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The Morphological Study of Maxillary Edentulous Ridges and Palates by Moiré Topography

TETSUYA TAKAMATA and KIMIAKI OCHIAI

Department of Complete and Partial Denture Prosthodontics, Matsumoto Dental College
(Chief : Prof. K. Hashimoto)

SAKAE NAGASAWA

Department of Dental Technology, Matsumoto Dental College
(Chief : Prof. S. Takahashi)

Summary

It is important to study the morphology of upper and lower edentulous ridges and palates on the basis of complete denture construction. We have conducted a mathematical method to classify the cross-section of the palatal vault and/or the alveolar crest line form with Fourier analysis without using methods of inspection which have already been discussed. As a result, the cross-section of palatal vaults can be divided into Ovoid, Square and V-shaped types, and alveolar crest lines can be divided into 4 types. This method appears to be an objective one avoiding the researcher's subjective point of view.

Introduction

An anthropological and anatomical study of the oral structures, the basis for dentistry, has been considered many times over the past years. The morphological study of the dental arch form and its classification was first described by Broca (1873) and has been examined by various authors from different points of view.[1~8] The study of the palatal vault form and its cross-section has also been reported by various investigators and not only morphology, but also anthropology, ethnology and genetics have been considered.[9~19] Several methods have been described for determining the size of palates and alveolar ridges. Most investigators have used various methods and instruments for measuring the height, breadth and length at certain reference points.[20~25] Only a few methods describe three-dimensional measurements on casts of edentulous jaws, probably because of difficulty in finding reliable reference points for mounting the casts in a measuring apparatus. Such methods have been surveyed by Carlsson,[26] who used a three-dimensional measuring apparatus based on a stereocomparator. Photogrammetric techniques for measuring the jaws have been tried by Hallert[27] and Nyquist and Tham.[28]

In spite of a significant amount of attention to residual arch form and palatal vault form, the

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problem of reproducible description still remains. There are obvious pitfalls of subjective analysis in describing the residual arch and palatal vault geometrically ; for example, in ovoid, hyperbollic, or V-shaped situations. In dentistry, there have been many investigations of the measurement of three-dimensional analysis by Moirê topography. Further, the various morphological studies in the range of Oral Anatomy, Pedodontics, Orthodontics and Oral Surgery by Moirê topography have been reported.[29~38] In prosthetic dentistry, however, it has not been so utilized. Thus, as was stated previously, we are interested in using Moirê topography for the morphological studies of edentulous arches and palates.

The purpose of this investigation was to measure by Moirê topography the dimensional variables of the upper edentulous casts and comparison with already reported results, and to classify the cross-section of palatal vault and alveolar crest line form with Fourier analysis.

Materials and Methods

The Moirê effect is an optical phenomenon that occurs when two similar arrays of alternately opaque and transparent lines or dots are superimposed upon each other. Suppose an object is illuminated with a point source through a plane grating and the object is observed through the grating. Superimposition of two such similar, but slightly different, line arrays causes interference between the arrays, the interference results in the formation of Moirê fringes. When the height of viewpoint is equal to that of light source, the depth of each fringe is constant. The depth of the *N*th bright fringe is obtained as follows:

$$h_N = N l / (d/s - N)$$

where l is the height of light source and viewpoint from the grid surface, S is the pitch of the grating, and d is the distance between the light source and the viewpoint, measured normal to the ruling of the grating. An apparatus for Moirê topography (FM 3011, Fuji Shashin Koki Inc., Ohmiya, Japan) was used for photographing and measuring the upper casts. Specifications of the apparatus are as follows:

Type	: Semi-dark room operation type
Grating size	: 410 × 260mm
Measurable area	: 300 × 250mm
Resolution	: 1.0, 1.5, 2.0mm (for single fringe)
Light source	: Hologen lamp (AC 100V, 1Kw)
Camera	: FUSICA ST 801 or 901
Dimension	: Approx. 1700(H) × 800(W) × 900(D)mm
Weight	: Approx. 130kg
Power supply	: AC 100V, 50/60Hz

