

# Sliding Mode Control with Gain-Scheduled for Magnetic Levitation System

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## ABSTRACT

This paper presents a study of controlling magnetic levitation object using conventional sliding mode control (CSMC) and sliding mode control (SMC) with gain-scheduled. SMC with gain-scheduled aims to improve the robustness of the control system from disturbance. The CSMC simulation results show that output can follow the set point if there is no interference, but if the disturbance happens then there is overshoot and undershoot of 0.034 mm and 0.07 mm for disturbance  $10 * \sin(t)$  and  $20 * \sin(t)$ . Then SMC with gain-scheduled shows excellent performance because the output can follow the reference even if it got disturbance of  $10 * \sin(t)$  and  $20 * \sin(t)$ .

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## 1. INTRODUCTION

Magnetic levitation or maglev system is a technique to make objects can float in the air using magnetic fields. The force produced by the magnetic field counteracts the gravitational force in order to the object does not fall. The Maglev system has advantages such as no friction with other objects so that the efficiency of movement becomes faster and requires low energy consumption. For example, the maglev train system can run at a top speed of 603 kilometers per hour [1]. However, the maglev system has high nonlinear dynamics, where nonlinear systems are more challenging to control than linear systems. The research conducted was controlling an object that can float a stable at a certain height. Illustration of the maglev system shows in Fig. 1. The system requires an electromagnetic coil, around iron object, a sensor (phototransistor and LED to detect the position of an object), a driver (to adjust the current and voltage needed by an electromagnetic coil), a microcontroller (as a controller). Several studies have used various methods to control object of a maglev system, including controlling maglev objects using sliding mode control [2], but the maglev equation is changed from nonlinear to linear so that if implemented on the plant the controller will not be optimal. Then, control of a magnetic levitation system using PD (Proportional Derivative) and PID (Proportional Integral Derivative) controller [3]. The PID control system demonstrates better performance in steady-state error and settling time rather than PD control system. However, this paper did not consider disturbance in magnetic levitation. In addition, there are also other methods such as fuzzy logic controller [4], feedback linearization [5], LQR [6], neural network [7][8][9]. Fuzzy logic controller for magnetic levitation system shows better performance than PID controller when adding mass as disturbance [4]. Then, feedback linearized magnetic levitation system using Sum of Squares (SOS) method explain about the stability of maglev, based on calculations using Lyapunov obtained negative definite or the system was globally asymptotically stable [5]. However, this study did not explain the output response in graphical form. Furthermore, LQR controller for a nonlinear maglev system, that shows oscillation response in the beginning and stable at 8 seconds [6]. The paper with title "neural network adaptive state feedback control of a magnetic levitation system", proposes combine neural network adaptive control and state feedback control based on RBFNN. In simulation show that

adaptive state feedback controller based on RBF has better stability than conventional PID [7]. However, it is known that neural networks need learning that can affect the controller system to become slower.

The nonlinear maglev system is better controlled by nonlinear control than linear control. Example of linear control is PI, PID, and LQR. Research carried out using nonlinear control, namely sliding mode control. Sliding control mode consists of equivalent control and switching control [10]. The difference in controllers used with the others lies in the addition of gain-scheduled on the switching control. The function of gain-scheduled is to set the gain value automatically on the switching control, where a high gain can increase the robustness. The idea arises to overcome the disturbance, if there is a disturbance, the gain in switching control will change to high, and the switching frequency will increase so that the system can be more robust.

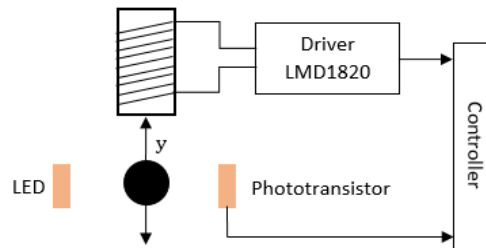


Fig. 1. Model of maglev object

## 2. RESEARCH METHOD

### 2.1 Mathematical model of magnetic levitation system

Mathematical model of a maglev object has been obtained using the Lagrangian method. The Lagrange method uses the principle of potential energy and kinetic energy [11]. The symbol of Lagrange is  $L$ , then potential energy is  $T$  and kinetic energy is  $V$ .

