

Morphology of Distant Galaxies

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Abstract

Binarized CCD images obtained from the original monocolor print in Tyson-Seitzer (Astrophys.J.335,552,1988) are image processed with personal image analyzing system(PIAS). Items of image processing for each galaxy are contour line length, area, lengths of major and minor axes of the approximating ellipse, the maximum diameter length, circularity, two factors concerning roundness, the ratio of contour line length to area, the ratio of major axis to minor axis, slant angle of major axis of the approximating ellipse, slant angle of maximum diameter, slant angle of major axis as rectangle enveloping the galaxy, the ratio of occupation area to the rectangle. These 14 factors are calculated for each galaxy at one threshold intensity in each binarized image.

The average value of each factor in each binarized image is compared with that of different threshold intensity to get the statistical information of a distant galaxy.

CCD images of distant galaxies show the tendency of alignment of maximum diameters towards east-west direction, which may be spurious due to some asymmetric device in the CCD observation or in printing process of the journal. The principal component analysis made with these 14 factors reveals that the complexity of shape is the first principal component correlating with the distance. And the average of circularity of low threshold image is larger than that of high threshold image. This means that a galaxy is the more distant, the shape of galaxy is the more complicated.

Key Words: Distant galaxies, Image analysis, Principal component analysis

1 Introduction

In previous report(Ohara et al 1990) we classified galaxies. the peculiarity of distant galaxies in 11 kinds using the binarization of image input into Personal Image Analyzing System (PIAS) by CCD camera from the 12 J band images that were monocolor photoprinted in Tyson-Seitzer(1988).

In this study we have analyzed the same data to get several morphological elements (the contour line length, the area, the circularity etc.) using PIAS(Personal Image Analyzing System). We have analyzed statistically these elements for galaxies in each binarized image to get quantitative information about the morphology of distant

- 1) Binarization is made at three brightness threshold levels 70, 135, 200 for 12 images from Tyson-Seitzer (1988).
- 2) The binarized image of each galaxy is image analyzed for 14 factors. They are: contour line length, area, length of major and minor axes of approximating ellipse, the maximum diameter length, circularity, the shape factors 2 and 3 (explained later), ratio of contour line length to area, ratio of minor axis

to major axis of the approximating ellipse, slant angle of major axis of the approximating ellipse, the slant angle of maximum diameter, the slant angle of diagonal line of vertical rectangle enveloping the galaxy, ratio of occupation area to the enveloping rectangle.

- 3) Next, statistical calculations for each factor mentioned above are made to give average, standard deviation, and frequency distribu-

tion and to provide data for the principal component analysis.

- 4) The same image analysis and statistical calculations are applied also to an artificial analog simulation image obtained by superimposing copies of image of a standard cluster of galaxy in different hierarchical scales. The result is compared to the result of the observed distant galaxies.

2 Image Analysis

On binarization at threshold 135 and 200 (In the PIAS, brightness scale increases from 0 to 255 showing distant galaxies in the large threshold value), the image merging is appears between galaxies due to extended appearance of low brightness area around each galaxy. Superimposed images which extend to the edge of figure are omitted from the present image analysis, and only independent islands in the background sea are taken into measurement. So, the average areas of galaxy treated in binarized image at these thresholds are not much larger in size than that of threshold 70. Measurements are made in PIAS in each isolated galaxy in the following quantities,