And an apparatus (FUSINON OPTICAL PATTERN ANALYZER MC5000) was used for obtaining the sectional shape of the palatal vault on Moirê topography photographs. The study was carried out on a series of 139 artificial stone casts of the edentulous upper jaws of 67 males (from 40 to 80 years of age) and 72 females (from 30 to 70 years of age). These casts were selected at random time intervals in our labolatory. The reasons for observing a number of upper casts by their Moirê fringes is that it is easy to see the anatomical landmarks, and the morphological features of the upper casts may be compared distinctly with those of the lower casts. To secure reasonably reproducible mountings of each cast in the measuring apparatus, three readily identifiable reference points were marked with a carbon marker using a stage table that is adjustable to any desired anterior, posterior, or lateral tilt. The reference points were located at the incisive papilla (A) and the deepest part of the hamular notch on both sides (H,I) in 139 casts, because these parts of the maxilla are not greatly influenced by resorption. A modified "technical stage", which is being applied to laser holography, was used for the mounting apparatus. This consists of a vertical support and a horizontal frame that could be raised and lowered and locked at the desired level. Casts were oriented with the gratings parallel to the standared plane which included three reference points using a "tecnical stage" which had already been adjusted to be parallel to the plane of the gratings, and the highest portion of the ridges was brought to approximately 1.0 mm from the gratings. The lens should be stopped down to F 1/8 to get good fringe visibility at the maximum depth of the fields, but in most cases F 1/4 is satisfactory. The shutter speed depends upon the sensitivity of the film, and the appropriate condition is F 1/4 at 1/8 sec. with ASA 400 (KODAK TRI X PAN) development and pitch of the grating is 1.0 mm. The Moiré topography photographs were taken under these conditions and were marked and lined in accordance with the three reference points (Figs.1 and 2). All of the photographs were then measured with a slide caliper (1/20 mm), a semicircular protractor and a curvimeter to obtain the dimensional variables. But as the photographs were 1.12 times the size of the substances, it should be corrected by calculation.

Results

The values were compared with the means ($\bar{x}$), the standared deviations (s) and the statistical analysis of measurement results in each portion. The following findings were then obtained.

Dimension of palatal vault breadth and length (Tables 1 and 2)

Among the breadth of each portion (BC : anterior, DE : middle, FG : posterior) , the anterior palatal vault breadth was the smallest. The length of the straight line between the highest portion

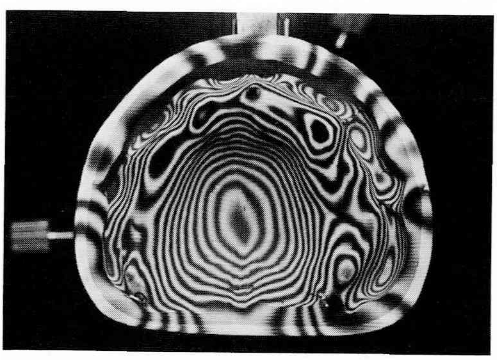

Fig. 1. Typical Moire' topography photograph (pitch : 1.0 mm)

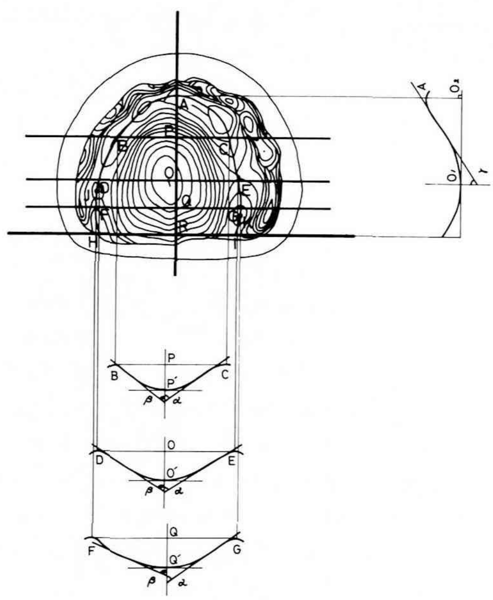

Fig. 2. Portion of cross-section, measurement of breadth and height and palatal wall angle

of the alveolar tubercles on both sides ($\overline{JK}$) was the largest, but there was not a definite difference with the posterior palatal vault breadth ($\overline{FG}$). In the comparative evaluation from 50 to 70 years of age, the difference between the average in each portion was not great, but the palatal vault length ($\overline{AR}$) was larger than the palatal vault breadth ($\overline{HI}$). Although the difference of palatal vault length between males and females was not clear, it seems to be correct to consider that the former

Table 1. Dimension of palatal vault breadth and length (in males)

