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THE SOLUTION OF SPACECRAFT NAVIGATION PROBLEMS WITH A HELP OF NONLINEAR PARAMETRIC MODELS

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Abstract. *In the article the solution of spacecraft navigation problem with a help of nonlinear parametric models for the initial conditions of movement of the surveillance object is considered. The technique of construction of nonlinear models with a use of the method of equal dimension in non-Taylor's shifted differential transformations is proposed. An example of the application of developed technique proves its efficiency for rapid and accurate determination of the initial conditions of spacecraft motion.*

Keywords: differential transformations, navigation, nonlinear model, spacecraft.

Introduction

It is difficult to identify areas of applied science, engineering or economics wherever the results, obtained in the space industry, are used. This is communication and navigation, remote sensing of the Earth and meteorology, television etc [1; 2].

Quality of target-oriented solution of problems of space systems largely depends on the efficiency of navigation spacecraft: observation of the motion trajectory, determination of its location (the initial conditions of motion, and the formation of control commands. That is why, we put stringent performance requirements of spacecraft navigation and according to the efficiency and accuracy of the initial conditions of their movement. Hence, there is a problem of rapid and highly-precise determination of the initial conditions of spacecraft motion.

Analysis of existing approaches

Definition of the initial conditions of spacecraft motion is carried by statistical processing of the sample experimental data, that's is discrete object measurements of selected observations. The use of adequate mathematical model of the process is the base for the effective implementation of statistical algorithms of data processing. According to the nonlinear nature of spacecraft motion, especially on the areas of apogee, perigee, with adequate corrective maneuvers his motion model that will be used for statistical data processing algorithms will have complex, nonlinear form. The problem of building mathematical models of processes, studied by the experimental data, was solved in the work of many authors [2–6].

This presentation of nonlinearity of experimental process is achieved by using polynomial smoothing, nonlinear parametric models with further linearization or numerical solution of nonlinear equations. However, polynomial smoothing does not include a priori information about the view of a nonlinear model of experimental process. The traditional approach for building the nonlinear models has some disadvantages that reduce the accuracy of the original information. So, not enough attention was paid to the reduction the impact of random errors and linearized dynamic models on nonlinear parametric model. There is a large computational complexity of the procedures for parameters of nonlinear models using numerical solutions and the dependence of accuracy of obtained solutions on the capabilities of applied numerical methods. This lack of finite analytical smoothing algorithm does not allow to form unified form of the construction of nonlinear mathematical models. Moreover, existing approaches to the construction of nonlinear parametric models are difficult to apply for the models with significant nonlinearities, which are defined by differential equations.

Thus, the purpose of the article is to develop approaches to the construction of the nonlinear parametric models of spacecraft motion by experimental data.

The main material

In general there are two basic approaches to the research of simulation process, namely: purely theoretical and purely empirical approaches [7].

The first approach uses analytical methods and applies when the laws that describe the change of the investigation process were put. It provides high accuracy of general conclusions, caused by the use of detailed mathematical models.

However, the absence of the connection with real processes, including the complexity of analytical models of the form, does not ensure their effective application. At the same time a purely empirical approach connect the results to a particular process and has worse prognostic features. Thus, theoretical and experimental approach that takes into account the information about the analytical description of the process and experimental data for the model parameters determination, is the most appropriate for developing mathematical models. So, the use of the method of differential transformations (DT) for the study of complex nonlinear processes enables the realization of that problem.

Differential transformations – is the operational method, which was established by G.E. Pukhov, member of National Academy of Sciences of Ukraine, and which is based on the transfer of the original image in the region of images through the operation of differentiation. Differential changes in the general case is a functional transformation in the form of [8; 9]:

$$Z(k) = P\{z(t)\}_{t^*} = \frac{H^k}{k!}\left[\frac{d^k z(t)}{dt^k}\right]_{t^*}; \qquad (1)$$

$$z(t) = f(t,c), \qquad (2)$$

where t^* – a meaning of argument in which the conversion is carried out;

$Z(k)$ – discrete function of integer argument $k = 0,1,2,\dots$;

H – the argument section, on which function $z(t)$ is considered;

$f(t,c)$ – restoring or approximating function;

c – a set of free coefficients.

