Estimation of Distances to the Red Giant Stars

Manabu Yuasa, Wasaburo Unno
RIST, Kinki University, Higashi-Osaka-shi, Osaka-577, Japan
Takayuki Ichino
Faculty of Science and Technology,
Kinki University, Higashi-Osaka-shi, Osaka-577, Japan

(Received January 18, 1995)

Abstract

Based on the Principal Component Analysis and Oort's Galactic rotation model, the distances to the red giant stars are estimated. The 157 red giant stars are devided into two groups, then the distances are estimated as $\log d(\mathrm{Kpc}) = 0.471z_2 + 0.241$ for Group 1 and $\log d(\mathrm{Kpc}) = 0.446z_2 + 0.532$ for Group 2, where z_2 is the principal component corresponding to the distance.

Key words: Stellar distance; red giant star; principal component analysis.

1 Introduction

The distances to the red giant stars are, so far, determined mostly on more or less hypothetical absolute magnitudes of each star. The absolute magnitude effect of spectral lines depends not only on the absolute magnitude but also on turbulence, chemical composition and others, even if the effective temperature could be determined very accurately. We have proposed a new method of distance determination on the basis of the double principal component analysis (Unno et al. 1989, 1990). In the present analysis we use 157 red giant stars which are observed by IRAS at the wave length 12μm, 25μm, 60μm and 100μm and are also observed by radio telescopes for identifying the circumstellar gas expanding velocity Ve and the stellar radial velocity V from the emission lines of CO and HCN molecules. The distance d is represented as $\log d = C_0 + C_1 * z_2$, where z_2 is the pricipal component corresponding to the distance. The scaling constant C_1 is determined from the principal component analysis leading to the z_2-vector as shown below, reflecting the fact that the observed IR brightness is inversely proportional to the square of the distance. The zero point distance C_0 was hitherto estimated from a crude theoretical model of a radiation driven expanding molecular shell, which is by no means accurate. The purpose of the present paper is to calibrate C_0 by using the Galactic rotation model.

2 Principal Component Analysis

We adopt three colors

$$Q_1(I) = \log(F_{12}(I)/F_{25}(I)),$$
$$Q_2(I) = \log(F_{25}(I)/F_{60}(I))$$

and

$$Q_3(I) = \log(F_{60}(I)/F_{100}(I)) \quad (I = 1, \ldots, 157)$$

as the variables for the preliminary principal component analysis, where $F_{12}(I)$, $F_{25}(I)$, $F_{60}(I)$ and $F_{100}(I)$ are the IRAS radiative fluxes of 157 stars at the wave length 12μm, 25μm, 60μm and 100μm.

Then normalized variables $q_1(I)$, $q_2(I)$ and $q_3(I)$ can be calculated such that the mean value is 0 and the standard deviation is 1. After the principal component analysis with the normalized variables $q_1(I)$, $q_2(I)$ and $q_3(I)$, we get eigen values and eigen vectors as follows:

eigen value	eigen vector		
1.528	(0.667,	0.715,	0.208)
1.029	(−0.355,	0.060,	0.933)
0.443	(−0.655,	0.696,	−0.294).

From these values we get three principal components z_1, z_2 and z_3 as follows:

principal component

$$z_1 = 0.667q_1 + 0.715q_2 + 0.208q_3$$
$$z_2 = -0.355q_1 + 0.060q_2 + 0.933q_3$$
$$z_3 = -0.655q_1 + 0.696q_2 - 0.294q_3 \quad .$$

Investigating the distribution of the first principal component z_1, we devide the 157 stars into two groups. The stars which have concentrated negative z_1 values are classified as Group 1 and the stars which have positive z_1 values with no concentration are classified as Group 2. The z_1 value is considered to indicate the evolutionary stage or the size of the shell and, therefore, the classification brings the increase of the accuracy of the distance determination. After the classification, we apply the first principal component analysis to the variables $q_1(I)$, $q_2(I)$ and $q_3(I)$ in the two groups respectively. The results are as follows:

