The Use of Finite Element Method in Analysing the Dynamic Characteristic of a Rotor System

Amir Zaki Mubarak
Department of Mechanical Engineering, Faculty of Engineering, Syiah Kuala University
Jl. Tgk. Syech Abdurrauf, No. 7, Darussalam, Banda Aceh 23111
E-mail: amir_zaki_mubarak@yahoo.com

Abstract

Vibration analysis is very essential to be considered in designing a rotor system. The failure in rotor system parts can be avoided by understanding the dynamic characteristic. The objective of this research is to investigate the dynamic characteristic of rotor systems by using finite element method. Finite element method is used to reveal the system characteristic which is derived from the kinetic and potential energy equations. Some models of rotor systems are analysed to understand the dynamic characteristic. The natural frequencies are obtained from the eigen values. The simulation is carried out with the distance of disk from the support as the variable. The result shows that the distance between the disk and the bearing affects the natural frequency of the system.

Keywords: dynamic characteristic, natural frequency, finite element method.

Nomenclatures

u = translation displacement vector in X direction
w = translation displacement vector in Y direction
θ = rotation displacement vector in X direction
ψ = rotation displacement vector in Y direction
T_D = kinetic energy of disk
m_d = mass of disk
I_{dx} = disk inertia moment in X direction
I_{dy} = disk inertia moment in Y direction
Ω = angular velocity
L = length of shaft
T_S = kinetic energy of shaft
ρ = density
S = area of shaft cross-section
I = inertia moment of area of shaft cross-section
U_S = strain energy of shaft
E = Modulus of elasticity
F_0 = axial force

1. Introduction

The failure of a rotor system is mostly found at the shaft which is caused by vibration. Vibration analysis is very important to be considered in designing a rotor system. A lot of studies have been conducted related the application of rotor systems [1], [2].

The failure of the system can be avoided by knowing the dynamic characteristic. The dynamic characteristic analysis involves carefulness and accuracy to get the best result which requires more time. The use of computer which is facilitated by many programming software lets the analysis to be simple and shortens the span of time in analysing it.

A computer program to analyse the dynamic characteristic of a rotor system by using finite element method has been developed [3], [4], [5]. The objective of this research is to investigate further about the dynamic characteristic of rotor systems by using the computer program. The focus is to study the effect of disk position relative to the bearing to system natural frequency.

2. Input Parameter of the Plant

There are several cases of rotor system to be analysed with a variation of disk position. In addition, the other variables are maintained to be similar to get equivalent comparison.

The data of the shaft is:

1. Density (ρ_p) = 7800 kg/m^3
2. Modulus elasticity (E_p) = 2×10^{11} N/m^2
3. Length (L_p) = 0.4 m
4. Cross-sectional diameter (d_p) = 0.02 m
5. Poisson's ratio (v) = 0.3

The data of the disk is:

1. Inner diameter (d_1) = 0.02 m
2. Outer diameter (d_2) = 0.3 m
3. Density (ρ_d) = 7800 kg/m^3

3. Methodology

The equation of motion is derived based on kinetic and potential energy in the system. The system is modelled as a flexible shaft with a disk attached on it, see figure 2.

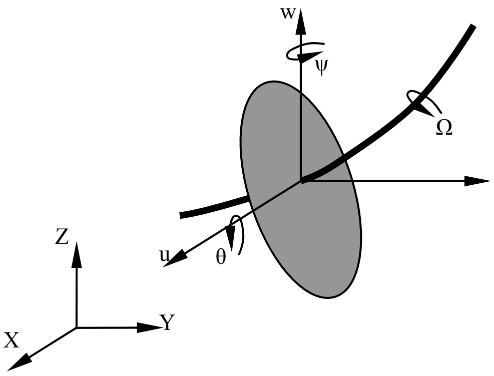

Figure 1. Rotor model [5].

The kinetic energy, the potential energy of the shaft and the kinetic energy of disk have been briefly derived [6], [4]. Therefore, the equation of kinetic energy of the disk is:

$$T_D = \frac{1}{2} m_d (\dot{u}^2 + \dot{w}^2) + \frac{1}{2} I_{dx} (\dot{\theta}^2 + \dot{\psi}^2) + \frac{1}{2} I_{dy} \Omega^2 \qquad (1)$$

The equation of kinetic energy of the rotor shaft is:

$$T_s = \frac{\rho S}{2} \int_0^L (\dot{u}^2 + \dot{w}^2) dy + \frac{\rho I}{2} \int_0^L (\dot{\psi}^2 + \dot{\theta}^2) dy + \rho I L \Omega^2 \qquad (2)$$

The potential energy equation of the shaft is:

$$U_s = \frac{EI}{2} \int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2} \right)^2 + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right] dy + \frac{F_0}{2} \int_0^L \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] dy \qquad (3)$$

The equation of motion is derived from the energy equations. By applying Lagrange's equation, the inertia and stiffness matrices are obtained that the dynamic characteristic can be resolved directly.

