A Model for Fracture in Fibrous Materials

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ABSTRACT

A fiber bundle model in (1 + 1)-dimensions for the breaking of fibrous composite matrix is introduced. The model consists of N parallel fibers fixed in two plates. When one of the plates is pulled in the direction parallel to the fibers, these can be broken with a probability that depends on their elastic energy. The mechanism of rupture is simulated by the breaking of neighbouring fibers that can generate random cracks spreading up through the system. Due to the simplicity of the model we have virtually no computational limitation. The model is sensitive to external conditions as temperature and traction time-rate. The energy *vs.* temperature behaviour, the diagrams of stress *vs.* strain and the histograms of the frequency *vs.* size of cracks are obtained.

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I. INTRODUCTION

Fracture is an important problem in material sciences and engineering. The response of a solid under load depends on the features of the material, the external conditions (temperature, humidity etc) and how the load is applied (uniaxial, radial, shear etc). The main features of the fracture processes can be found in the classical Young's experiment. Let us consider a homogeneous bar of initial length L_o and cross section S pulled by an uniaxial force F parallel to the length. In the $\sigma = F(t)/S(t)$ *vs.* $\delta = \Delta L/L_o$ diagram one can observe an elastic (linear and nonlinear) region and a plastic/deformation one. The elastic region occurs in the beginning of the traction when the material returns to L_o if the traction is stopped. On the other hand, the material acquires a permanent deformation when the force vanishes in the plastic/deformation region. If the material breaks in the elastic regime the fracture is called brittle (like glass at room temperature). Otherwise, if the material breaks in the plastic/deformation region the fracture is called ductile (like a school-rubber).

The presence of disorder in the material is an important feature that determines the rupture processes [**?**]. These inhomogeneities strongly influence the mechanical behaviour of the material and are responsible for the patterns obtained experimentally. In the last decade, some models taking into account this feature were proposed to simulate the breaking processes of disordered media [**?**]. The material, in general, is represented by a network of structural units whose rate of rupture depends on the local conditions and inhomogeneities. These models, which were proposed to simulate the rupture of polymer fibers or thin films (models of lattice springs) [**?**,**?**,**?**] and to study the interface properties of breaking processes [**?**,**?**], have been studied mostly by computational experiments . However, these models provide just a partial description of the problem. At most, only the fracture pattern and the stress *vs.* strain diagram can be obtained. These models do not allow an analysis of the dependence of rupture features with traction velocity and temperature because they are sensitive to changes only in one of the external conditions.

In this paper a fiber bundle model to simulate the failure processes of fibrous material is introduced. Fracture of fiber-reinforced materials is an important field of investigation, because these materials have a higher Young's modulus and other different mechanical properties than unreinforced ones [?,?]. Fiber bundle models were introduced to study the strength of material where fibers are held together by friction forces. They are also used to study the breaking of composite materials where the fibers of the material are joined together by other homogeneous material, as fiberglass-reinforced composite. When a fiber fails, the load that it carries is shared by intact fibers in the bundle. An important effort to study these models was carried out by the calculation of the cumulative breaking probability of the chain of fiber bundles [?,?,?].

Our model considers the amount of elastic energy into the material, the spread of a local crack and the fusion of cracks as the breaking mechanism. Some features already proposed in the literature are used in the definition of our model – the computation of breaking probability from the elastic energy of a fiber [?,?,?] and a deformation limit for an isolated fiber like the threshold in the random fuse network model [?,?]. In addition, we adopt the cascade of breaking fibers as the mechanism to form the cracks into the fiber bundle. This last characteristic is clearly inspired in the self-organizing criticality [?]. Our attention is focused on computational simulation for the breaking of a fiber bundle when we have an uniaxial force (parallel to the fibers) in $(1+1)$ dimensions. The fracture processes are described by the energy of the rupture process *vs.* temperature, the diagram stress *vs.* strain and by the size of the cracks that occurs in the breaking. This paper is organized as follow: in section II, the model is presented; the results of the computational experiments are shown and discussed in section III; finally, the conclusions are given in the last section.

II. THE MODEL

Our model consists of N_o parallel fibers, each of them with the same elastic constant k. These fibers are fixed in parallel plates as is shown in Figure 1. Note that the first and last fiber contact with only one neighbour while the inner fibers have two neighbours. For convenience one plate is fixed and the other is pulled by a force F in the direction parallel to the fibers with constant velocity v. It means that at each time step τ the amount of deformation of the non broken fibers is equal to ($\Delta z = v \times \tau$), where v is the velocity (in our units $\tau = 1$). When the deformation is z, the elastic energy for each fiber is given by

$$\epsilon = \frac{1}{2} k \; z^2 \quad . \tag{2.1}$$

We define the critical elastic energy for each fiber as

$$\epsilon_c = \frac{1}{2} k \; z_c^2 \quad , \tag{2.2}$$

where z_c is imposed as the maximum deformation supported by an individual fiber. We assume that an isolated fiber has a purely linear elastic behaviour with a breaking probability which grows up with the deformation z of the fiber, being equal to unity at $z = z_c$. The probability of rupture of the fiber i is

$$P_i(z) = \frac{1}{(n_i + 1)} \exp[\frac{1}{t}(\delta^2 \; - \; 1)] \quad . \tag{2.3}$$