(A-1) Group 1

eigen value	eigen vector		
1.881	(−0.624,	0.624,	0.469)
0.751	(0.333,	−0.331,	0.883)
0.368	(0.707,	0.707,	−0.001)

principal component

$$z_1 = -0.624q_1 + 0.624q_2 + 0.469q_3$$
$$z_2 = 0.333q_1 - 0.331q_2 + 0.883q_3$$
$$z_3 = 0.707q_1 + 0.707q_2 - 0.001q_3$$

(A-2) Group 2

eigen value	eigen vector		
1.706	(0.663,	0.599,	−0.449)
0.859	(0.119,	0.508,	0.853)
0.435	(0.739,	−0.619,	0.265)

principal component

$$z_1 = 0.663q_1 + 0.599q_2 - 0.449q_3$$
$$z_2 = 0.119q_1 + 0.508q_2 + 0.853q_3$$
$$z_3 = 0.739q_1 - 0.619q_2 + 0.265q_3 \quad .$$

Next we introduce the fourth variable

$$Q_4(I) = -\log(F_{12}(I)) + 4\log(Ve(I)),$$

where Ve(I) indicates the expanding velocity of circumstellar gas. The Ve data will be presented in a paper under preparation (Unno et al. 1995). In this case also, the normalized fourth variable $q_4(I)$ can be calculated such that the mean value is zero and the standard deviation is 1. The standard deviations of $Q_4(I)$ are 0.878 for Group 1 and 0.795 for Group 2, which are used later to calculate the scale of the distances C_1 for the two groups respectively. Then the second principal component analysis is performed with the four variables $q_1(I)$, $q_2(I)$, $q_3(I)$ and $q_4(I)$ for the two groups. The results are as follows:

(B-1) Group 1

eigen value	eigen vector			
1.885	(−0.615,	0.628,	0.472,	0.062)
1.078	(0.309,	0.088,	0.162,	0.933)
0.736	(0.240,	−0.396,	0.865,	−0.192)
0.301	(0.684,	0.664,	0.048,	−0.297)

principal component

$$z_1 = -0.615q_1 + 0.628q_2 + 0.472q_3 + 0.062q_4$$
$$z_2 = 0.309q_1 + 0.088q_2 + 0.162q_3 + 0.933q_4$$
$$z_3 = 0.240q_1 - 0.396q_2 + 0.865q_3 - 0.192q_4$$
$$z_4 = 0.684q_1 + 0.664q_2 + 0.048q_3 - 0.297q_4$$

(B-2) Group 2

eigen value	eigen vector			
1.716	(0.662,	0.599,	−0.436,	0.114)
1.034	(0.011,	0.133,	0.433,	0.892)
0.817	(0.111,	0.500,	0.741,	−0.435)
0.434	(0.741,	−0.611,	0.272,	−0.050)

principal component

$$z_1 = 0.662q_1 + 0.599q_2 - 0.436q_3 + 0.114q_4$$
$$z_2 = 0.011q_1 + 0.133q_2 + 0.433q_3 + 0.892q_4$$
$$z_3 = 0.111q_1 + 0.500q_2 + 0.741q_3 - 0.435q_4$$
$$z_4 = 0.741q_1 - 0.611q_2 + 0.272q_3 - 0.050q_4$$

The variables $q_1(I)$, $q_2(I)$ and $q_3(I)$ are colors of three kinds, so they do not intrincically depend on the distance. On the other hand the fourth variable $q_4(I)$ must depend on the distance very strongly. If we compare the foregoing results (A-1) and (A-2) with the results (B-1) and (B-2), the first, the second and the third principal components in (A-1) and (A-2) are similar to the first, the third and the fourth ones in (B-1) and (B-2) respectively. Moreover, in the results of the four variables analysis (B-1) and (B-2), we can find that the second eigen values are 1.078 and 1.034 respectively, and that the fourth components of the second eigen vectors are 0.933 and 0.892 respectively. These four values are all approximately 1. Consequently, we can conclude that the pricipal components z_1, z_2 and z_3 in the first principal component analysis(PCA) (A-1) and (A-2) are identified with the principal components z_1, z_3, and z_4 in the second PCA (B-1) and (B-2), and the second principal components z_2 in (B-1) and (B-2) does represent the distance without any ambiguities.

