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Variance of the decay intensity of superdeformed bands*

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We present analytic formulae for the energy average and variance of the intraband decay intensity of a superdeformed band.

1. Introduction

The intensity of the collective γ–rays emitted during the cascade down a superdeformed (SD) band remains constant until a certain spin is reached where-after it drops to zero within a few transitions. The sharp drop in intensity is believed to arise from mixing of the SD states with normally deformed (ND) states of identical spin [1]. The model of Refs. [2,3] attributes the suddenness of the decay-out to the spin dependence of the barrier separating the SD and ND minima of the deformation potential. Refs. [4,5,6] discuss the effect of the chaoticity of the ND states on the decay-out.

In the present paper we present analytic formulae for the energy average (including the energy average of the fluctuation contribution) and variance of the intraband decay intensity of a superdeformed band in terms of variables which usefully describe the decay-out [7,8,9]. In agreement with Gu and Weidenmüller [8] (GW) we find that average of the total intraband decay intensity can be written as a function of the dimensionless variables $\Gamma^{\downarrow}/\Gamma_S$ and Γ_N/d where $\Gamma^{\downarrow}$ is the spreading width for the mixing of an SD state with ND states of the same spin, d is the mean level spacing of the latter and Γ_S (Γ_N) are the electromagnetic decay widths of the SD (ND) states. Our formula for the variance of the total intraband decay intensity, in addition to the two dimensionless variables just mentioned, depends on the dimensionless variable $\Gamma_N/(\Gamma_S + \Gamma^{\downarrow})$.

2. Energy average and variance of the decay intensity

The total intraband decay intensity has the form [6,8]

$$I_{\text{in}} = (2\pi\Gamma_S)^{-1} \int_{-\infty}^{\infty} dE |A_{00}(E)|^2, \tag{1}$$

where $A_{00}(E)$ is the intraband decay amplitude and Γ_S is the electromagnetic decay width of superdeformed state $|0\rangle$. The energy average of Eq. (1) may be written as the incoherent

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sum [9,10]

$$\overline{I_{\rm in}} = I_{\rm in}^{\rm av} + \overline{I_{\rm in}^{\rm fl}}, \tag{2}$$

where

$$I_{\rm in}^{\rm av} = \overline{I_{\rm in}^{\rm av}} = (2\pi\Gamma_S)^{-1}\int_{-\infty}^{\infty} dE\,\left|\overline{A_{00}(E)}\right|^2 \tag{3}$$

and

$$\overline{I_{\rm in}^{\rm fl}} = (2\pi\Gamma_S)^{-1}\int_{-\infty}^{\infty} dE\overline{\left|A_{00}^{\rm fl}(E)\right|^2}, \tag{4}$$

where we have written the decay amplitude as $A_{00} = \overline{A_{00}} + A_{00}^{\rm fl}$ where $\overline{A_{00}}$ is a background contribution and $A_{00}^{\rm fl}$ is the fluctuation on that background. In [9] the background is taken to be

$$\overline{A_{00}} = \frac{\Gamma_S}{E - E_0 + i(\Gamma_S + \Gamma^{\downarrow})/2}. \tag{5}$$

Eq. (5) exhibits the structure of an isolated doorway resonance. The doorway $|0\rangle$ has an escape width Γ_S for decay to the SD state with next lower spin and a spreading width $\Gamma^{\downarrow}$ for decay to the ND states with the same spin which are reached by tunnelling through the barrier separating the SD and ND wells.

In [9] it is shown that the auto-correlation function of the decay amplitude is given by

$$\overline{A_{00}^{\rm fl}(E)A_{00}^{\rm fl}(E')^*} \;\approx\; 2\,(2\pi\Gamma_N/d)^{-1}\,(\Gamma^{\downarrow}/\Gamma_S)^2\;\overline{A_{00}(E)}^2\;\frac{i\Gamma_N}{E - E' + i\Gamma_N}\overline{A_{00}(E')^*}^2. \tag{6}$$

When $E' = E$ this reduces to

$$\overline{\left|A_{00}^{\rm fl}\right|^2} = 2\,(2\pi\Gamma_N/d)^{-1}\,\frac{{\Gamma_S}^2{\Gamma^{\downarrow}}^2}{[(E-E_0)^2 + (\Gamma_S + \Gamma^{\downarrow})^2/4]^2}, \tag{7}$$

which is the average of the fluctuation contribution to the transition intensity.