	Breadth					Length	
Portion	$\overline{BC}$	$\overline{DE}$	$\overline{FG}$	$\overline{HI}$	$\overline{JK}$	$\overline{AR}$	$\widehat{JK}$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
40 n = 5	35.1±2.8	43.1±2.4	47.3±2.4	44.8±3.4	47.5±3.0	45.2±3.5	97.1±14.6
50 n =15	37.2±6.8	45.3±5.3	46.8±3.7	44.7±3.7	47.1±4.2	42.7±2.8	89.8±12.3
60 n =20	35.7±3.1	45.6±4.5	47.1±3.9	43.1±3.2	47.5±3.4	46.4±4.2	97.4±11.1
70 n =21	35.0±3.9	44.3±4.1	46.3±4.1	42.5±4.5	46.7±3.9	45.0±3.3	95.8± 6.6
80 n = 6	37.6±2.2	47.2±3.6	48.1±3.3	44.0±4.2	48.5±3.6	44.3±3.5	93.4± 4.1
Total n=67	35.9±4.3	45.1±4.4	46.9±3.7	43.4±3.8	47.2±3.7	44.9±3.7	94.8±10.2

mm.

Table 2. Dimension of palatal vault breadth and length (in females)

	Breadth					Length	
Portion	$\overline{BC}$	$\overline{DE}$	$\overline{FG}$	$\overline{HI}$	$\overline{JK}$	$\overline{AR}$	$\widehat{JK}$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
30 n = 3	35.2 ± 0.8	37.9 ± 9.9	45.3 ± 5.7	42.4 ± 6.0	45.0 ± 4.7	44.4±2.6	94.1±4.1
40 n =10	35.0 ± 2.5	43.1 ± 2.6	45.5 ± 3.0	42.6 ± 3.1	46.9 ± 6.9	43.4±4.6	92.7±9.7
50 n =24	36.2 ± 2.6	42.9 ± 2.4	44.9 ± 2.3	41.7 ± 2.7	44.6 ± 2.3	44.5±2.9	93.7±6.0
60 n =25	33.0 ± 4.1	41.7 ± 3.7	44.5 ± 2.8	41.3 ± 2.7	44.6 ± 2.9	43.8±3.2	89.7±7.3
70 n =10	34.0 ± 2.7	41.8 ± 1.9	44.6 ± 1.8	40.1 ± 2.8	45.0 ± 1.6	44.6±3.6	91.0±4.3
Total n=72	34.5 ± 3.6	42.1 ± 3.5	44.8 ± 2.5	41.5 ± 2.9	44.9 ± 2.6	44.1±3.3	91.8±2.5

mm.

is larger than the latter. As to the dimension of the alveolar crest line ($\overset{\frown}{JK}$), there were differences between males and females.

Dimension of palatal vault height (Tables 3 and 4) and the length of cross-section line (Tables 5 and 6)

There were slight diferences from 50 to 70 years of age, similar to the results of palatal vault breadth and length. The palatal vault height which is the perpendicular distance between the line segment $\overline{DE}$ and point O, was the largest as compared with other portions. As was expected, the perpendicular distance between point A′ and O was the largest. Generally, the average of palatal vault height, ranged from 0.04 mm to 0.80 mm. As related to length of the cross-section line, FG was the largest in both males and females. In all of the measurement values, males were larger than females, and these results were proportional to the values of the palatal vault breadth. In the average of saqittal cross-section line lengths ($\overset{\frown}{AO_1}$) males were larger than females and this difference was about 2.8 mm.

Table 3. Dimension of palatal vault height (in males)

Portion	PP'	OO'	QQ'	AO_2
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
40 n = 5	11.3 ± 2.4	14.4 ± 2.1	14.2 ± 3.5	16.8 ± 3.9
50 n =15	9.7 ± 3.2	12.4 ± 3.9	11.9 ± 3.0	13.9 ± 2.3
60 n =20	9.7 ± 2.9	13.1 ± 2.5	12.9 ± 2.3	14.9 ± 2.4
70 n =21	8.2 ± 1.8	10.7 ± 2.5	10.2 ± 2.5	13.1 ± 2.9
80 n = 6	11.0 ± 3.5	12.7 ± 2.0	12.1 ± 1.6	13.0 ± 1.2
Total n=67	9.5 ± 2.4	12.3 ± 2.9	12.0 ± 2.7	14.1 ± 2.7

mm.