3 Scale of the Distance C_1

The principal components corresponding to the distance are $z_2 = 0.309q_1 + 0.088q_2 + 0.162q_3 + 0.933q_4$ for Group 1 and $z_2 = 0.011q_1 + 0.133q_2 + 0.433q_3 + 0.892q_4$ for Group 2. Furthermore, the standard deviations of $Q_4(I)$ are 0.878 for Group 1 and 0.795 for Group 2. On the other hand, Unno et al. showed under some assumptions the distances were propotinal to the quantity $(F_{\rm bol})^{-1/2}(Ve)^2$, where $F_{\rm bol}$ indicates bolometric fluxes (Unno et al. 1989). Then the distance d must be expressed as follows:

$$\log d(\mathrm{Kpc}) = C_0 + C_1 z_2,$$

where C_0 is a constant to be determined in the present paper and C_1 is another constant which equals to (1/2)*(standard deviation of $Q_4(I)$)/(fourth component of the eigen vector corresponding to the distance). Therefore, from the foregoing analysis, the scale of the distance C_1 is given as follows:

$$C_1 = (1/2) * (0.878/0.933) = 0.471(\mathrm{Group1})$$
$$C_1 = (1/2) * (0.795/0.892) = 0.446(\mathrm{Group2}).$$

4 Zero Point Distance C_0

To determine the zero point distance C_0, we introduce Oort's Galactic rotation model. According to the model, the radial velocity V of the observed star at the Galactic longitude l is written as follows:

$$V = 2A(R_0 - R)\sin l,$$

where A is the Oort's constant, R_0 is the distance of the Sun from the Galactic center and R is the distance of the observed star from the Galactic center. The values, A = 15 km/sec/Kpc and R_0 = 8.59 Kpc are adopted. On the other hand R can be expressed as follows:

$$R = (R_0^2 + d^2 - 2R_0 d\cos l)^{1/2}.$$

Then the distance d of the observed star from the Sun is given by the following formula:

$$d = R_0 \cos l$$
$$\pm\, (R_0^2 \cos^2 l + V^2/(4A^2 \sin^2 l) - R_0 V/(A \sin l))^{1/2}.$$

At this stage we have to exclude some stars from the calculated d by the following reasons,

1. No observed radial velocity (number of stars = 5)

2. Calculated d is imaginary (number of stars = 10)

3. Two calculated d are both negative (number of stars = 14)

4. Caluculated d has poor accuracy due to $|l| \sim 0$ or $|l - \pi| \sim 0$ (number of stars = 14).

Moreover we impose the following two assumptions:

