SPECTRAL RESPONSE SURFACES AND THE RINGING RESPONSE OF OFFSHORE STRUCTURES

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Abstract

A method, called spectral response surface method, is proposed for the study of the ringing phenomenon of offshore structures. Newman's results for diffraction force on a column were reformulated in terms of the frequency components of the ocean surface and their Hilbert transforms. Using a first-order reliability method together with a dynamic model for the structural response, it is straightforward to calculate the ocean surface history most likely to generate an extreme ringing response and the ringing response history.

Keywords: ringing, spectral response surface, first-order reliability method

1. Introduction

Ringing response usually features a rapid build up and slow decay of energy at the resonant frequency of the structure. It may be possible that ocean waves excite ringing in an offshore structure. Indeed, it has been observed in small scale experiments on a vertical column during the passage of steep waves [1, 2]. The natural frequency of the structure and the observed response were relatively high compared to the dominant wave frequencies. However, some frequencies in the exciting force must have coincided with the natural frequency of the structure. This implies a non-linear loading model is required to predict the forces at the sum frequencies of the ocean wave field.

Newman [3] studied the interaction between a body and random waves in the long wavelength regime. He concluded that non-linearity in loading is due more to fluid-structure interaction than to non-linear features in the incident waves. In particular, if one considers only the loads which occur at the sum of the wave frequencies, the incident wave field is described adequately by linear theory for loads up to third order. Newman's expression for horizontal force on a column is consistent with the slender body results of Rainey [4]. We shall use Newman's formulation in this paper.

Ringing becomes important if it influences significantly the extreme response of a structure to a random sea. Therefore, a stochastic analysis is required and we shall use the spectral response surface method. The random sea is represented as a sum of many frequency components, each obeying a normal distribution. In many cases, a structural response can be expressed as a function of these components and their Hilbert transforms. The Hilbert transform is the signal phase shifted by 90 degrees. A first order reliability method (FORM) type of analysis is then applied, treating a surface of constant response level as a limit state [5, 6]. The method has been used in several ocean engineering problems [7, 8].

Consistent with a linear description of the incident waves, we assume that the ocean surface elevation is a sum of many, random, narrow banded components. Each component is normally distributed. All the components are independent and uncorrelated. Thus, the ocean surface at a fixed point is

$$\eta(t) = \sum_j \eta_j = \sum_j \underline{A}_j \sin(\omega_j t - \underline{\vartheta}_j) \tag{1}$$

where $\underline{A}_j$ is random amplitude, ω_j is frequency, $\underline{\theta}_j$ is random phase angle, and t is time. The frequency components can be transformed into standardized (unit-variance, zero mean) variables by dividing each by its standard deviation σ_j. Thus,

$$x_j = \frac{\eta_j}{\sigma_j}, \ \tilde{x}_j = \frac{\tilde{\eta}_j}{\sigma_j} \tag{2}$$

where η_j is the jth spectral component and $\tilde{\eta}_j$ is its Hilbert transform. The joint density function of the

standardized variables, x_j, $\tilde{x}_j$, is then unit-variance normal.

Newman [3] calculated the diffraction forces generated by such an incident sea on a vertical, cylindrical column in the limit where the wavelength is long compared with the column diameter. In a coordinate system with the x and y axes in the mean water surface, x oriented in the wave direction and z vertical, the first order force on the column is

$$F_1 = \frac{1}{2}\pi D^2 \rho \int_{-\infty}^{0} u_t \, dz \tag{3a}$$

This is the 'Morison inertia load' below mean water line.

The second-order force is

$$F_2 = \frac{1}{4}\pi D^2 \rho \int_{-\infty}^{0} \left(2ww_x + uu_x\right) dz + \frac{1}{2}\pi D^2 \rho u_t \eta_1 \tag{3b}$$

The third-order force is

$$F_3 = F_3^{(1)} + F_3^{(2)} \tag{3c}$$

where

$$F_3^{(1)} = \frac{1}{4}\pi D^2 \rho \left[\eta_1\left(u_{tz}\eta_1 + 2ww_x + uu_x - \frac{2}{g}u_t w_t \right) \atop -\left(u_t/g\right)\!\left(u^2 + w^2\right) \right]$$

and

$$F_3^{(2)} = \left(\pi D^2 \rho / g\right) u^2 u_t + O\!\left(\varepsilon^6\right)$$

Both the third order terms are point forces acting at mean water line.