1. Contour line length: PIAS counts the numbers of pixels which form the contour line of each object.
2. Area: PIAS counts the numbers of pixels which form the object.
3. Length of major axis of an ellipse which approximates an isolated galaxy.
4. Length of minor axis of an ellipse which approximates an isolated galaxy.
5. Length of maximum diameter which is the maximum distance between two pixels on the contour line of the object.
6. Circularity: This factor is given by the following formula, $\alpha_1 = 4\pi \times \text{area} / (\text{contour length})^2$. This value is equal to 1 if the object is a circle, and approaches zero for thinner objects.
7. Shape factor 2: This factor is given by the following formula, $\alpha_2 = \pi \times (\text{major axis of ellips} + \text{minor axis of ellips}) / (2 \times \text{contour length})$, approaches to 1 if the shape of galaxy becomes closer to a circle.
8. Shape factor 3: This factor is given by the following formula, $\alpha_3 = \pi \times (\text{maximum axis length})^2 / (4 \times \text{area})$. This value means the filling rate of object in the circle of maximum axis length as diameter.
9. Ratio of contour line length to area: This factor is given by the following formula, $\beta = \text{contour line length} / \text{area}$ This value tends to be larger when the shape of an object is complicated.
10. Ratio α_4 of minor axis to major axis of an approximating ellipse.
11. Occupation rate of object area to the rectangle, given by the next formula. $\alpha_5 = \text{area} / \text{area of the rectangle}$.
12. Slant angle of major axis of the approximating ellipse. This is the slant angle θ of major axis of the ellipse with respect to the direction of horizontal axis ($-\pi/2 \leq \theta \leq \pi/2$).
13. Slant angle of maximum diameter: This is the slant angle of maximum diameter of the object against the direction of horizontal axis ($0 \leq \theta \leq \pi$).