Expression (1) determines the direct conversion, which allows you with a help of original image $z(t)$ to find the image $Z(k)$. Inverse transformation that restores the original $z(t)$ in the form of approximating functions is defined by the expression (2).

Differential image $Z(k)$ is called a differential spectrum (DS), or P-spectrum, and the meaning of function $Z(k)$ with the particular values of the argument k – is DS-discretes, or P-discretes. In the simplest case, restoring function $f(t,c)$ looks like a polynomial, and the recovery of the original is reduced to the summation of discrete P-spectrum in the form of Taylor segment series. In this case differential transformations are called the principal or Taylor's differential transformations [8; 9].

Their disadvantage is the small interval of precise recovery process through a limited radius of convergence of Taylor series. In order to enhance the capacity of the solutions which were obtained by DT [10], the restoration of the originals in the form of arbitrary approximating functions was introduced. In this case, DT is called non-Taylor's differential transformations (NDT). Free coefficients c_i of restoring function $f(t,c)$ for non-Taylor's differential transformations can be determined by minimizing the deficiency $\varepsilon(t)$ between the original and approximating functions for the selected criteria, which in the agreed notation has the form [10]:

$$\left(\left[P\{z(t)\}_{t^*} \Rightarrow Z(k)\right] - \left[P\{f(t,c)\} \Rightarrow F(k,c)\right]\right) = \\ = \left[P\{\varepsilon(t)\}_{t^*} \Rightarrow E(k)\right] \to \min, \qquad (3)$$

where $E(k) = Z(k) - F(k,c)$ characterizes the deficiency of DS.

For general DT the definition of P-spectrum by the expression (1) is for zero values of argument t^*. On the one hand, it simplifies the subsequent conversion, on the other hand - reduces prognostic properties of obtained models. In order to enhance the properties of the models which were obtained by DT, in [11] shifted DT are proposed, that provide a precise solution for every argument functions. Shifted differential transformation are transformed basic DT (1) and (2) have the following form

$$Z(k,t_v) = P\{z(t_v + \tau)\}_\tau = \\ = \frac{H^k}{k!}\left[\frac{d^k z(t_v + \tau)}{d\tau^k}\right]_\tau, \qquad (4)$$

$$\bar{Z}(k,t_v) = P\{z(t_v - \tau)\}_\tau = \frac{(-H)^k}{k!}\left[\frac{d^k z(t_v - \tau)}{d\tau^k}\right]_\tau,$$

$$z\left(t_v+\tau\right)=f\left(t_v+\tau,c\right),$$

$$\bar{z}\left(t_v-\tau\right)=f\left(t_v-\tau,c\right),$$

where $\bar{Z}\left(k,t_v\right)$ – direct and inversed P-spectra of the initial function;

τ – local argument the value of which is chosen within $H\geq\tau\geq 0$ τ;

$z\left(t_v+\tau\right)$, $\bar{z}\left(t_v-\tau\right)$ – direct and inversed model.

Simultaneous usage of direct and inverse P-models in the NDT-scheme provides the compensation of disadvantages of simplified approximating functions.

Thus, while determining the parameters of nonlinear models with experimental data to reduce the impact of measurement errors on the simulation results, it is expedient to use dislocated DT on the basis of non-Taylor's differential transformations.

The essence of the problem of mathematical change of the coordinates for the model determination of the initial conditions of motion of his movement has such view. Let an experimental polynomial model $z\left(t\right)$ with the measured data of selected coordinate of the spacecraft according to the algorithm of the classical method of the least squares is built. Nonlinear (theoretical) model of the process is considered to be a priori known in the form of $f\left(t,c\right)$. It is necessary to define the parameters of the model $f\left(t,c\right)$ with a help of well-known experimental function $z\left(t\right)$.

In order to solve the problem we use the criterion of the method of equal planes (MEP) in the NDT (3) [10]. This choice is justified when the construction of a nonlinear model of the process is in a class of smooth functions.