Table 4. Dimension of palatal vault height (in females)

Portion	PP'	OO'	QQ'	AO_2
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
30 n =3	9.8 ± 1.4	12.4 ± 2.1	12.3 ± 1.5	13.0 ± 1.9
40 n =10	11.2 ± 2.1	12.9 ± 3.2	12.6 ± 3.4	13.8 ± 3.6
50 n =24	10.0 ± 1.9	11.9 ± 2.3	11.8 ± 2.3	12.9 ± 2.2
60 n =25	9.7 ± 2.3	12.0 ± 2.8	11.6 ± 3.2	13.9 ± 2.1
70 n =10	11.7 ± 4.3	12.9 ± 3.5	12.1 ± 3.3	14.5 ± 3.3
Total n=72	10.3 ± 2.5	12.4 ± 2.7	12.1 ± 2.5	13.9 ± 2.5

mm.

Table 5. The length of cross-section line (in males)

Portion	$\overset{\frown}{BC}$	$\overset{\frown}{DE}$	$\overset{\frown}{FG}$	$\overset{\frown}{AO_1}$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
40 n = 5	45.1 ± 4.2	55.5 ± 4.2	58.2 ± 4.3	33.6 ± 4.6
50 n =15	45.2 ± 7.2	53.7 ± 4.7	56.4 ± 5.3	32.3 ± 6.1
60 n =20	45.3 ± 4.7	55.7 ± 4.6	57.6 ± 4.2	35.7 ± 6.5
70 n =21	43.3 ± 6.0	50.3 ± 6.1	54.9 ± 4.5	31.6 ± 3.3
80 n = 6	47.7 ± 5.2	57.8 ± 4.5	57.5 ± 3.8	33.8 ± 3.6
Total n=67	44.8 ± 5.8	53.7 ± 5.6	56.5 ± 4.6	33.3 ± 5.4

mm.

Table 6. The length of cross-section line (in females)

Portion	$\overset{\frown}{BC}$	$\overset{\frown}{DE}$	$\overset{\frown}{FG}$	$\overset{\frown}{AO_1}$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
30 n = 3	40.2 ± 2.1	42.7 ± 15.8	56.4 ± 4.6	33.0 ± 6.8
40 n =10	42.7 ± 3.5	51.1 ± 9.9	55.3 ± 4.8	31.8 ± 3.3
50 n =24	44.3 ± 0.1	52.6 ± 5.9	54.2 ± 6.0	30.4 ± 5.1
60 n =25	40.1 ± 6.4	49.6 ± 7.4	52.4 ± 6.7	30.9 ± 5.6
70 n =10	45.1 ± 6.8	51.2 ± 4.7	53.5 ± 4.8	28.1 ± 3.9
Total n =72	42.6 ± 6.1	50.7 ± 7.5	53.7 ± 5.9	30.5 ± 5.0

mm.

Angle of lateral and saqittal palatal wall (*Tables 7 and 8*)

In the mean value of the anterior portion (BC), right (β) and left (α) lateral inclination were nearly equal (about 40 degrees) in males. On the other hand, the left lateral inclination (α) was larger than the right lateral inclination (β) in males and the difference was approximately 4 degrees. In the middle portion (DE), right and left lateral inclination was about 50 degrees in males and 44 degrees in females. In the posterior portion (FG), the difference of right and left inclination was not so distinct in either males or females. In the saqittal palatal wall angle (γ), males were larger than females about 5 degrees. In the comparative evaluation from 50 to 70 years of age, there were not clear differences in each portion, and it can been seen that the angle of α is larger than the angle of β in both males and females. On the other hand, it was shown that the angle of γ was inclined to increase with age in males, but on the contrary, it was disposed to decrease in females.

Classifications of palatal vault sections (*Table 9*) *and alveolar crest line forms* (*Table 10*)

In each portion, the sectional shapes can be classified into Ovoid, V-shaped, Square types (Fig. 3). As related to the sections of anterior palatal vault, Ovoid appeared most often in males (about 40%), and V-shaped was most common in females (about40%). In the middle palatal vault, the V-shaped form was most common in males and Ovoid was most common in females. Ovoid appeared most often in both males and females (about 44%) in the posterior palatal vault. Alveolar

Table 7. Angle of lateral (α, β) and sagittal (γ) palatal wall (in males)