In Eqs (3a), (3b) and (3c) D is the column diameter, ρ is the mass density of the water, g is the gravitational acceleration, u and w are respectively the horizontal and vertical orbital velocities, and η_1 is the first-order estimate of the ocean surface.

Given linear wave kinematics, we can use Eq. (2) to express Newman's forces in terms of the standardized variables, x_j, $\tilde{x}_j$. The following expressions result:

$$F_1 = \frac{1}{2}\pi D^2 \rho g \sum_j \sigma_j \tilde{x}_j \tag{4a}$$

$$F_2 = \frac{1}{4}\pi D^2 \rho \sum_j \sum_k \left(\frac{2\omega_j \omega_k^3 - \omega_j^3 \omega_k}{\omega_j^2 + \omega_k^2} + 2\omega_j^2 \right) \tag{4b}$$
$$\sigma_j \sigma_k \tilde{x}_j x_k$$

$$F_3 = \frac{1}{4}\pi D^2 \rho \frac{1}{g}\left[\sum_j \sum_k \sum_l \left(\omega_k^4 + 2\omega_k \omega_l^3 - \omega_l \omega_k^3 \atop + 2\omega_k^2 \omega_l^2 + 3\omega_j \omega_k^2 \omega_l \right) \right.$$
$$\sigma_j \sigma_k \sigma_l x_j \tilde{x}_k x_l$$
$$\left. -\sum_j \sum_k \sum_l \omega_j^2 \omega_k \omega_l \sigma_j \sigma_k \sigma_l \tilde{x}_j \tilde{x}_k \tilde{x}_l \right] \tag{4c}$$

2. Methods

The dynamic behaviour of the structure is modelled as a single degree of freedom system. The load is applied directly on the mass and the response is the force transmitted by the spring. The dynamic amplification factor $D(\omega)$ and the phase shift $\theta(\omega)$ are given respectively as

$$D(\omega) = \frac{1}{\left\{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\dfrac{\omega}{\omega_n}\right]^2\right\}^{1/2}} \tag{5a}$$

$$\tan(\theta) = -\frac{2\zeta\dfrac{\omega}{\omega_n}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2} \tag{5b}$$

In Eqs (5a) and (5b) ω_n is the natural frequency of the system and ζ is a non-dimensional damping parameter (relative damping).

The application of the transfer function (5a) and (5b) to Eqs (4a) to (4c) leads to the corresponding responses R_1, R_2, R_3.

The first order response is

$$R_1 = \frac{1}{2}\pi D^2 \rho g \sum_j \sigma_j \left[\tilde{x}_j D_j \cos(\theta_j) \atop - x_j D_j \sin(\theta_j)\right] \tag{6a}$$

where $D_j = D(\omega_j)$ and $\theta_j = \theta(\omega_j)$, are the dynamic amplification factor and the phase shift at frequency ω_j.

The second order response is

$$R_2 = \frac{1}{8}\pi D^2 \rho \sum_j \sum_k \left(\frac{2\omega_j \omega_k^{\,3} - \omega_j^{\,3}\omega_k}{\omega_j^{\,2} + \omega_k^{\,2}} + 2\omega_j^{\,2} \right)$$

$$\sigma_j \sigma_k \left\{ \tilde{x}_j \tilde{x}_k \left[D_{j+k} \sin(\theta_{j+k}) - D_{j-k} \sin(\theta_{j-k}) \right] \right.$$
$$+ \tilde{x}_j x_k \left[D_{j+k} \cos(\theta_{j+k}) + D_{j-k} \cos(\theta_{j-k}) \right]$$
$$+ x_j \tilde{x}_k \left[D_{j+k} \cos(\theta_{j+k}) - D_{j-k} \cos(\theta_{j-k}) \right]$$
$$\left. + x_j x_k \left[-D_{j+k} \sin(\theta_{j+k}) - D_{j-k} \sin(\theta_{j-k}) \right] \right\} \quad (6b)$$

where $\quad D_{j\pm k} = D\left(\omega_j \pm \omega_k\right) \quad$ and

$\theta_{j\pm k} = \theta\left(\omega_j \pm \omega_k\right)$ are the dynamic amplification

factor and the phase shift at frequency $\left(\omega_j \pm \omega_k\right)$.