According to the MEP, criterion of minimizing the deficiency between the experimental and theoretical models is formed as follows:

$$\int_a^b \varepsilon(t)dt=0\Rightarrow\int_a^b\left(z(t)-f\left(t,c\right)\right)dt\Rightarrow$$

$$\Rightarrow\int_a^b z(t)dt=\int_a^b f\left(t,c\right)dt. \qquad (5)$$

The essence of MEP criteria for the construction of nonlinear parametric models of the experimental processes is the requirement of equality planes limited by the curves of experimental and theoretical models on a limited range. To simplify the operation of integration we transfer the criterion (5) into the range of images by using the basic DT (1):

$$H\sum_{k=0}^{k=\infty}\left[\left(\frac{b}{H}\right)^{k+1}-\left(\frac{a}{H}\right)^{k+1}\right]\frac{Z\left(k\right)}{k+1}=$$

$$=H\sum_{K=0}^{k=\infty}\left[\left(\frac{b}{H}\right)^{k+1}-\left(\frac{a}{H}\right)^{k+1}\right]\frac{F\left(k,c\right)}{k+1},$$

where $z\left(k\right)$, $F\left(k,c\right)$ are DS originals $z\left(t\right)$ and $f\left(t,c\right)$ respectively.

In order to determine the parameters $c=\left\{c_i\right\}$ of nonlinear model of the system $f\left(t,c\right)$, it is necessary to make the system with m (the number of unknown parameters of theoretical model is known) equations, following the criterion (5).

In order to receive the system of equations, the solution of which are unknown parameters of nonlinear model, the interval of integration $\left[a,b\right]$ is broken into m subinterval. So, if common interval $\left[0,H\right]$ is considered, the subintervals are

$$\left[0,b_1\right],\left[b_1,b_2\right],\ldots,\left[b_{m-1},b_m=H\right].$$

The division of the limit construction of nonlinear models to the subintervals allows to apply technology and advantages of shifted DT. According to this we can receive the system of equation in the form of

$$\begin{cases} H\sum_{k=0}^{k=\infty}\left[\left(\frac{0}{H}\right)^{k+1}-\left(\frac{b_1}{H}\right)^{k+1}\right]\frac{Z\left(k\right)}{k+1}= \\ \quad =H\sum_{k=0}^{k=\infty}\left[\left(\frac{0}{H}\right)^{k+1}-\left(\frac{b_1}{H}\right)^{k+1}\right]\frac{F\left(k,c\right)}{k+1}, \\ H\sum_{k0}^{k=\infty}\left[\left(\frac{b_1}{H}\right)^{k+1}-\left(\frac{b_2}{H}\right)^{k+1}\right]\frac{Z\left(k\right)}{k+1}= \\ \quad =H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_1}{H}\right)^{k+1}-\left(\frac{b_2}{H}\right)^{k+1}\right]\frac{F\left(k,c\right)}{k+1},\ldots, \\ H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_{m-1}}{H}\right)^{k+1}-\left(\frac{b_m=H}{H}\right)^{k+1}\right]\frac{Z\left(k\right)}{k+1}= \\ \quad =H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_{m-1}}{H}\right)^{k+1}-\left(\frac{b_m=H}{H}\right)^{k+1}\right]\frac{F\left(k,c\right)}{k+1}. \end{cases} \qquad (6)$$

In the system (6) DS functions $z(t)$, $f(t,c)$ are presented in assumption of the basic DT (1). In this case the increase of imaging accuracy of obtained analytical solutions is provided by using NDT basis as nonlinear model $f(t,c)$. In order to satisfy opportunities for shifted DT a number of successive actions is usually applied. Let the observation interval of experimental process is equal to $[-H,\ H]$, we define the direct and inversed model of functions $z(t)$ and $f(t,c)$. According to the shifted DT (4) P-spectrum of models $z(t)$, $f(t,c)$ in the initial point $t_v = 0$ is equal to

$$Z(k,t_v) = Z_v(k), \qquad (7)$$
$$F(k,t_v,c) = F_v(k,c).$$

We can write direct and inversed analogues respectively in points $t_v = \pm H$ while using the DT of model functions (7) for the initial point $t_v = 0$ including transformation (4).

$$Z(k,t_v + H) = Z_v(k),$$
$$\bar{Z}(k,t_v - H) = (-1)^k Z_v(k),$$
$$F(k,t_v + H,c) = F_v(k,c), \qquad (8)$$
$$\bar{F}(k,t_v - H,c) = (-1)^k F_v(k,c).$$

Here the notion of direct and inversed models is used to explain the direction of arguments change of functions which are considered: direct model - with the change of argument from left to right, inversed - with the change of argument in the opposite direction to a direct model of order.

