Relatório Técnico

A Cognitive Approach to Mental Representation in Mathematical Problem Solving

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Abstract

In this paper we investigate mental-representation strategies applied to solve contextual problems that involve mathematical calculus. We are interested in formal procedures, algorithms, and strategies that could be used by three groups of peoples: with specific mathematical knowledge (*the expert group*); without this knowledge (*the control group*); and without this specific knowledge but acquainted with the designed problem context (*the familiar group*).

Our investigation demonstrates that the reasoning developments of these three groups are quite different from one to another: in the expert group the reasoning is only based on algorithmic operations; in the control group the reasoning combines algorithm with other mathematical strategies; and the familiar group uses a more intuitive reasoning, probably influenced by their familiarity with the problem context and by their special mental strategies.

In this research we aim to come through with Cognitive Science studies on the nature of human knowledge. Mathematical problem resolutions in general and arithmetic calculus in particular are tools very interesting to recognize the difference between expert and beginner knowledge and reasoning.

Keywords
Mental representation; mathematical reasoning; cognitive science.

Introduction

We are interested in educational approaches involving problem solving and epistemological strategies. Our actual researches concern cognitive aspects on mental representations occurring in mathematical operations. In this paper we describe some theoretical paradigms supporting the experimental tools we have applied to our last experiment concerning mathematical reasoning on arithmetic.

We discuss about cognitive aspects associated with this experiment. This research is based on (Carraher, Schliemann and Carraher, 1995) and on some results obtained by (Tversky and Kahneman, 1983). In this paper we also take in account the *abstract rules theories* (Braine, 1978) and the *modal theories* (Eysenck and Keane, 1994)

For instance, the *abstract rules theories* apply a set of rules -as in lambda calculus- in order to obtain deductive inferences; the *modal theories* use the syllogistic reasoning. Some researches performed by (Gentner and Stevens, 1983) and (Johnson-Laird, 1983) point out an interesting theoretical direction with regards to problem solving abilities and reasoning. Sternberg's results (Sternberg, 1982) shows the interdependency between *intelligence*, *reasoning* and *problem solving*.

The definition of *intelligence*, for instance, is fundamentally based on the roles assumed by the *reasoning* and *problem solving* involved in human strategies present in global processes of real problem resolution (Mayer, 1983), (Johnson-Laird, 1983, (Anderson, 1978), (Roy d'Andrade 1982), (Tversky and Kahneman, 1983), (Kahneman and Tversky, 1982).

The Experiment

Our experiment has as central goal to know about all conditions that could facilitate the mathematical reasoning, specially the arithmetic one. In our recent researches we have tried to demonstrate how humans without a mathematical formation and training can solve mathematical problems correctly. Our central goal is to verify if the strategies applied by these peoples are very different from those applied by peoples having a more specific formation, which generally apply formal procedures properly.

Our hypotheses are:

- Peoples with a specific formal training in mathematics – *specialist group*, which could use only formal procedures in order to solve any mathematical problem;
- Peoples without a specific formal training in mathematics – *control group*, which could not use formal procedures in order to solve a mathematical problem;
- Peoples with a specific formal education in mathematics – *familiar group*, which could have some familiarity with inductive reasoning from data of the problem to be solved;
- If all peoples involved in the experiment sometimes use formal procedures and sometimes use others strategies, then we could affirm that formal procedures are not the dominant in the mental representation of a problem. This means that there are additional strategies influencing the solving approach.

Methods

We have used eight problems whose respective contexts are inserted in real situations. Solving these problems require cognitive strategies based on arithmetical calculus. We grouped the peoples in three sets, each one with ten peoples. The proposed problems have been solved using pencil and paper associated with a synchronous oral explanation about the strategy being used in all steps of the resolution process (*thinking aloud strategy*).

The issues from each people and the ones from each group were comparatively analyzed. The immediate consequence of this method is the possibility of to obtain a profile of the mental representations that are frequently elaborated by peoples solving mathematical problems. We have analyzed the oral and the written protocols in a synchronous way. We have seen that in formal reasoning there is a predominance of calculus based on arithmetic operations or then based on concepts concerned to specific mathematical contexts. In the alternative reasoning predominates the use of arithmetical operations associated with others strategies –denoted *auxiliaries*– such as those proportioned by rules that are frequently applied to inductive reasoning.

These strategies can call 'images' from several solutions based on intellectual intuitions without the required "cognitive way" (such as formulas, rules, mathematical concepts etc.) previously established.

Results

By using the *Test(t) Student Method* we have analyzed the correct results and the wrong ones. It was confirmed a difference from more 1% of correct results than wrong results. This fact means that the cognitive strategies used in the associated problem resolutions, although been diversified, were quite well applied to. The comparative analysis of the three groups and the three kinds of reasoning by *Ducan's Amplitude Test* has confirmed that the formal reasoning is predominant (1%) in the specialist group over both the control group and the familiarity group (such as we have supposed in our hypotheses).

There is predominance of the alternative reasoning (1%) in the familiarity group over both control group and the specialist group (obeying our hypotheses). It was also found a significant predominance (1%) of the inductive reasoning in the familiarity group with regards to both the control group and the specialist group (obeying our hypotheses).

Conclusions and Perspectives

The global analysis of the obtained results has confirmed that correct results are more significant than the wrong ones. This means that the cognition strategies used in a problem solving process are generally well utilized. The results concerning the kinds of reasoning have confirmed that it is significantly superior the predominance of formal reasoning in both the specialists group and the familiarity group.

This last result points out to the need of future researches. The development of formal reasoning in real application is done in a natural way, but the scholar level also influences it. We could admit too that the scholar level influences in several ways, but under some contexts the familiarity with certain tasks could produce more efficiency. It is important to note that university's students that used predominately the alternative reasoning, applying general strategies, composed the control group.

We could assume that the results obtained in our research are the basis to future researches we are actually developing. In particular we are interested in peculiar characteristics of the familiarity group behavior. We are especially interested in the investigation about mental representation proprieties, which are more predominant in this group.

We want also investigate about mental representation involving the main kinds of mathematical skills. An important issue of this research could be to know how the humans could participate of a more general mathematical learning process.

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