The expression for the third order response, R_3, is similar, but rather long. It can be found in Suastika [9].

The total response is $R = R_1 + R_2 + R_3$. The condition R = constant defines a surface in the space of the standardized variables, x_j, $\tilde{x}_j$.

3. Results and Discussion

Using the expressions above, it is possible to generate surfaces of constant response in the space of the spectral components of the ocean surface. We recall that the spectral components and their Hilbert transforms are un-correlated, linear processes. They obey a joint normal distribution with zero cross correlation. Thus, surfaces of constant probability density are concentric spheres in the space of the standardised variables. The probability density is highest at the origin and falls monotonically as a function of distance from the origin. Under these circumstances, it is straightforward to treat the response surfaces as limit states in a FORM type of analysis. The point on a surface of constant response where the distance to the origin is shortest and the probability density greatest is called the "design point." This is, to a good approximation, the point where a response maximum is most likely. We find the design point using Lagrange's method of undetermined multipliers [5].

Since the design point (x_j*, $\tilde{x}_j*$) defines the amplitude and phase of the frequency components at the instant when the extreme occurs, it allows us to deduce the time histories of the response and related variables around the time of the extreme. These histories are those most likely to be associated with a response maximum. Thus, in the case of the ringing problem, we can identify if ringing determines the extreme and identify the type of applied load history that excites it. In addition, we find the ocean surface history that generated it all. Finally,

we can estimate the exceedance probabilities of extreme ringing responses very efficiently.

We studied the case of a uniform 10 m diameter column in a sea with a significant wave height of 15 m and a zero crossing period of 13.5 s that obeys a JONSWAP spectrum. The peak of the surface energy spectrum is approximately 0.36 rad/s. The water depth is 300 m. The dynamics are modeled with a damping coefficient that is 2.5 % of critical and a natural frequency of 1.57 rad/s. We shall discuss the case of the maximum negative or "swing back" response, that is in the direction opposed to the waves. We select this since it is more severe than the swing forward response. We describe here the solutions we found for an extreme response level such that the probability that an individual maximum exceeds it is 10^{-4}. Results of the study have been presented in the 1998 OTRC Symposium in Houston, Texas, USA [10].

We investigated the effects of using the Newman load model to first, second and third order. The response time histories are plotted in Figs 1 to 3. The contribution to the dynamic response from the higher order forces is very significant. There is some evidence of ringing from second order forcing. With third-order forcing the ringing is unmistakable. The corresponding surface elevation is shown in Fig. 4.
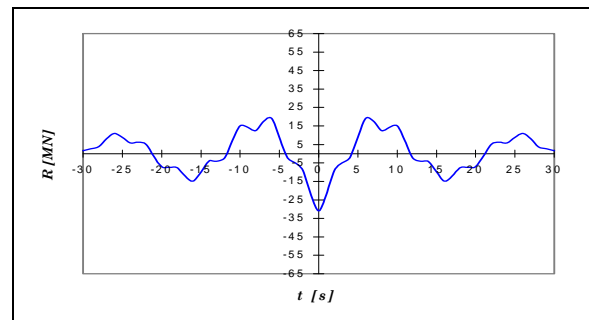

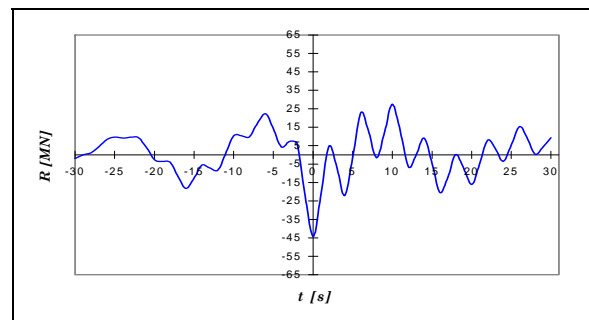**Figure 1. Dynamic response with only first-order excitation**