The integrals in Eqs. (3) and (4) may be carried out using the calculus of residues. One obtains

$$I_{\rm in}^{\rm av} = \frac{1}{1 + \Gamma^{\downarrow}/\Gamma_S}, \tag{8}$$

for the average background contribution and

$$\overline{I_{\rm in}^{\rm fl}} \;=\; 2\,(\pi\Gamma_N/d)^{-1}\,\frac{\left(\Gamma^{\downarrow}/\Gamma_S\right)^2}{(1 + \Gamma^{\downarrow}/\Gamma_S)^3} = 2\,(\pi\Gamma_N/d)^{-1}\,I_{\rm in}^{\rm av}\,(1 - I_{\rm in}^{\rm av})^2, \tag{9}$$

for the average fluctuation contribution to the average decay intensity. Eq. (9) for $\overline{I_{\rm in}^{\rm fl}}$ is plotted in Fig. 2 and for comparison we have also plotted a fit formula which was obtained by GW,

$$\overline{I_{\rm in}^{\rm fl}} \;=\; \left[1 - 0.9139\,(\Gamma_N/d)^{0.2172}\right]\exp\left\{-\frac{\left[0.4343\ln\left(\frac{\Gamma^{\downarrow}}{\Gamma_S}\right) - 0.45\left(\frac{\Gamma_N}{d}\right)^{-0.1303}\right]^2}{(\Gamma_N/d)^{-0.1477}}\right\}. \tag{10}$$

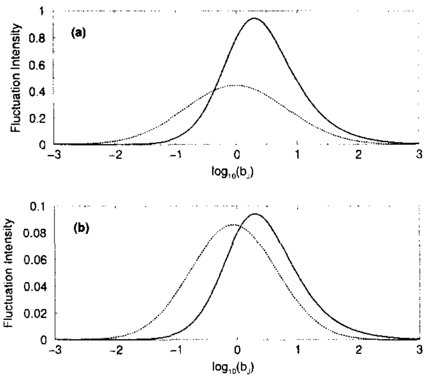

Figure 1. Average of the fluctuation contribution to the intraband intensity $\overline{I_{\rm in}^{\rm fl}}$ vs. $\log_{10}(b_J)$ where $b_J \equiv \Gamma^{\downarrow}/\Gamma_S$. The solid lines were calculated using Eq. (9) and the dotted lines by GW's fit formula, Eq. (10). The variable Γ_N/d took the value 0.1 for graph (a) and 1 for graph (b).

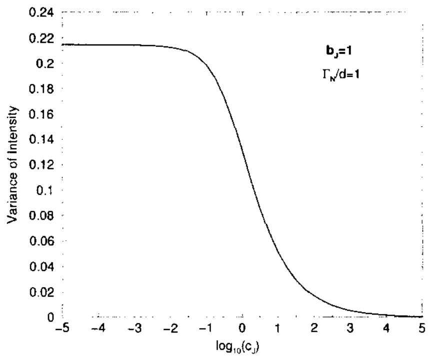

Figure 2. The standard deviation of the decay intensity $\sqrt{\overline{(\Delta I_{\rm in})^2}}$ vs. $\log_{10}(c_J)$ where $c_J \equiv \Gamma^{\downarrow}/\Gamma_N$ plotted using Eq. (12) for fixed $b_J = \Gamma^{\downarrow}/\Gamma_S$ and Γ_N/d.

Qualitative agreement is seen between the two formulae. Our results are strictly valid only when $\Gamma_N/d \gg 1$. The dependence of $\overline{I_{\rm in}^{\rm fl}}$ (and that of $I_{\rm in}^{\rm av}$) on $\Gamma^{\downarrow}/\Gamma_S$ results from the resonant doorway energy dependence of the decay amplitude $\overline{A_{00}(E)}$ [Eq. (5)]. This energy dependence also manifests itself in the average of the fluctuation contribution to the transition intensity $\overline{|A_{00}^{\rm fl}(E)|^2}$ [Eq. (7)]. GW include precisely the same energy dependence in their calculation by use of an energy dependent transmission coefficient to describe decay to the SD band. This is the reason for our qualitative agreement with GW concerning $\overline{I_{\rm in}}$.

