction to interpret all your complexity with base, just, in formal logical principles.

In the second group of experiments, for your time, clear differences were evidenced between beginners and specialists in mathematics. In our evaluation, this difference exist because in the problems of IC, the beginners don't get to see the internal structure of the problem, in other words, they don't get to see what is besides the words of the statement, and, this way they don't get to accomplish an appropriate translation[2] of the exposed problem.

As they don't get to identify in an exact way that type of the problem is solving, the beginners have great difficulties to access, in the memory, the knowledge mathematical necessary for resolution. The specialists, for their time, sustained, maybe, by a larger experience, accomplish a good translation of the problem and access with larger precision the pertinent knowledge.
In short, the obtained results appear for a categorization guided by principles in the specialists' case and a categorization guided by superficial elements found in the statement in the beginners' case.

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2 translation here has the sense of transforming the information described in the statement in mathematical expressions that make possible the resolution of the problem.

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