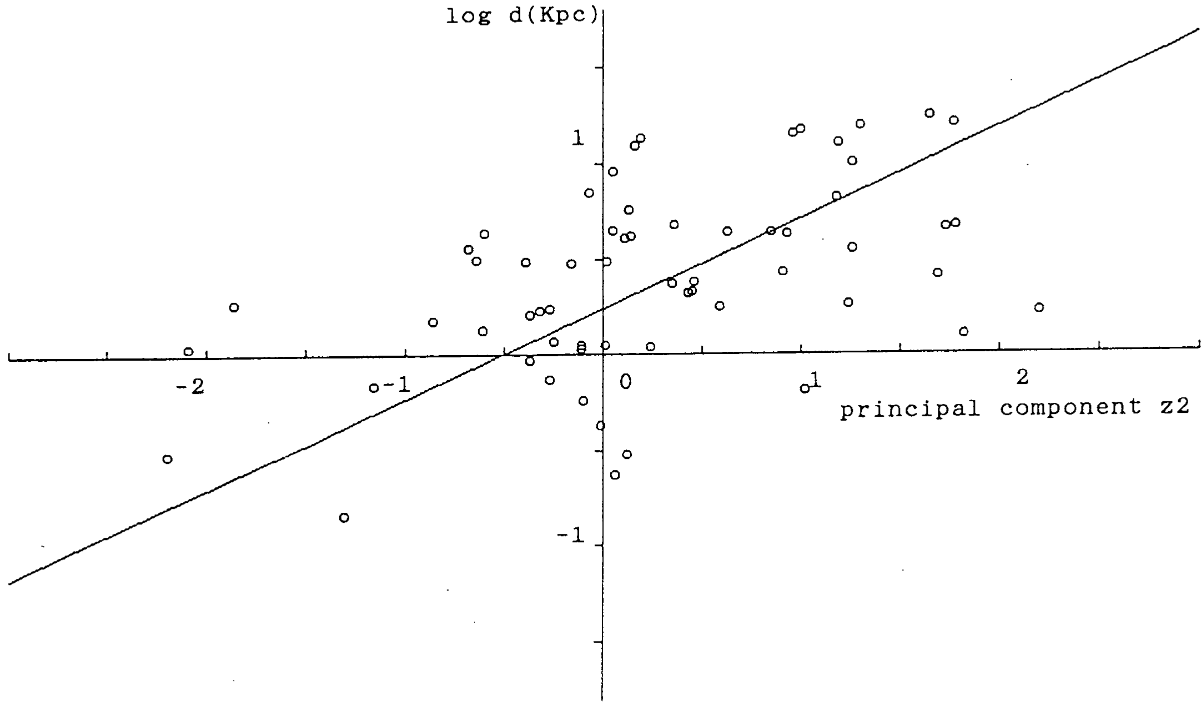

Figure. 1: The correlation between the principal components corresponding to the distance and the distances which are determined from the Galactic rotation model (Group 1).

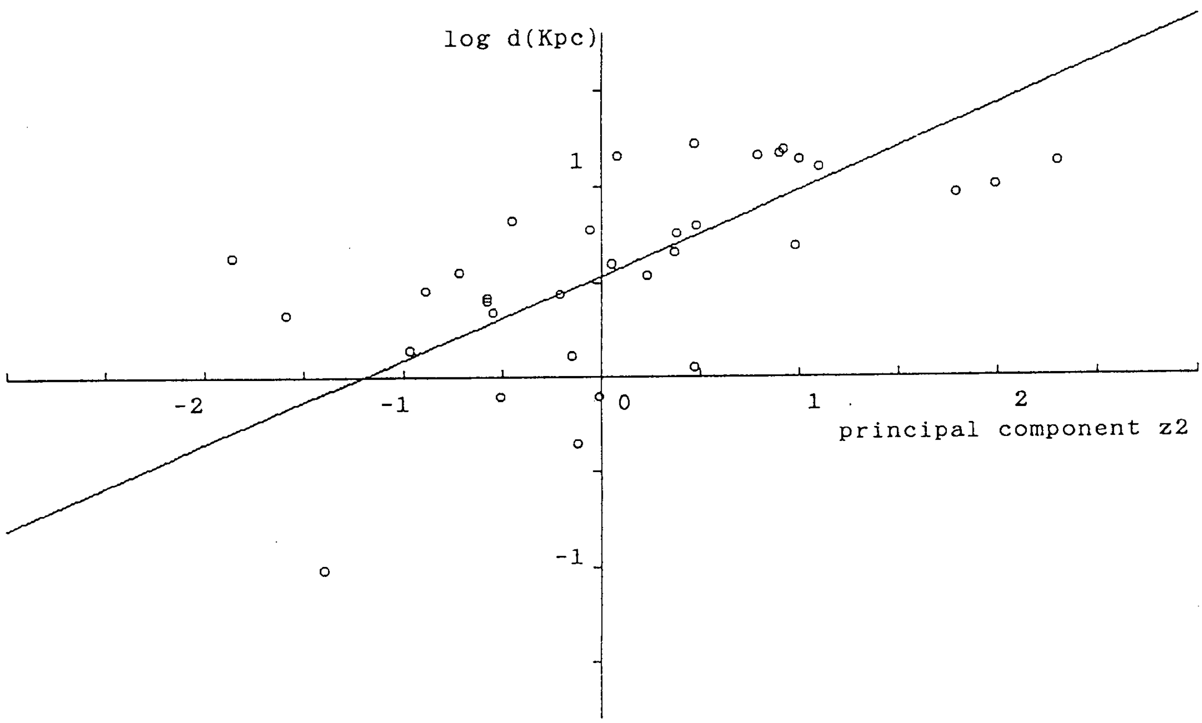

Figure. 2: The correlation between the principal components corresponding to the distance and the distances which are determined from the Galactic rotation model (Group 2).