$$L = T - V. \quad (1)$$

In the first step to getting the Lagrange equation from the maglev system is to determine the general coordinates of the system. The coordinates are  $y_1 = y$ ,  $\dot{y}_1 = \dot{y}$ ,  $\dot{y}_2 = i$ , where  $y$  is an object position from the electromagnetic coil in a vertical axis;  $\dot{y}$  is the velocity of the object;  $i$  is current on the electromagnetic coil. Based on Fig. 1. kinetic energy equations and potential energy are found in (2).

$$\left. \begin{aligned} T &= \frac{1}{2}L(y)i^2 + \frac{1}{2}m\dot{y}^2, \\ V &= -mgy, \end{aligned} \right\} \quad (2)$$

where  $L(y) = L_i + \frac{L_0 y_0}{y}$ ,  $L(y)$  is coil inductivity,  $m$  is mass of an object, and  $g$  is gravity.  $L_0 y_0$  equal with  $2k$ , where  $k$  is a magnetic force. Finally, the Lagrangian formula as follows:

$$L = \frac{1}{2}\left(L_i + \frac{2k}{y}\right)i^2 + \frac{1}{2}m\dot{y}^2 + mgy. \quad (3)$$

The Lagrangian equation defined as [12]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_i}\right) - \frac{\partial L}{\partial y_i} = 0, \quad 1 \leq i \leq n. \quad (4)$$

Based on (4), we can analyze the Lagrangian equation on a maglev system

$$\left. \begin{aligned} \frac{\partial L}{\partial y_1} &= \frac{\partial L}{\partial y} = m\dot{y} \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) &= m\ddot{y} \\ \frac{\partial L}{\partial y_1} &= \frac{\partial L}{\partial y} = mg - k\frac{i^2}{y^2} \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) - \frac{\partial L}{\partial y_1} &= f \\ \ddot{y} &= \frac{f}{m} + g - k\frac{i^2}{my^2} \end{aligned} \right\} \quad (5)$$

and

$$\left. \begin{aligned} \left(\frac{\partial L}{\partial y_2}\right) &= \frac{\partial L}{\partial i} = L(y)i \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) &= -i\frac{2k}{y^2}\dot{y} + L\frac{di}{dt} \\ \left(\frac{\partial L}{\partial y_2}\right) &= 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) - \left(\frac{\partial L}{\partial y_2}\right) &= u - iR \\ i &= \frac{u}{L} - \frac{iR}{L} + i\frac{2k}{Ly^2}\dot{y}. \end{aligned} \right\} \quad (6)$$

State-space of the maglev system chooses  $y_1 = y, y_2 = \dot{y}, y_3 = i, u = e$ ; state-space showed on (7)

$$\left. \begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= g - \frac{ky_3^2}{my_1^2}, \\ \dot{y}_3 &= \frac{2ky_2y_3}{Ly_1^2} + \frac{u}{L} - \frac{Ry_3}{L}. \end{aligned} \right\} \quad (7)$$

We need nonlinear change coordinates, which presented in (8).

$$\left. \begin{aligned} c_1 &= y_1, \\ c_2 &= y_2, \\ c_3 &= g - \frac{ky_3^2}{my_1^2}. \end{aligned} \right\} \quad (8)$$

Then we get state space of the maglev system on new coordinates

$$\left. \begin{aligned} \dot{c}_1 &= c_2, \\ \dot{c}_2 &= c_3, \\ \dot{c}_3 &= -\frac{2k}{m}\left(\frac{y_3\dot{y}_3}{y_1^2}\right) + \frac{2k}{m}\left(\frac{y_3^2\dot{y}_1}{y_1^3}\right). \end{aligned} \right\} \quad (9)$$

Substituting  $\dot{y}_3$  in (7) into  $\dot{c}_3$  in (9)