Here n_i is the number of non broken neighbours fibers of the fiber i (in this paper n_i could be 0, 1 or 2),

$$t = \frac{k_B T}{\epsilon_c} \tag{2.4}$$

is the normalized temperature, k_B is the Boltzmann constant (in our unity system it is equal 1) and

$$\delta = \frac{z}{z_c} \tag{2.5}$$

is the strain of the material. The dependence on the non broken neighbours fibers simulates the existence of an interaction between the fibers. This dependence is responsible for the distribution of the load between neighbouring fibers and allows a fiber having an elastic energy greater than ϵ_c. In this sense, one can observe fibers with $z > z_c$ if they have at least one non broken neighbouring fiber.

Initially all the fibers have the same length and zero deformation. In each time step of the simulation the system is pulled by Δz, and we randomly choose $N_q = q \times N_o$ fibers that can be broken, where q is a positive number. It means that the probability of rupture for the material does not depend on the number of fibers in the fiber bundle. This assumption is in agreement with the observation that systems with different sizes must have the same rupture features for the same external conditions (temperature and traction velocity). Obviously the force and the energy needed to break the bundle must depend on the system size but not the stress *vs.* strain diagrams or the size of the cracks that arises in the breaking processes. This assumption makes also possible the appearance of cracks in different parts of the material for the same deformation. Let us consider a chosen fiber. The breaking probability is evaluated and compared with a random number in the interval $[0, 1)$. If the random number is less than the breaking probability, the fiber breaks. The load spreads to the neighbour fibers and the breaking probability of them increases because of the decreasing of the parameters n_{i-1} and n_{i+1}. This procedure describes the propagation of the crack through the fiber bundle. Then, the same steps are done for one of the neighbouring fibers. Note that if it breaks, a cascade begins. It stops in a given fiber, when the test of the probability does not allow its rupture, or when a hole in the bundle is found (an old crack). The propagation of the crack is done in either "left" or "right" directions, perpendicular to the force applied on the system. When the cascade process stops, other fiber in the N_q set is chosen and all steps already described are repeated. After the N_q trials, we pull the system to a new displacement Δz and the breaking procedure begins again. The simulation continues until the rupture of the system, when no more entire fibers exists.

III. RESULTS AND DISCUSSIONS

At $t = 0$ it is easy to see that the model breaks at $\delta = 1.0$ with a maximum force $F = N_o k z_c$. All the fibers break at the same time and we have just one crack spreading in the entire system (the limit of a brittle fracture). For finite temperatures different behaviours are observed when the traction velocity is varied. The number of fibers is chosen in such a way that it does not affect the propagation of the cracks. It means that a crack greater than or equal to the size of the system, for the values of t and v used in the simulation, occurs with a negligible probability. In order to investigate this picture we have performed simulations in systems $N_o = 10^3 - 10^6$. This probability is controlled by determining the distribution of cracks *vs.* the sizes of the cracks arising in the process of fracture. We have used the following values for the parameters: $q = 0.1$, $N_o = 10^4$, $z_c = 1$ and $k = 1$.

Preliminary we have obtained the stress *vs.* strain diagrams for different temperatures and traction velocities. When the deformation of the bundle is z and the number of non broken fibers is N, the stress σ is defined as

$$\sigma = \frac{Nkz}{N_o} \quad . \tag{3.1}$$

The strain δ was defined in expression (2.5). We compare our results with the description obtained experimentally in order to classify the fracture as brittle or ductile [**?**]. Figure 2 shows the result of a computational simulation carried out in just one fiber bundle. In this case, averages are avoiding. For $t = 0.1$ one observes a brittle behaviour, i. e., the fiber bundle breaks in the elastic region. Note that the $\sigma \times \delta$ plot is purely linear for the highest velocity ($v = 0.1$). At $t = 1.0$ and for high and intermediate velocities the fracture occurs in the brittle/ductile transition region. The rupture of the material is ductile for low velocities and it occurs in the plastic/deformation region. For high temperatures ($t = 4.0$), the shape of the stress *vs.* strain plot is typically ductile for intermediate and low velocities. For high velocities the fracture occurs in the transition region brittle/ductile.

Now let us discuss the behaviour of the energy of rupture as function of temperature. This energy is defined as the work done to break the material and it can be obtained from the stress *vs.* strain diagrams. It is well known that the breaking of materials has strong dependence with temperature. In general, some materials break brittle at low temperature and ductile at high ones. It means that the energy of the fracture process has a small value in the brittle region and a greater value in the ductile one. The results of the normalized averaged energy of the breaking process *per* fiber $< E_f >$ *vs.* the normalized temperature t are shown in Figure 3. We have considered 10^3 samples with 10^4 fibers, with velocities $v = 0.001,\ 0.002$ and 0.005 in the simulations. At low temperatures the energy of the fracture becomes independent of the traction velocity. For velocity $v = 0.005$ the energy increases with temperature. On the other hand, for slow traction ($v = 0.001$) the energy grows up to a maximum (near $t \sim 0.5$) and for $t > 0.5$ it decays smoothly. For an intermediate value of the velocity ($v = 0.002$), the energy remains closely constant at high temperatures.