So, obtained direct models $Z(k)$, $F(k,c)$ characterize experimental process for identifying the range of its observations. Properties of shifted DT allow to form analytical functions of the experimental process beyond an interval of observation process in the form of their inversed analogues $\bar{Z}(k)$, $\bar{F}(k,c)$. Usage of direct and inversed models in the calculations, allows to expand the overall recovery interval parameters of these experimental processes and compensate the errors of complete solutions. The usage of inversed models is similar to the usage of additional information channel. In order to combine the properties of shifted and NDT, including obtained the P-spectra of the models (8), the system (6) is transformed to the form

$$\begin{cases} H\sum_{k=0}^{k=\infty}\left[\left(\frac{0}{H}\right)^{k+1} - \left(\frac{b_1}{H}\right)^{k+1}\right]\frac{Z(k,t_v+H)}{k+1} = \\ = H\sum_{k=0}^{k=\infty}\left[\left(\frac{0}{H}\right)^{k+1} - \left(\frac{b_1}{H}\right)^{k+1}\right]\frac{F(k,t_v+H,c)}{k+1}, \\ H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_1}{H}\right)^{k+1} - \left(\frac{b_2}{H}\right)^{k+1}\right]\frac{\bar{Z}(k,t_v-H)}{k+1} = \\ = H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_1}{H}\right)^{k+1} - \left(\frac{b_2}{H}\right)^{k+1}\right]\frac{\bar{F}(k,t_v-H,c)}{k+1}, \dots, \\ H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_{m-1}}{H}\right)^{k+1} - \left(\frac{b_m=H}{H}\right)^{k+1}\right]\frac{\bar{Z}(k,t_v\pm H)}{k+1} = \\ = H\sum_{k=0}^{k=\infty}\left[\left(\frac{b_{m-1}}{H}\right)^{k+1} - \left(\frac{b_m=H}{H}\right)^{b+1}\right]\frac{\bar{F}(k,t_v\pm H,c)}{k+1}. \end{cases} \qquad (9)$$

Solving the system (9) relatively to c, we determine the unknown parameters of nonlinear model $f(t,c)$. In this case we formulate the techniques of the construction nonlinear models of experimental process by using MEP in the shifted NDT.

1. Determination of direct and inversed P-model of the form (8), experimental $z(t)$ and non-linear model $f(t,c)$ according to expressions (1), (4).

2. Formation of a system of equations in the form (9) by equating the values of integrals for functions $z(t)$, $f(t,c)$ on m (with the number of unknown parameters of nonlinear model c) of observation subintervals of experimental process. While forming the system (9), the use of direct and inversed models for subinterval sequentially alternates.

3. The solution of formed techniques of the equation system in paragraph 2 relatively to unknown parameters of nonlinear models.

Example of the application of the developed techniques concerns the solution of the problem of determining the initial conditions of spacecraft motion and demonstrates the practical side of the obtained results. In practice, the navigations of spacecraft is carried on the ground points of the signal from the board of the spacecraft – the command-measuring systems using the results of measured arrays, for example the Doppler's frequency of a signal transferred in radial velocity. Let obtain polynomial model, including the results of processing a sample of radial velocity, which characterizes the motion of spacecraft in circular orbit:

$$r_p(t) = -5,09671 - 0,01604 \cdot t + 0,00192 \cdot t^2 - \\ -0,01039 \cdot 10^{-3} \cdot t^3 + 0,25773 \cdot 10^{-8} \cdot t^4.$$

It is known that in the interval of observation changes the model radial velocity can be described by nonlinear parametric theoretical model of the form

$$r_n(t) = a \cdot arctg(\omega t),$$

where a, ω are unknown parameters that characterize the change of the experimental process.