Portion	BC			DE			FG			AO_1
	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\gamma°$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
40 n=5	44.2 ± 3.6	40.9 ± 2.6	85.1 ± 4.7	46.0 ± 5.7	44.8 ± 9.8	90.8 ± 12.5	52.6 ± 11.8	59.5 ± 6.6	102.0 ± 17.2	45.3 ± 7.5
50 n=15	55.7 ± 9.6	45.4 ± 8.7	101.2 ± 15.2	51.9 ± 10.2	49.2 ± 11.4	101.2 ± 17.9	54.5 ± 9.1	51.0 ± 11.3	105.5 ± 18.1	43.5 ± 10.9
60 n=20	49.6 ± 11.8	48.7 ± 9.7	98.2 ± 17.9	47.4 ± 7.2	45.5 ± 8.8	93.0 ± 14.4	50.8 ± 8.7	45.3 ± 8.4	96.1 ± 15.8	48.3 ± 9.4
70 n=21	56.1 ± 14.1	51.6 ± 8.9	107.7 ± 12.9	53.1 ± 9.5	53.1 ± 9.8	106.2 ± 16.6	56.0 ± 9.6	51.4 ± 9.4	107.4 ± 17.8	49.6 ± 7.1
80 n=6	47.4 ± 8.1	49.3 ± 14.2	96.7 ± 20.0	49.0 ± 8.3	53.3 ± 9.5	102.4 ± 17.6	49.4 ± 7.0	53.2 ± 3.8	102.7 ± 9.4	52.5 ± 3.7
Total n=67	52.4 ± 9.9	48.3 ± 9.6	100.7 ± 16.1	50.2 ± 10.6	49.4 ± 10.6	99.6 ± 16.5	53.2 ± 9.2	49.5 ± 9.3	102.8 ± 16.9	47.1 ± 10.5

Table 8. Angle of lateral (α, β) and sagittal (γ) palatal wall (in females)

Portion	BC			DE			FG			AO_1
	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\alpha°$	$\beta°$	$\alpha°+\beta°$	$\gamma°$
Age	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s	$\bar{x}$ s
30 n=3	46.1 ± 6.0	42.0 ± 4.3	88.0 ± 9.5	45.1 ± 5.6	48.1 ± 14.3	93.3 ± 32.8	36.1 ± 8.3	36.3 ± 7.5	72.4 ± 10.1	51.1 ± 7.1
40 n=10	38.5 ± 9.9	37.2 ± 6.6	75.2 ± 14.5	46.1 ± 12.4	41.9 ± 10.4	88.1 ± 21.7	47.7 ± 8.6	39.6 ± 11.7	87.3 ± 19.4	49.7 ± 10.7
50 n=24	42.8 ± 8.1	41.4 ± 7.3	84.2 ± 13.6	43.0 ± 7.9	43.5 ± 10.2	86.5 ± 17.0	43.4 ± 11.1	41.4 ± 10.6	84.9 ± 20.1	46.1 ± 22.4
60 n=25	40.6 ± 7.8	40.0 ± 7.5	80.7 ± 12.7	45.0 ± 9.5	42.3 ± 10.2	87.3 ± 17.2	44.0 ± 9.9	42.7 ± 8.0	86.8 ± 16.2	42.1 ± 8.5
70 n=10	36.2 ± 11.9	38.6 ± 13.7	74.9 ± 25.2	44.9 ± 14.6	43.9 ± 14.1	88.8 ± 27.8	46.4 ± 10.6	44.2 ± 11.0	90.6 ± 20.9	41.7 ± 9.1
Total n=72	40.6 ± 8.9	39.9 ± 8.2	80.6 ± 17.9	44.5 ± 11.7	43.1 ± 10.6	87.6 ± 19.6	44.3 ± 10.2	41.8 ± 9.8	86.1 ± 18.3	43.8 ± 8.6

Table 9. Classifications of palatal vault sections

Portion		Ovoid	Square	V-Shaped	Another Type
Anterior Cross Section	Male	38.8	24.9	35.7	0.6
	Female	36.9	21.1	41.0	1.0
Middle Cross Section	Male	31.3	19.4	47.7	1.6
	Female	40.2	22.2	31.9	5.7
Posterior Cross Section	Male	56.7	19.8	22.3	1.2
	Female	43.8	36.9	18.3	1.0 %

Table 10. Classifications of alveolar crest line form

Sex \ Type	I	II	III	IV	V
Male	31.2	43.2	13.4	11.9	0.3
Female	28.6	43.9	8.2	19.1	0.2 %