A measure of the dispersion of the calculated $I_{\rm in}$ is given by the variance

$$\overline{(\Delta I_{\rm in})^2} = \overline{\left(I_{\rm in} - \overline{I_{\rm in}}\right)^2}. \tag{11}$$

It is shown in [9] that

$$\overline{(\Delta I_{\rm in})^2} = \overline{I_{\rm in}^{\rm fl}}^2 f_1(\xi) + 2 I_{\rm in}^{\rm av} \overline{I_{\rm in}^{\rm fl}} f_2(\xi), \tag{12}$$

where the variable ξ is defined by

$$\xi \equiv \frac{\Gamma_S + \Gamma^{\downarrow}}{\Gamma_N} = \frac{\Gamma_S}{\Gamma_N}(1 + \Gamma^{\downarrow}/\Gamma_S) = \frac{\Gamma_S}{\Gamma_N} I_{\rm in}^{\rm av\,-1} = \frac{\Gamma^{\downarrow}}{\Gamma_N}(1 + \Gamma_S/\Gamma^{\downarrow})^{-1} = \frac{\Gamma^{\downarrow}}{\Gamma_N}(1 - I_{\rm in}^{\rm av}). \tag{13}$$

and

$$f_1(\xi) = \frac{1}{(1+\xi)} + \frac{\xi}{(1+\xi)^2} + \frac{\xi^2}{2(1+\xi)^3} \quad \text{and} \quad f_2(\xi) = \frac{1}{2(1+\xi)}. \tag{14}$$

Since the variance depends only on $(\Gamma_S + \Gamma^{\downarrow})/\Gamma_N$ in addition to $\Gamma^{\downarrow}/\Gamma_S$ and Γ_N/d, upon fixing the latter two variables the variance may be considered a function of any *one* of $\Gamma^{\downarrow}/\Gamma_N$, Γ_S/Γ_N, $\Gamma^{\downarrow}/d$ or Γ_S/d [see Eq. (13)]. Fig. 2 shows a plot of the standard deviation, $\sqrt{\overline{(\Delta I_{\rm in})^2}}$, [Eq. (12)] as a function of $\Gamma^{\downarrow}/\Gamma_N$ for fixed $\Gamma^{\downarrow}/\Gamma_S$ and Γ_N/d. Ultimately, the variance like the intensity is a function of the spin of the decaying nucleus and could provide an additional probe to the spin dependence of the barrier separating the SD and ND wells which is contained in the spreading width $\Gamma^{\downarrow}$ [1,2,3]. Our result for the variance of the decay intensity, $\overline{(\Delta I_{\rm in})^2}$ [Eq. (12)] has a structure reminiscent of Ericson's expression for the variance of the cross section [11]. In the case compound nucleus scattering, extraction of the correlation width from a measurement of cross section autocorrelation function permits the determination of the density of states of the compound nucleus [12]. In the present case the variance supplies a second "equation" besides that for $\overline{I_{\rm in}}$. Both equations are functions of $\Gamma^{\downarrow}$ and d, since the electromagnetic widths are measured. Thus both $\Gamma^{\downarrow}$ and d can be unambiguously determined.

3. Conclusions

In conclusion, we have presented analytic formulae for the energy average and variance of the intraband decay intensity of a superdeformed band. These formulae were derived by making assumptions and approximations which are strictly valid only in the strongly overlapping resonance region, $\Gamma_N/d \gg 1$. However, these formulae work well when $\Gamma_N/d=1$ and provide a qualitative description even when $\Gamma_N/d=0.1$. We have revealed that the variance of the decay intensity depends on the correlation length $\Gamma_N/(\Gamma_S+\Gamma^{\downarrow})$ in addition to the two dimensionless variables $\Gamma^{\downarrow}/\Gamma_S$ and Γ_N/d on which the average of the decay intensity depends.

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