Figure 2. Response with first and second-order excitation
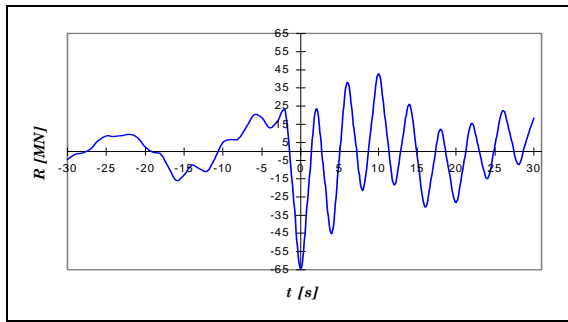
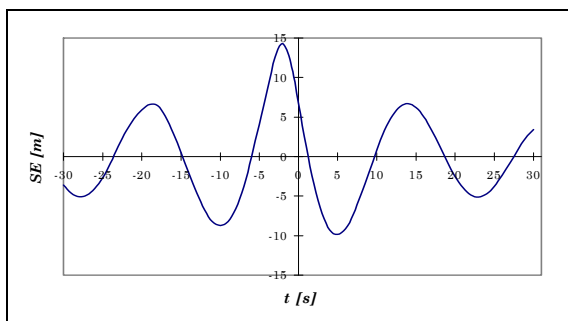**Figure 3. Response with first, second and third-order excitation**

Figure 4. Surface elevation generating the response time series plotted in Fig. 3
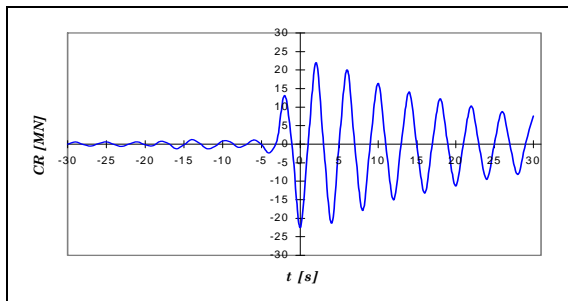

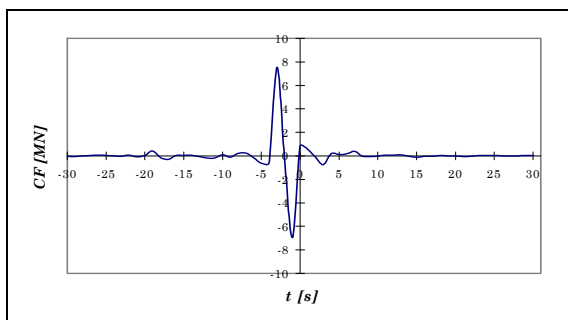**Figure 5. Third-order component of the dynamic response**

Figure 6. Third-order component of the force excitation

To gain insight into the processes, we look more closely at the third order contribution to the response. From the design point we have calculated, it is possible to reconstruct the time series of all the components of the response and of the excitation. The time history of the third order component of the response is plotted in Fig. 5 and the corresponding force excitation in Fig. 6. The third-order component of the response resembles the response of a linear oscillator to an impulse excitation: a rapid increase of energy at the natural frequency followed by a slow decay due to damping. The corresponding exciting force in Fig. 6 supports this view. In fact, the excitation takes the form of a double impulse. The double impulse is associated with a wave crest; the positive impulse immediately precedes the crest and the negative impulse follows it. Apart from the isolated double impulse, the third order excitation is remarkably small. Though not plotted here, similar, but less pronounced, effects are seen at second order.

Finally, we note that the results provided here involved a few minutes of computer simulation. To achieve similar results by time domain calculation of random waves would involve many hours of computing.

4. Conclusion

The ringing generated by non-linear terms in Newman's model can have a significant effect on the dynamic response of offshore structures such as gravity base structures (GBS) or tension leg platforms (TLP). Some ringing effects can be found in the response with only second order excitation. Ringing arises from an impulse generated by non-linear loading. The spectral response surface method is tractable and much faster than a random time-domain simulation.

Acknowlegement

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