$$\dot{c}_3 = n(y)u + m(y), \quad (10)$$

where

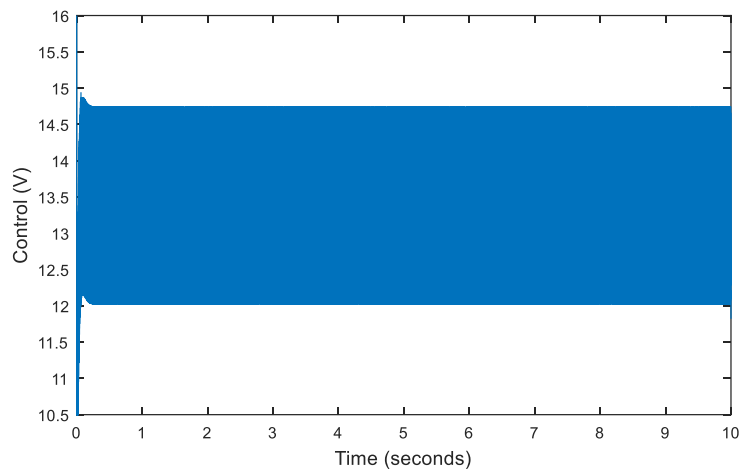
$$\left. \begin{aligned} m(y) &= -\frac{4k^2y_2y_3^2}{mLy_1^4} + \frac{2kRy_3^2}{mLy_1^2} + \frac{2ky_3^2y_2}{my_1^3}, \\ n(y) &= -\frac{2ky_3}{my_1^2L}. \end{aligned} \right\} \quad (11)$$

## 2.2 Conventional sliding mode control

Conventional sliding mode control (CSMC) has advantages such as robustness against small disturbance and uncertainty. However, CSMC also has weaknesses in switching controls, which can lead to chattering phenomena. Chattering phenomena is a high-frequency movement that makes the state trajectories quickly oscillate around the sliding surface [13]. Chattering phenomena shows in Fig. 2. Where produces high-frequency control and high amplitude. Chattering phenomena is minimized by changing the sign to sat [14].

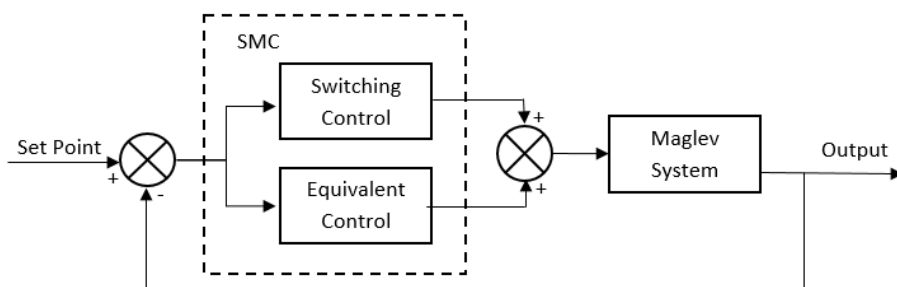
$$\text{sign}(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0, \end{cases} \quad (12)$$

$$\text{sat}\left(\frac{s}{\varphi}\right) = \begin{cases} 1, & \frac{s}{\varphi} > 1, \\ \frac{s}{\varphi}, & \left|\frac{s}{\varphi}\right| \leq 1, \\ -1, & s < -1. \end{cases} \quad (13)$$



**Fig. 2.** High chattering phenomena

Designing a conventional sliding mode control requires several stages [15][16]. Stage one, designing a sliding surface; stage two, designing the switching control which serves to find the sliding surface; stage three, designing equivalent control functions to maintain control to always move on the sliding surface. Equivalent control obtained by a derivative of the equation of a sliding surface. System of CSMC shows in Fig. 3. In the CSMC controller there are equivalent control and switching control. The output is used as feedback to the input to determine the error that occurred.



**Fig. 3.** CSMC

The first stage, determine the sliding surface of the maglev system

$$s = \dot{c}_1 + \gamma \cdot error \dot{c}_1 + \delta \cdot error c_1. \tag{14}$$

The substituting (8) into (14)

$$s = g - \frac{ky_3^2}{my_1^2} + \gamma(y_2 - y_{2d}) + \delta(y_1 - y_{1d}) \tag{15}$$

The second stage, design of switching control

$$u_s = -G \text{ sat } (s), \tag{16}$$

where  $G$  is gain.

The third stage is designed of equivalent control by differentiating of (4)

$$\left. \begin{aligned} \dot{s} &= \ddot{c}_1 + \gamma \cdot error \ddot{c}_1 + \delta \cdot error \dot{c}_1, \\ \dot{s} &= n(y)u_{equi} + m(y) + \gamma \left( g - \frac{ky_3^2}{my_1^2} \right) + \delta(y_2 - y_{2d}). \end{aligned} \right\} \tag{17}$$

The equivalent control show in (18).

$$u_{equi} = \frac{1}{n(y)} \left( m(y) - \gamma \left( g - \frac{ky_3^2}{my_1^2} \right) - \delta(y_2 - y_{2d}) \right). \quad (18)$$