Table 1 (Group 1)

STAR NAME	ALPHA & DELTA	DISTANCE (Kpc)	STAR NAME	ALPHA &DELTA	DISTANCE (Kpc)
T Cas	00205+5530	0.182	SW Vir	13114−0232	0.423
GL168	01085+3022	3.295	R Hya	13261−2301	0.160
Z Psc	01133+2530	0.357	W Hya	13462−2807	0.125
S Cas	01159+7220	2.543	X TrA	15094−6953	0.817
GL278	01556+4511	52.257	V CrA	15477+3943	0.505
W Ant	02168−0312	0.233	ST Her	15492+4837	2.393
R For	02270−2619	2.246	X Her	16011+4722	0.657
GL349	02316+6455	2.608	+40283	16269+4159	0.371
GL357	02351−2711	1.165	GL1922	17049−2440	3.123
TW Hor	03112−5730	0.419	TW Oph	17267−1926	1.105
+50096	03229+4710	1.541	MW Her	17334+1537	10.379
V Eri	04020−1551	1.556	T Dra	17556+5813	1.835
GL595	04307+6210	1.778	GL2135	18194−2708	2.932
ST Cam	04459+6804	1.539	IRC10365	18349+1023	1.715
TX Cam	04566+5606	0.937	+20370	18397+1738	1.490
R Lep	04573−1452	2.772	+00365	18398−0220	11.815
W Ori	05028+0106	1.299	+10374	18413+1354	4.946
32SSS	05104+2055	10.725	GL2259	18475+0926	6.343
****	05136+4712	4.782	−30398	18560−2954	1.252
GL724	05151+6312	3.715	GL5552	18595−3947	1.528
S Aur	05238+3406	3.480	V Aql	19017−0545	0.872
W Pic	05418−4628	1.144	−20540	19059−2219	7.129
Y Tau	05426+2004	1.612	−10502	19175−0807	5.132
TU Gem	06077+2601	2.325	UX Dra	19233+7627	0.950
GL935	06230−0930	4.652	+30374	19321+2757	6.815
GL954	06291+0319	6.103	R Cyg	19354+5005	2.001
UU Aur	06331+3829	1.347	GY Aql	19474−0744	1.323
GL971	06342+0328	0.897	+30395	19486+3247	0.108
CL Mon	06529+0626	6.840	RR Aql	19550−0201	0.497
R Vol	07065−7256	3.432	RT Cap	20141−2128	1.450
****	07217−1246	10.902	V Cyg	20396+4757	0.808
VY CMa	07209−2540	2.862	*****	20435+3825	11.376
Y Lyn	07245+4605	0.524	GL2686	20570+2714	6.266
39HCI	07582−1933	6.655	*****	21032−0024	2.076
GL1235	08088−3243	1.411	T Ind	21168−4514	0.724
X Cnc	08525+1725	1.980	Y PAV	21197−6956	2.132
GL5254	09116−2439	1.168	+40485	21320+3850	1.964
IW Hya	09429−2148	2.024	S Cep	21358+7823	4.757
IRC10216	09452+1330	19.823	*****	21373+4540	4.388
X Vel	09533−4120	0.968	U Cep	21419+5832	1.728
CIT 6	10131+3049	0.682	EP Aqr	21439−0226	0.483

Table 1 (Group 1)—Continued—

U Hya	10350−1307	1.202	PQ Cep	21440+7324	5.269
VY Hya	10416+6740	1.855	PW Peg	22017+2806	1.085
V Hya	10491−2059	1.757	SV Peg	22035+3502	1.530
R Crt	10580−1803	0.829	CV Cep	22097+5647	0.904
****	11308−1020	2.569	GL2901	22241+6005	18.972
GL4136	11461−3542	0.751	GL2999	22556+5833	12.011
SS Vir	12226+0102	1.109	GL3011	23585+6402	12.448
Y CVn	12427+4542	0.410	*****	23279+5336	1.458
RU Vir	12447+0425	1.840	+40540	23320+4316	2.825
RY Dra	12544+6615	1.302	TX Psc	23438+0312	1.236
RT Vir	13001+0527	0.582			