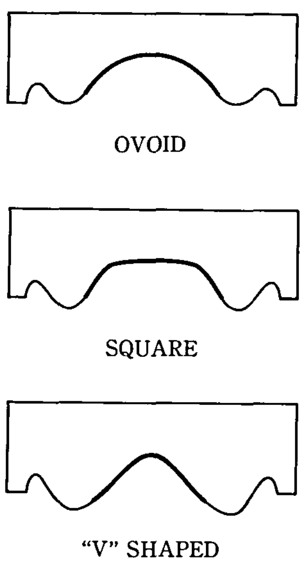

Fig. 3. Classifications of palatal vault sections

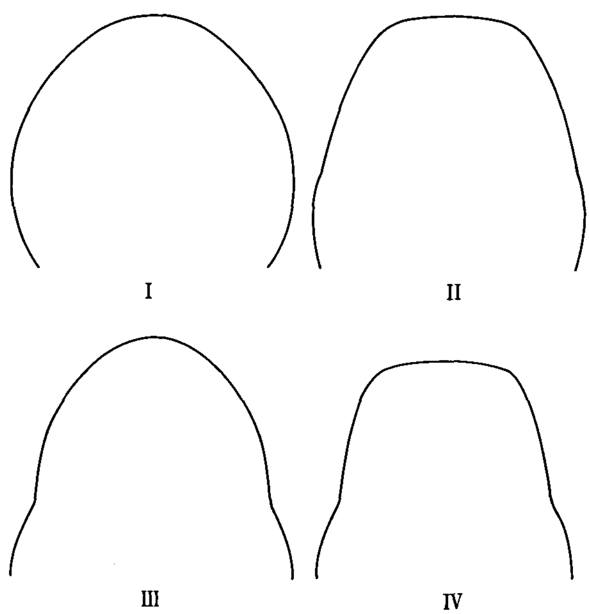

Fig. 4. Classifications of alveolar crest line form

crest line form can be divided into five types (Fig. 4). V type was very small in number and can not be classified into I to IV types. II type appeared most often in both males and females (about 43%).

Discussion

In prosthetic dentistry, it is very important to research the basic morphology of edentulous ridges and palates, because it is closely related to problems between biomechanical considerations and the impression-making and support available for complete dentures. The size of the maxillae and mandible is involved in the margin of error in impression-making. If the jaws are small, the impressions must be as accurate as possible because a small error would be relatively larger than the same error in an impression of a large mouth. Instead, impression requirements for small jaws are more critical than those for large ones. The study of edentulous ridges and palates has already been described by various researchers. The paucity of reports on lateral and saqittal inclination of a palatal wall encouraged us to investigate it.

Regarding the measurement result of palatal vault

We compared present measurement results and previously reported results. Even though the measured points in each portion of palatal vault varied by researchers, it was very difficult to compare them with already reported results. However, from Table 11, it can be seen that the value of palatal vault breadths which were measured by each researcher, and the length of the straight line between right and left upper tuberosity resemble each other very much, and also the differences in sex distinction showed a similar tendency. The length of the straight line connecting the right and left buccal frenum position which were measured by Kobayashi et al.[39] had similar value as that measured in our experiment that was obtained anterior palatal vault breadth. It was difficult to compare the results in another items, because their measuring points were different from this study.

The statistical analysis of these results (mean value) is shown in Table 12. The analysis of variance for the palatal vault breadth showed that the variation between the ages in males and females was significant at 1%. The results showed that no significant difference occured between the sexes, and between the ages. And as to the palatal vault height the variation between the ages was significant at 5%, but that the variation between the sexes and a measurement portion was insignificant.

Although not statistically significant, there was a slight difference between the sexes. A similar analysis for the cross-section length showed that the variation between the ages was 1%, and the variation between the sexes was 5%.

As to the inclination of lateral palatal wall, there were definite differences between BC and DE in male, and it was clear that the average values of males were larger than females in each portion. Generally, in the average of the lateral wall angle, males were more than 90 degrees and females were less than 90 degrees. In the comparative evaluation of 50 to 70 years of age, there were not clear differences in each portion, and it can been seen that the angle of α is larger than the angle of β in both males and females. On the other hand, it was shown that the angle of γ was inclined to increase in females. There were no clear differences between the mean values of expanded lateral wall angles in the anterior, middle and posterior portion in males and little difference was observed between the anterior and middle portion in females. A statistically significant difference was found between the ages and the sexes at 1%.