Finally, we get complete CSMC ( $u_s + u_{equi}$ )

$$u_{CSMC} = -G \text{ sat}(s) + \frac{1}{n(y)} \left( m(y) - \gamma \left( g - \frac{ky_3^2}{my_1^2} \right) - \delta(y_2 - y_{2d}) \right). \quad (19)$$

### 2.3 Sliding Mode Control with Gain-Scheduled

Sliding Mode Control (SMC) with gain-scheduled is CSMC that is given gain-scheduled in the switching control. So that the equivalent control equation in SMC with gain-scheduled is the same as CSMC. The difference is that the switching control gain in SMC with gain-scheduled can automatically change according to the disturbance given to the system [17]. System of SMC with gain-scheduled shows in Fig. 4. In this simulation, (20). used as gain-scheduled.

$$Gain = \begin{cases} 2, & dist = 0, \\ 4, & dist > 0 \ \& \ dist < 6, \\ 10, & dist \geq 6 \ \& \ dist < 11, \\ 13, & dist \geq 11 \ \& \ dist < 16, \\ 16, & dist \geq 16. \end{cases} \quad (20)$$

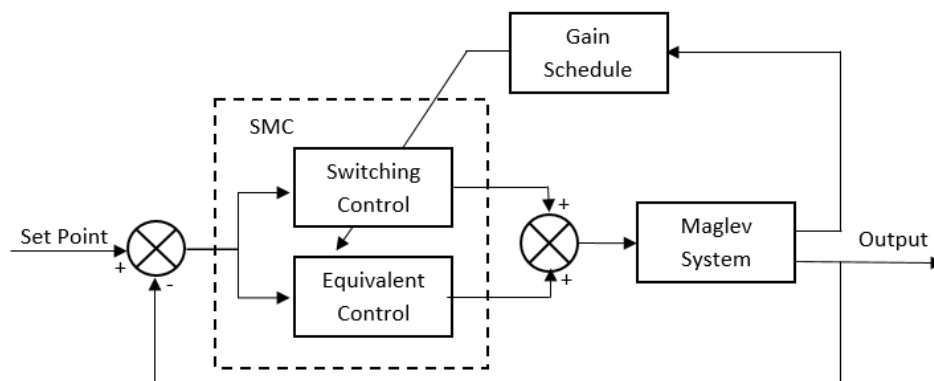


Fig. 4. SMC with Gain-Scheduled

### 3 RESULTS AND DISCUSSION

The results obtained by simulating the maglev system and the SMC controller via Simulink. The simulation uses the following parameters:  $m = 300 \text{ g}$ ,  $g = 9.81 \text{ m/s}^2$ ,  $R = 8 \ \Omega$ , magnetic force constant  $10 \cdot 10^{-5} \text{ Nm}^2/\text{A}^2$ , and  $L = 0.089 \text{ H}$ . The simulations result of CMSC shows in Fig. 5. until Fig. 9. and simulations results of SMC with gain-scheduled that shows in Fig. 10. until Fig. 14. The maglev system that gave a disturbance shows in Fig. 5. and Fig. 10. we can see that the output can follow the set point ie the object floats at an altitude of 0.01 meters without overshoot and steady-state error. Then, in Fig. 6. response of CSMC has overshoot (0.034 mm) and undershoot (0.034 mm) when adding disturbance  $10 \cdot \sin(t)$ . In Fig. 8. the CSMC add disturbance  $20 \cdot \sin(t)$ , the output showed that have overshoot (0.07 mm) and undershoot (0.07 mm). The control of CSMC shown in Fig. 7. and Fig. 9. have low chattering. The response of SMC with gain-scheduled (Fig. 11. and Fig. 13.) could be following the reference without overshoot, undershoot, and steady-state error although add disturbance  $10 \cdot \sin(t)$  and  $20 \cdot \sin(t)$ . In Fig. 12. and Fig. 14. are shown the controller have chattering with amplitude 0.28 V and 0.21 V respectively. That chattering still tolerated because of its small value. Based on simulation results, controller of SMC with gain-scheduled is more robust than

CSMC controller because SMC with gain-scheduled can overcome the disturbance. In CSMC, the gain value in the switching control does not change even though there is disturbance in the system. Whereas in SMC with gain-scheduled, specifically the gain in switching control can automatically change according to the magnitude of disturbance so that the robustness of the controller can be maintained. The results obtained are following the switching control equation in CSMC and SMC with gain-scheduled. The switching control CSMC in (16) showed that the gain is constant (20). shows the gain value can change according to disturbance. In simulation of CSMC with disturbance  $10 * \sin(t)$  and  $20 * \sin(t)$ , the output has overshoot and undershoot where the higher the disturbance can cause overshoot and undershoot to increase. In SMC with gain-scheduled, showed that the output could follow the reference without overshoot and undershoot because the gain in switching control could automatically update become 10 if adding disturbance  $10 * \sin(t)$  and update to 16 if adding disturbance  $20 * \sin(t)$ . This update is obtained according to (20).