Figure 4 shows the frequency of the cracks H_c *vs.* the size of the cracks S_c that arises in the breaking process. The frequency of the cracks is averaged over the samples (10^3 ones in this simulation). Two features can be observed in this figure. For a low temperatures ($t = 0.1$, typically brittle fracture) one observes cracks of very different sizes. For low velocities one observe cracks with a maximum size $\sim 10^2$. For high velocities ($v \approx 0.1$) the size of the cracks tends to the entire system (10^4 fibers). This means that the system was pulled essentially non broken until a certain time when a big crack arises in the material. After this big crack, small ones are observed because of the rupture of the remained fibers. The brittle process is characterized by the existence of cracks with different sizes and a remarkable feature is the presence of cracks with sizes near to the system size. The curves that represent H_c *vs.* S_c have a maximum at $S_c = 1$. Note that at the beginning H_c goes down linearly. In order to verify this last feature, we adjust the data using

$$H_c \sim S_c^{\alpha} \quad . \tag{3.2}$$

A good fit for this linear part is obtained with $\alpha \sim 1.02$ (see Figure 5).

As long as the temperature is increased the size of the cracks becomes smaller. When one has many cracks of small sizes the fracture is clearly ductile. These cracks appear in different parts of the material and the shape of the curve H_c *vs.* S_c changes. The curve has a maximum at $S_c = 2$, instead $S_c = 1$ that occurs for brittle fractures.

A quite different feature can be observed for a slow traction ($v = 0.001$) and high temperatures ($t > 0.5$) when we have obtained a ductile fracture with an unusual low energy of rupture (see Figure 3). Only cracks of small size are present in the system ($S_c < 12$). Now the system fails when a force smaller than the one needed to break it at higher velocities is applied. This could indicate that the system is in a different state at high temperatures and that we have observed it in disaggregation.

IV. CONCLUSION

We have introduced a fiber bundle model for simulate fractures in fibrous materials. The model is sensitive to external conditions which are present in some problems of material sciences: traction velocity and temperature. The simplicity of the model allows us to perform computations on very large systems. Because of this feature we can explore all pictures of the failure processes.

We have obtained stress *vs.* strain diagrams showing features of the two principal types of fractures: brittle and ductile. For low temperatures the system breaks brittle, independent of the traction velocity. When the temperature increases, the fracture is influenced by the traction velocity. We can observe a transition from the brittle regime to the ductile one. The amount of energy needed to rupture the material is dependent on the traction velocity. For high velocities more energy is needed. This comes from the fact that the size of the cracks depends on the temperature. For high temperatures and low velocities we observe a curious behaviour. In this case the energy is smaller than that needed to break the material in brittle regime. This could indicate that we have a disaggregation process at this temperature and the interaction between the fibers exerts a small influence in the rupture process. These results are independent on the number of fibers in the fiber bundle, because we have chosen values of the parameters v and t for which the maximum crack sizes obtained in our simulations are less than N_o.

Several questions remain opened. The first one is the behaviour of this model in $(2+1)$-dimensions for several lattices topologies. This could allow the comparison of our results with that observed in realistic systems. The behaviour of the model in high temperatures and low velocities needs a more accurate investigation. It is interesting to verify if those features remains in $(2+1)$-dimensions.

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Figure captions

Figure 1: A schematic representation of our model. We have filled in deep gray the fibers fixed on a plate at rest (bellow) with the high extremity fixed on a moving plate, pulled with constant displacement.

Figure 2: Strees *vs.* strain plots for different normalized temperatures (indicated in the diagrams) and velocities ($v = 0.001$ - full line; $v = 0.01$ - dashed line; $v = 0.1$ - long-dashed line). The pictures were made with just one simulation with a sample of 10^4 fibers for each pair of parameters.

Figure 3: Energy of fracture process *per* fiber $< E_f >$ *vs.* normalized temperature t. The value of the velocities for each curve are shown in the inner box.

Figure 4: Frequency of cracks H_c *vs.* crack sizes S_c for different temperatures and velocities. The simulations were performed in 10^3 samples of 10^4 fibers for each pair of parameters. Pictures at the top have been calculated for $t = 0.1$, at the middle for $t = 1.0$ and at the bottom for $t = 4.0$. Full line represents $v = 0.001$, dashed line $v = 0.01$ and long-dashed line $v = 0.1$.

Figure 5: Regression (dotted lines) for the diagrams frequency of cracks H_c *vs.* crack sizes S_c for $t = 0.1$ for different velocities (indicated in the inner box).