It is necessary to determine the parameters of nonlinear model $r_n(t)$. According to the proposed techniques, including input signs, we have.

1. Direct and inversed P-model of polynomial $r_p(t)$ and nonlinear functions $r_n(t)$ have the values given by expressions

$$R_p(0) = -5,09671,\ R_p(1) = -0,01604 \cdot H;$$
$$R_p(2) = 0,00192 \cdot H^2, R_p(3) = -0,01039 \cdot 10^{-3} H^3;$$
$$R_p(4) = 0,25773 \cdot 10^{-9} \cdot H^4; \qquad (10)$$
$$\bar{R}_p(0) = -5,09671,\ \bar{R}_p(1) = 0,01604 \cdot H;$$
$$\bar{R}_p(2) = -0,00192 \cdot H^2, \bar{R}_p(3) = 0,01039 \cdot H^3;$$
$$\bar{R}_p(4) = -0,25773 \cdot 10^{-9} \cdot H^4;$$
$$R_n(0) = 0,\ R_n(1) = awH,\ R_n(2) = 0;$$
$$R_n(3) = -(1/3) a w^3 H^3,\ R_n(4) = 0; \qquad (11)$$
$$\bar{R}_n(0) = 0,\ \bar{R}_n(1) = -awH,\ \bar{R}_n(2) = 0;$$
$$\bar{R}_n(3) = (1/3) a w^3 H^3,\ \bar{R}_n(4) = 0.$$

2. In order to find the parameters of nonlinear model – $a,\ \omega$ taking into account the general form of the equations system (9) and P-spectra (10) (11), we have

$$\begin{cases} 5762,01125 \cdot aw - \\ \qquad - 0,11066 \cdot aw^3 = -98,95855, \\ -17286,03375 \cdot aw + \\ \qquad + 0,166 \cdot 10^{-9} \cdot aw^3 = 10453,55658. \end{cases}$$

Solving formed system relatively unknown parameters $a,\ w$ of nonlinear models, it was defined $a_r = 5,34224,\ w_r = 0,02428$. Ideal values of the nonlinear model are assumed to be known and $a_i = 5,34322,\ w_i = 0,02424$.

The results of evaluation the value and dynamic error of reproduction experimental process of the change spacecraft radial velocity are given in table.

Results of evaluation the value and dynamic error

Parameter	Interval point of spacecraft		
	initial	medium	final
MLS, кm/c	-5,09640	-0,25382	5,05769
MEP, кm/c	-5,33694	-0,12969	5,33693
Stand. кm/c	-5,33393	-0,12950	5,33393
Δ_{MLS}	0,23753	0,12432	0,27652
Δ_{MEP}	0,00300	0,00018	0,00300
σ_{MLS}, кm/c	0,000457	0,000198	0,000502
σ_{MEP}, кm/c	0,000109	0,000022	0,000114

These results were obtained with the absence of random measurement errors of experimental data. For the real terms of determination of radial velocity measurements by Doppler's frequency the calculations were carried out with the value of standard deviation error of radial velocity $\sigma = 0.002$ km/c. (14) The value of random errors of the image of experimental process using the models obtained in accordance with the proposed techniques compared with classical MLS given in the last two rows of table.

Analysis of the results of research shows that compared with the traditional approach of using the proposed technique for modeling a nonlinear process changes, it allows to improve the accuracy of his image including dynamic, and a random error components. Getting a win in the dynamic accuracy of smoothing is explained by using nonlinear model for approximation of the measured values that are more adequate representation of the experimental process. The increase of accuracy smoothing according to a random error component is explained by the use of direct and inversed models in NDT, which provides partial compensation of random errors.

Conclusions

Thus, proposed technique is based on the use of MEP in the vicinity of experimental and theoretical functions. The distinction of this technique is to combine construction of nonlinear parametric models of experimental processes of positive opportunities of NDT and shifted DT within a single solution.

The results of applying the developed technique to solve the practical problem of determining the initial conditions of spacecraft motion proved its effectiveness by the criterion of accuracy of final results. In its turn, the increase of the accuracy of the initial conditions of motion provides quality improvement of solution of the spacecraft navigation problem.

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