The analysis is conducted in three cases. In case 1, the analysed system is a shaft with a disk attached to it. In this case the system is analysed by varying the disk position, which are 0.1 m, 0.13 m, 0.2 m, 0.27 m, and 0.3 m from the edge of the shaft. The thickness of the disk is also varied, which are 0.03 m and 0.015 m. Figure 2 shows the finite element model for case 1 analysis with the disk position of 0.13 m from the edge.

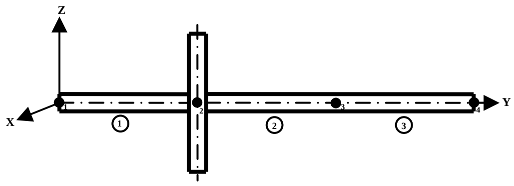

Figure 2. Monorotor finite element model.

In case 2, the analysed systems are dual rotor systems. The disks are located closed to the bearing. The analysis is carried out for two position of the disks which at 0.1 m and 0.13 m from the edges. The result of this analysis is compared with the result of case 1 which the disk is located at 0.2 m from the edge. Figure 3 shows the sample of dual rotor model in which the disks are located at 0.13 m and 0.27 m from the edge.

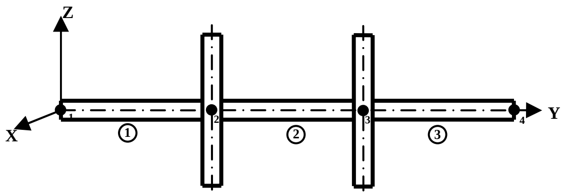

Figure 3. Dual rotor finite element model.

In case 3, there are three disks located 0.1 m, 0.2 m, and 0.3 m from the edge. Figure 4 shows the model.

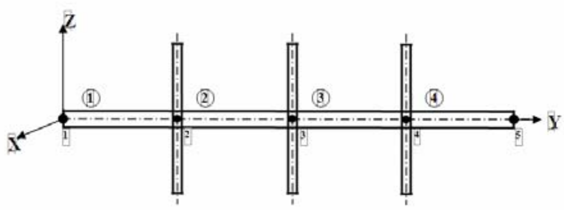

Figure 4. Finite element model of a rotor system with four disks.

4. Results and Discussions

This piece of writing explores the dynamic characteristic of rotor systems by using computer program. The program has been validated in previous papers [3], [4] and [5]. The program can be used to investigate the dynamic characteristic of rotor systems.

In case 1, there are ten data of natural frequency extracted from the program. There are five natural frequencies with the thickness of disk of 0.03 m and five of thickness of 0.015 m. The result of case one is summarised in table 1. The result shows that the natural frequencies of rotor systems with less disk thickness (0.015 m) are higher than the frequencies of thicker disk rotor systems. This result goes well with the statement of mubarak [5] that the bigger the mass of disk the lower the natural frequency.

Table 1. Natural Frequency result in case 1

No	Thickness of Disk (m)	Disk Position (m)	Natural Frequency (Hz)
1	0.03	0.1	55.2246
2	0.03	0.13	46.8969
3	0.03	0.2	41.8487
4	0.03	0.27	46.8969
5	0.03	0.3	55.2246
6	0.015	0.1	76.4186
7	0.015	0.13	65.2289
8	0.015	0.2	58.3678
9	0.015	0.27	65.2289
10	0.015	0.3	76.4186

The trend of the effect of disc position can be clearly seen in figure 5. The natural frequencies of the rotor system with disk thickness of 0.03 m are slightly higher than the one of thickness of 0.015 m for the corresponding disc position. Additionally both line graphs show similar trend that the natural frequency is more likely to be lower when the disk is located in the middle of the shaft while the shaft is simply supported at both its edge. Thus in designing a rotor system, it is advisable to consider the disc position relative the bearing to maintain the natural frequencies not to be closed to the excitation frequencies.