Classifications of palatal vault sections and alveolar crest line forms

Palatal vault vary considerably from patient to patient. The most favorable vault form is one that is a medium depth, with a well-defined incline of the rougae area in the anterior part of the palate. A flat palatal vault can present some difficulties if the bone in the anterior region is

Table 11. Comparative evaluation of the alveolar arch which had already reported results[39] ($\overline{CC'}$: dimension of between the highest portion of right and left upper tuberosity (palatal vault breadth), $\overline{BB'}$: dimension of between the right and left buccal frenum, $\overline{AD}$: perpendicular from incisive papilla (A) to $\overline{CC'}$ (palatal vault length)

	$\overline{CC'}$	$\overline{BB'}$	$\overline{AD}$	AC	$\overline{CC'}/\overline{AD}\times100$
Yoshiro Kondo & Giichiro Asano	4.83	——	4.03	4.63	——
	4.67	——	3.93	4.52	——
	4.75	——	3.97	4.58	——
Hisashi Sugisaki	4.780	——	4.175	4.856	114.8
	4.716	——	3.896	3.955	124.1
	4.748	——	4.035	4.385	119.4
Shunzo Kobayashi et al.	4.889	3.547	3.757	4.568	130.9
	4.696	3.543	3.625	4.371	130.8
	4.764	3.545	3.674	4.472	130.8

cm

Table 12. Statistical analysis of (a) palatal vault breadth and length, (b) palatal vault height, (c) cross-section line length and (d) lateral palatal wall inclination

a) Palatal vault breadth and length

S.V.	S.S.	D.F.	M.S.	*F*=ratio
Age(A)	224.7762	2	112.3881	113.2964**
Sex(B)	12.8044	1	12.8044	12.9079**
Portion(C)	40.6740	5	8.1348	8.2005**
A x B	244.5095	2	112.2548	123.2428**
A x C	10.3729	10	1.0373	1.0457
B x C	1.3135	5	0.2627	0.2648
e	9.9198	10	0.9920	

b) Palatal vault height

S.V.	S.S.	D.F.	M.S.	*F*=ratio
Age(A)	31.2325	2	15.6162	10.4105*
Sex(B)	4.5937	1	4.5937	3.0624
Portion(C)	9.3004	3	3.1001	2.0667
A x B	6.5341	2	3.2671	2.1780
A x C	4.8787	6	0.8131	0.5421
B x C	0.4354	3	0.1451	0.0967
e	9.0003	6	1.5000	

c) Cross-section line length

S.V.	S.S.	D.F.	M.S.	*F*=ratio
Age(A)	364.8865	2	182.4432	29.1368**
Sex(B)	58.3753	1	58.3753	7.7229*
Portion(C)	16.4950	3	5.4983	0.7274
A x B	138.5100	2	69.2550	9.1623*
A x C	30.4158	6	5.0693	0.6707
B x C	8.3102	3	2.7701	0.3665
e	45.3522	6	7.5587	

d) Lateral palatal wall inclination

S.V.	S.S.	D.F.	M.S.	*F*=ratio
Age(A)	74851.8721	2	37425.9360	502.0408**
Sex(B)	25822.7451	1	25822.7451	346.3927**
Portion(C)	607.1894	3	202.3965	2.7150
A x B	48940.5996	2	24470.2998	328.2506**
A x C	1800.8149	6	300.1358	4.0261
B x C	223.6428	3	74.5476	0.0000
e	38174.0767	17	0.0000	

$^{**}P<0.01$, $^{*}P<0.05$

severely resolved. The problem in this situation is one of in sufficient resistance to a forward movement of the maxillary dentures. Such a loss of stability will cause a loss of retention of the denture, especially during the masticatory function. Dentures in a mouth with a flat palatal vault are removed by a direct force or by rotating force. Therefore particular attention must be paid to balancing the occlusion. A high, narrow, V-shaped vault is also unfavorable for the retention of dentures. The tighter the denture presses, the sooner it will loosen and slip out of place. And also, it is very important to understand the relation between alveolar crest line form, the arch of artificial dentition and the condition of inter alveolar ridges. We have conducted a new method classify the cross-section of palatal vault and alveolar crest line form with Fourier analysis without using inspection which have already been discussed. Fourier transformation was divided into Fourier resolution and Fourier composition, which were based on the fact that was expressed with an

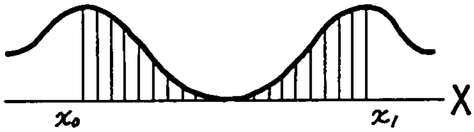

Fig. 5. Sectional shape of palatal vault was divided into twenty equal parts to substitute into equation (1)