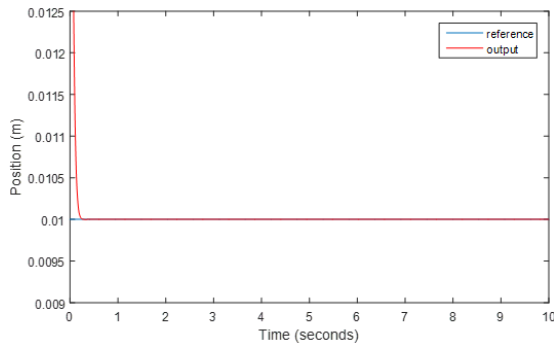


Fig. 5. CSMC response without disturbance

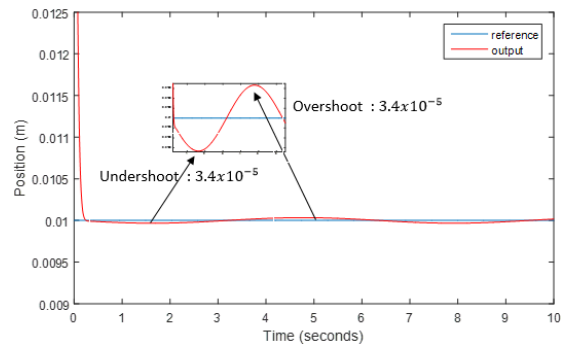


Fig. 6. CSMC response with disturbance  $10 * \sin(t)$

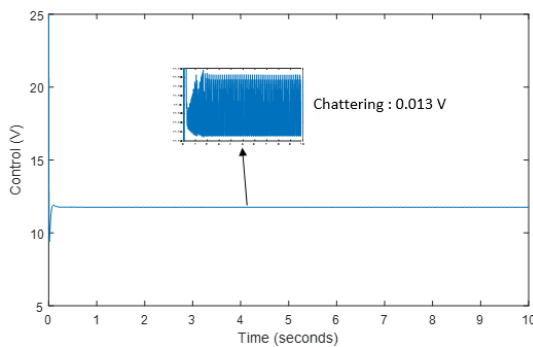


Fig. 7. CSMC with disturbance  $10 * \sin(t)$

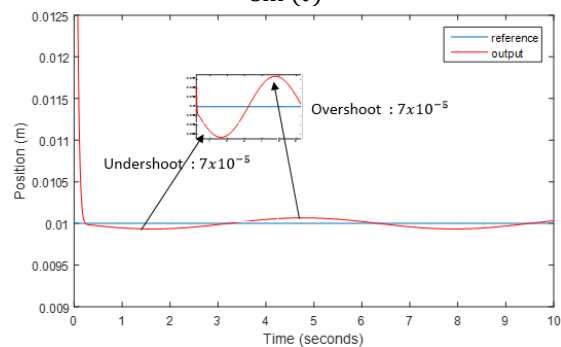


Fig. 8. CSMC response with disturbance  $20 * \sin(t)$

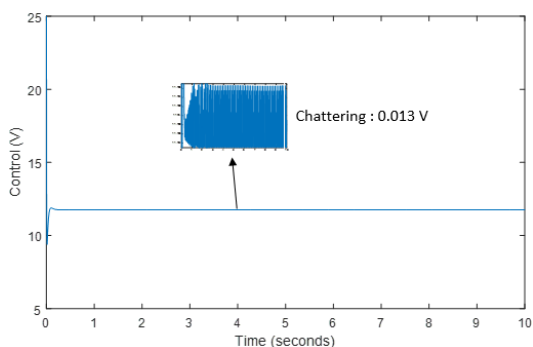


Fig. 9. CSMC with disturbance  $20 * \sin(t)$

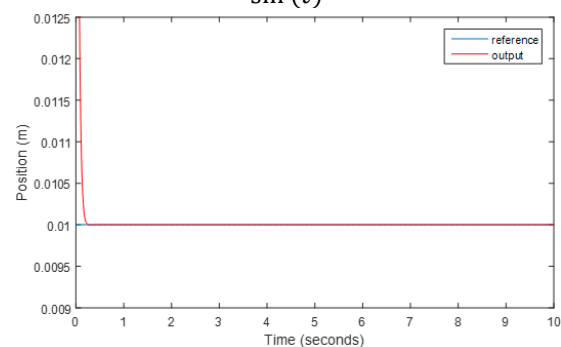
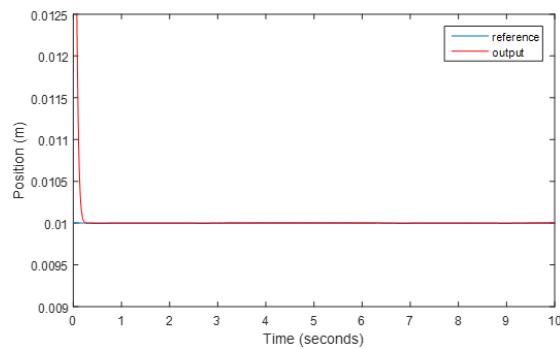
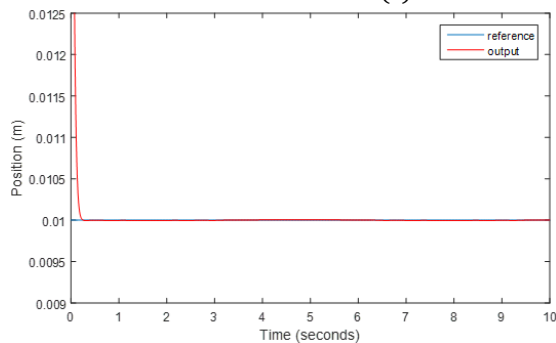


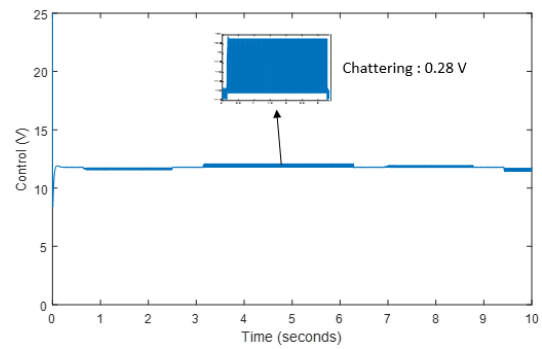
Fig. 10. Gain Scheduling SMC response without disturbance



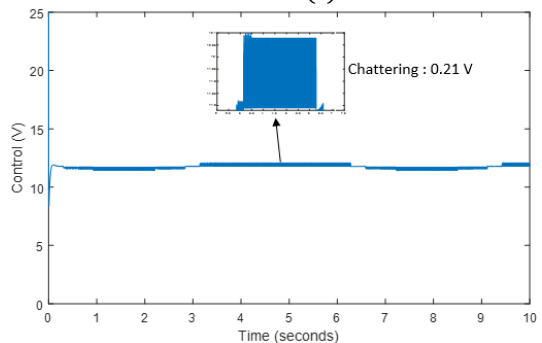
**Fig. 11.** Gain Scheduling SMC response with disturbance  $10 * \sin(t)$



**Fig. 13.** Gain Scheduling SMC response with disturbance  $20 * \sin(t)$



**Fig. 12.** Gain Scheduling SMC with disturbance  $10 * \sin(t)$



**Fig. 14.** Gain Scheduling SMC with disturbance  $20 * \sin(t)$

#### 4 CONCLUSION

SMC with gain-scheduled shows excellent performance, there is no steady-state error, overshoot, and undershoot in the output response. The robustness of SMC with gain-scheduled can be maintained because the gain in switching control can automatically change according to the magnitude of disturbance. Although there is chattering in the SMC with gain-scheduled, the chattering value is minimal, so it does not interfere with the control system. Then, CSMC control shows a performance that is not optimal if the maglev system is given a disturbance. The output response of CSMC has overshoot and undershoot when adding the disturbance. The greater the disturbance added can make the control system unstable because value of gain in switching control is constant so that it cannot maintain the robustness.

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