Table 2 (Group 2)

STAR NAME	ALPHA & DELTA	DISTANCE (Kpc)	STAR NAME	ALPHA &DELTA	DISTANCE (Kpc)
GL190	01144+6658	2.746	*****	18424+0346	8.537
R Scl	01246−3248	2.923	S Sct	18476−0758	8.767
****	02152+2822	0.806	+10401	19008+0726	2.845
GL341	02293+5748	1.206	R Aql	19039+0809	0.665
U Cam	03374+6229	3.865	*****	19068+0544	21.361
GL5102	03448+4432	1.257	*****	19075+0921	8.640
IRC50096	04530+4427	3.396	GL2343	19114+0002	24.365
GL807	05405+3240	4.196	+10414	19146+0959	36.081
****	06088+1909	10.545	GL2362	19161+2343	2.997
GX Mon	06500+0829	1.368	−20554	19162−1600	3.285
W CMa	07057−1150	5.487	AQ Sgr	19314−1629	3.371
13SAO	07134+1005	2.148	*****	19346+1209	4.220
****	08074−3615	1.878	GL2494	19594+4047	1.790
GL5250	08171−2134	2.007	*****	20028+3910	1.917
U Ant	10329−3918	5.014	GL2513	20072+3116	5.578
****	16105−4205	0.801	*****	20532+5554	1.941
NGC6302	17103−3702	9.269	*****	21147+5110	3.189
GL68155	17150−3224	9.592	*****	21223+5114	4.301
****	17217−3916	7.655	*****	21377+5042	0.106
****	17371−3021	2.725	V460Cyg	21399+3516	1.750
GL5379	17411−3154	1.193	RV Cyg	21412+3747	2.874
GL5416	17534−3030	7.673	*****	21449+4950	4.976
****	17581−1744	9.468	*****	21489+5301	3.579
GL2154	18239−0655	5.503	*****	21554+6204	1.886
****	18248−0839	3.697	*****	22272+5435	1.619
****	18269−1257	26.155	*****	22303+5950	9.270
GL5502	18308−0503	2.046	GL3068	23166+1655	0.503

1. In the case of two positive values of d, we adopt the large solution for the positive z_2 stars and adopt the small solution for the negative z_2 stars.

2. In the case of one positive and one negative value of d, if the absolute values of negative d are small, their stars are excluded. The reason is they have possibly small positive d due to the peculiar motion.

Under the above excludings and the assumptions, we construct the map whose horizontal axis is z_2 and the vertical axis is $\log d(\text{Kpc})$. In Figure 1, 61 stars are plotted belonging to Group 1 and in Figure 2, 33 stars are plotted belonging to Group 2.

Since each star has the peculiar motion, the plotted points distribute with rather large deviations. But, if the plotted number of stars is sufficiently large, the peculiar motions are expected to be cancelled each other. Then we approximate these plotted points by a straight line with the inclination C_1 (Group 1 : 0.471, Group 2 : 0.446) by the least square method. Consequently the zero point distance C_0 is determined as follows:

$$C_0 = 0.241(\text{Group1})$$
$$C_0 = 0.532(\text{Group2}).$$

5 Estimation of Distances

Since the zero point distance C_0 and the scale of the distance C_1 have been determined, we can compute the distances to the 157 red giant stars by the formula $\log d(\text{Kpc}) = C_0 + C_1 z_2$. The results are shown in Table 1 and Table 2.

6 Discussion

In Table 1 and Table 2, several stars have extremely large distances. Their observed data may have poor accuracies. We had better exclude them or at least re-analyse the whole system with small weights on them. With the increase of the number of data, our new method will give more improved distances with higher accuracies. The zero point C_0 will not be altered, however, with the improved data set.

References

[1] Unno,W., Yuasa,M., Hayashi,H., and Yamanaka,T., 1990, Science and Technology, Kinki University, 2, 77

[2] Unno,W., Koyama,K., Tsuji,T., and Izumiura,H., 1989, Publ. Astron. Soc. Japan, 33, 234

[3] Unno,W., Yuasa,M., Tsuji,T., Izumiura,H., 1995, in preparation