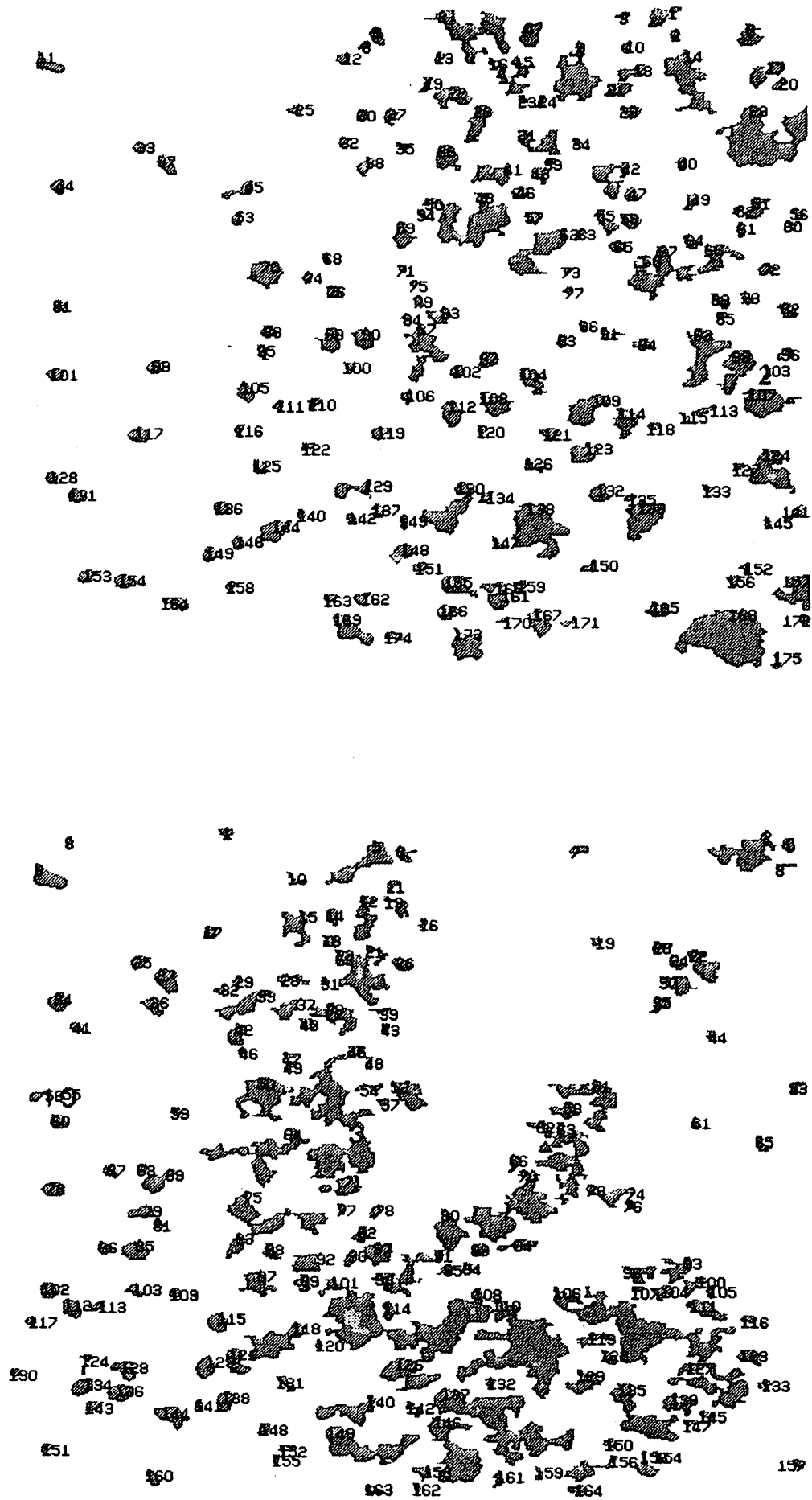


Figure 1a, 1b: The identified image of galaxies binarized at threshold 70(1a) and 135(1b) for T-S figure 23.

T-S figure no.	Figure 13			Figure 14			Figure 15				
	70 293 Avr	135 257 Avr	170 170 Avr	70 135 230 Avr	70 234 Avr	135 120 Avr	Std	Std	Std		
Contour length	3.60	3.88	3.42	3.66	3.68	3.72	0.69	0.54	0.73	0.78	0.71
Area	4.42	4.72	4.22	4.48	4.44	4.49	1.00	0.85	1.05	1.07	1.00
Major axis	2.61	2.78	2.43	2.60	2.58	2.62	0.56	0.46	0.58	0.59	0.54
Minor axis	2.05	2.19	2.02	2.13	2.10	2.11	0.49	0.44	0.51	0.53	0.50
Maximum diameter	2.71	2.92	2.54	2.73	2.72	2.77	0.58	0.47	0.60	0.61	0.57
Circularity	0.85	0.72	0.96	0.82	0.78	0.74	0.31	0.28	0.31	0.34	0.33
Shape factor 2	0.94	0.86	0.99	0.92	0.89	0.87	0.17	0.14	0.18	0.20	0.20
Shape factor 3	2.32	2.57	1.96	2.17	2.26	2.36	0.97	0.93	0.77	0.84	0.85
Contour to area	0.47	0.46	0.49	0.46	0.50	0.50	0.15	0.18	0.16	0.19	0.17
Minor to major	0.60	0.58	0.69	0.65	0.65	0.62	0.18	0.18	0.18	0.17	0.17
Major axis angle	-0.03	0.39	0.08	0.05	0.02	0.05	0.39	0.50	0.44	0.49	0.42
Max diam. angle	1.33	1.38	1.33	1.37	1.28	1.34	0.37	0.36	0.37	0.37	0.38
Diagonal angle	0.66	0.62	0.73	0.69	0.72	0.71	0.17	0.13	0.15	0.16	0.17
Occupation rate	0.64	0.60	0.68	0.64	0.62	0.60	0.13	0.13	0.14	0.14	0.13

T-S figure no.	Figure 16			Figure 17			Figure 18				
	70 288 Avr	135 238 Avr	252 252 Avr	135 202 Avr	70 264 Avr	135 281 Avr	Std	Std	Std	Std	
Contour length	3.60	3.78	3.60	3.78	3.57	3.89	0.65	0.68	0.77	0.61	0.90
Area	4.37	4.54	4.40	4.57	4.42	4.72	0.93	0.99	1.09	0.94	1.26
Major axis	2.54	2.67	2.54	2.68	2.55	2.76	0.52	0.55	0.59	0.51	0.66
Minor axis	2.08	2.12	2.10	2.13	2.11	2.21	0.46	0.48	0.55	0.47	0.65
Maximum diameter	2.67	2.82	2.67	2.82	2.66	2.91	0.53	0.56	0.62	0.52	0.69
Circularity	0.82	0.71	0.83	0.72	0.88	0.69	0.31	0.31	0.31	0.28	0.31
Shape factor 2	0.92	0.85	0.92	0.86	0.95	0.84	0.18	0.17	0.19	0.15	0.20
Shape factor 3	2.17	2.50	2.12	2.44	2.03	2.45	0.75	0.74	1.00	0.64	0.86
Contour to area	0.49	0.50	0.48	0.49	0.45	0.47	0.18	0.17	0.19	0.15	0.20
Minor to major	0.66	0.61	0.67	0.61	0.66	0.61	0.17	0.18	0.18	0.17	0.18
Major axis angle	0.01	0.49	0.02	-0.03	0.00	0.01	0.49	0.46	0.43	0.49	0.46
Diagonal angle	0.74	0.70	0.72	0.70	0.73	0.70	0.14	0.15	0.18	0.15	0.17
Occupation rate	0.62	0.57	0.63	0.59	0.65	0.58	0.14	0.14	0.14	0.12	0.13