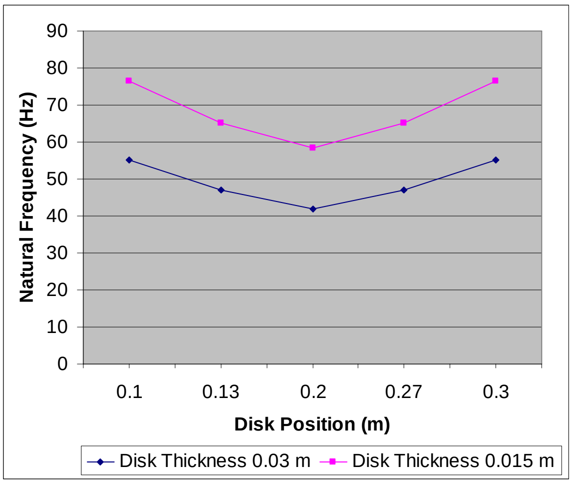

Figure 5. Natural Frequency relative to the disc position

The result of case 2 is summarised in table 2. Table 2 shows the natural frequencies of dual rotor systems. One of the rotor systems has both discs with thickness of 0.015 m and another is with thickness of 0.03 m. The table shows similar trends with case 1 in which the natural frequencies are lower for more disk thickness and the natural frequencies are higher when the discs are closed to the supports.

Table 2. Natural Frequency result in case 2

No.	Thickness of Disk (m)	Position of Disk 1 (m)	Position of Disk 2 (m)	Natural Frequency (Hz)
1	0.015	0.1	0.3	58.2803
2	0.015	0.13	0.27	48.3392
3	0.03	0.1	0.3	41.7838
4	0.03	0.13	0.27	34.5098

Comparing table 2 with table 1 we can see that the natural frequency of a rotor system with two discs of 0.03 m thickness located closed to the supports which is 41.7838 Hz is almost similar to the natural frequency of a rotor system with one disc of similar thickness located in the middle of the shaft which is 41.8487 Hz. The result number 4 in table 2 also goes well with the statement that the natural frequency becomes lower when both discs position are rather far from the supports. Considering the result number 1 and 2 in table 2, it is obvious that the natural frequencies are higher when having two discs with half of the thickness located closed to the supports.

In case 3 the system is a multi rotor system consisting three discs with similar thickness of 0.01 m. The result shows that the natural frequency is 51.1977 which is higher than the result number three in table 1 and 2, but lower than the result number one in table 2. This means that there are correlations among the thickness of disk (mass of disk), number of disk and the distance from the support and the natural frequency.

Thus, it is reasonable to consider the best position of the disk, the thickness or the mass of the disk in designing a rotor system. If it is necessary, splitting the disk into two or three pieces can be an option to get the best result.

5. Conclusion

There are correlations among the disk thickness which is related to the mass, number of disk, the distance relative to the support with the natural frequency.

The bigger is the mass of the disk, the lower the natural frequency.

The natural frequency of a rotor system is higher the closer the disk to the support.

In designing a rotor system, it is considerable to have two or more discs close to the support rather than one which is far from the support. The decision is taken based on the dynamic analysis of the system.

References

[1] Thelen, RF, dkk, 2007, Testing of A 3 MW High Speed Generator and Turbine Drive for A Hybrid Vehicle Propulsion System, *Proceedings of GT2007, ASME Turbo Expo 2007: Power for Land, Sea and Air*, Montreal, Canada, 14-17 Mei.

[2] Murphy, BT, dkk, 2001, Rotordynamics design and test results for a model scale compulsator rotor, *IEEE Transactions on Magnetics*, Vol 37, no 1.

[3] Mubarak, AZ, Balia, FN dan Akhyar, 2007, Aplikasi Metode Elemen Hingga untuk Menentukan Frekuensi Pribadi dan Modus Getar Poros Rotor, *Prosiding SNTTM VI*, Banda Aceh.

[4] Mubarak, AZ, Juni 2012, Pengembangan Algoritma untuk Menganalisis Frekuensi Pribadi Poros dengan Rotor Ganda, *Jurnal Teknik Mesin Unsyiah*, Vol. 1, No. 1.

[5] Mubarak, AZ, 2012, Algorithm Development to Predict the Dynamic Characteristic of a Multi Rotor System, *Proceeding of the 2nd AIC,* Banda Aceh.

[6] Lelanne. M, dan Ferraris. G, 1990, *Rotordynamics Prediction in Engineering*, John Wiley and Sons, England,.