OVOID	SQUARE	"V" SHAPED
15.00	20.00	17.00
11.74	20.00	15.21
8.98	20.00	13.42
6.64	15.00	11.63
4.69	10.00	9.84
3.11	5.00	8.05
1.87	.00	6.26
.95	.00	4.47
.34	.00	2.68
.04	.00	.89
.04	.00	.89
.34	.00	2.68
.95	.00	4.47
1.87	.00	6.26
3.11	5.00	8.05
4.69	10.00	9.84
6.64	15.00	11.63
8.98	20.00	13.42
11.74	20.00	15.21
15.00	20.00	17.00

P01	COS	SIN	P02	COS	SIN	P03	COS	SIN
0	9.6539	.0000	0	16.8421	.0000	0	17.0471	.0000
1	5.9982	.0000	1	11.5674	.0000	1	6.9056	.0000
2	1.6815	.0000	2	2.3033	.0000	2	.0484	-.0000
3	.8012	.0000	3	-1.5654	.0000	3	.7814	-.0000
4	.4856	.0000	4	-0.5557	-.0000	4	.0526	.0000
5	.3400	.0000	5	-0.0520	.0000	5	.2918	.0000
6	.2633	.0000	6	-0.5506	.0000	6	.0609	.0000
7	.2203	.0000	7	-0.1029	.0000	7	.1574	.0000
8	.1968	.0000	8	0.4294	.0000	8	.0756	.0000
9	.1863	.0000	9	0.1057	.0000	9	.1027	.0000

POWER (P0100)	POWER (P0200)	POWER (P0300)
58.96	67.13	81.47
16.53	13.37	.57
7.88	9.08	9.22
4.77	3.22	.62
3.34	.30	3.44
2.59	3.20	.72
2.17	.60	1.86
1.93	2.49	.89
1.83	.61	1.21

Fig. 6. Power spectrum of typical cross-section form

arbitrary function by superimposed sinusoidal waves.

Fourier resolution can be expressed by means of the following formula:

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n cos2\pi nf_1t+b_n sin2\pi nf_1t)\cdots\cdots\cdots(1)$$

$$a_n=\frac{1}{T_1}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)cos2\pi nf_1tdt(n=0,1,2,\cdots\cdots\cdots)$$

$$b_n=\frac{1}{T_1}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)sin2\pi nf_1tdt(n=1,2,3,\cdots\cdots\cdots)$$

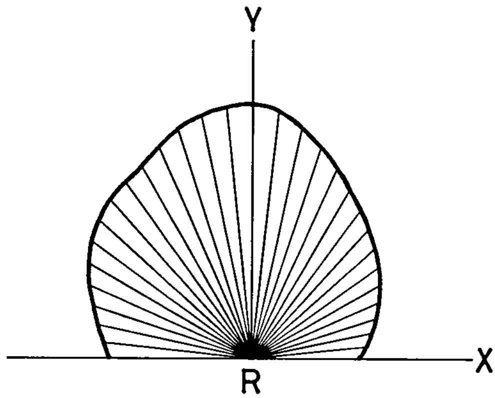

Fig. 7. Alveolar crest line was divided into thirty equal parts to substitute into equation (1).

where T_1 is a period time and $f\ (t)$ is periodical signal, and a_n, b_n is a modulus.

Firstly, the sectional shapes of palatal vaults were classified into three basic types, Ovoid, Square and V-shaped by means of inspection in order to distinguish them. To obtain the results of Fourier transformation and to substitute the mean value of middle palatal vault breadth and height for the formula (1) as before, these types were denoted by following equation:

Ovoid : $x^2+(y+b)^2=a^2$

Square : $x=\pm a, y=0, y=c(-a \leqq x \leqq a)$

V-shaped : $y=\pm ax+b(c \leqq x \leqq d)$

As shown in Fig. 5, the section of palatal vault in each portion was divided into twenty equal parts from x_0 to x_1, and the value of y with respect to x was measured by a universal projector with digital computer (Tokyo Kogaku Inc., Tokyo, Japan) in order to substitute it into equation (1). By using the value obtained from equation (1), and from the value for typical pattern of the sections, we can calculate least squares using the UNIVAC 1106 systems (Fig. 6). The alveolar crest line was divided into thirty equal parts with 6 degrees (Fig. 7) and was measured by slide caliper (1/20) from R to the alveolar crest line in each portion. The value of y with respect to x was obtained to substitute into equation (1). The classification was the same as described for the section of palatal vault. The reason for using Fourier transformation was that as the sectional shapes of palatal vault or alveolar crest line form can be resolved into sinusoidal waves, their differences of form of both palatal vault and alveolar crest line size were standardized simultaneously. One of the advantages of this classification was an objective method lacking the researcher's subjective point of view.

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