Table 1a-1b: Average and standard deviation of 13 factors for observed images of threshold 70 and 135 with the number of galaxies identified.

T-S figure no.	Figure 19			Figure 20			Figure 21		
	70 306 Avr	135 134 Avr	70 254 Avr	135 199 Avr	70 264 Avr	135 179 Avr	70 264 Avr	135 179 Avr	70 264 Avr
Threshold	3.75	3.77	3.54	3.80	3.73	3.95	3.73	3.95	3.73
Number of galax.	4.51	4.51	4.46	4.59	4.59	4.75	4.59	4.75	4.59
Contour length	2.66	2.64	2.58	2.69	2.65	2.80	2.65	2.80	2.65
Area	2.10	2.12	2.12	2.15	2.19	2.20	2.19	2.20	2.19
Major axis	2.81	2.81	2.71	2.84	2.78	2.96	2.78	2.96	2.78
Minor axis	0.72	0.71	0.82	0.72	0.78	0.63	0.78	0.63	0.78
Maximum diameter	0.87	0.85	0.91	0.86	0.89	0.81	0.89	0.81	0.89
Circularity	2.51	2.49	2.14	2.44	2.16	2.68	2.16	2.68	2.16
Shape factor 2	0.51	0.50	0.47	0.49	0.46	0.49	0.46	0.49	0.46
Shape factor 3	0.60	0.62	0.66	0.61	0.65	0.58	0.65	0.58	0.65
Contour to area	0.03	0.00	0.05	-0.01	0.04	0.00	0.04	0.00	0.04
Minor to major	0.70	0.71	0.72	0.69	0.73	0.67	0.73	0.67	0.73
Major axis angle	0.58	0.58	0.63	0.59	0.62	0.56	0.62	0.56	0.62
Diagonal angle	0.14	0.14	0.13	0.13	0.13	0.14	0.13	0.14	0.13
Occupation rate	0.80	0.80	0.83	0.70	0.89	0.87	0.89	0.87	0.89
	1.15	1.12	1.12	0.99	1.21	1.24	1.21	1.24	1.21
	0.60	0.60	0.60	0.55	0.63	0.65	0.63	0.65	0.63
	0.60	0.56	0.56	0.48	0.62	0.65	0.62	0.65	0.62
	0.64	0.66	0.66	0.57	0.68	0.68	0.68	0.68	0.68
	0.32	0.33	0.33	0.31	0.32	0.31	0.32	0.31	0.32
	0.19	0.21	0.21	0.18	0.22	0.20	0.22	0.20	0.22
	0.97	0.90	0.90	0.75	0.88	1.10	0.88	1.10	0.88
	0.20	0.16	0.16	0.16	0.19	0.21	0.19	0.21	0.19
	0.18	0.17	0.17	0.16	0.17	0.18	0.17	0.18	0.17
	0.44	0.44	0.44	0.45	0.46	0.43	0.46	0.43	0.46
	0.17	0.16	0.16	0.14	0.16	0.18	0.16	0.18	0.16
	0.14	0.14	0.16	0.13	0.14	0.14	0.14	0.14	0.13

T-S figure no.	Figure 22			Figure 23			Figure 24		
	70 251 Avr	135 215 Avr	70 249 Avr	135 238 Avr	70 264 Avr	135 172 Avr	70 264 Avr	135 172 Avr	70 264 Avr
Threshold	3.65	3.75	3.66	3.72	3.71	3.73	3.71	3.73	3.71
Number of galax.	4.44	4.47	4.40	4.40	4.40	4.50	4.40	4.50	4.40
Contour length	2.59	2.65	2.57	2.61	2.62	2.63	2.62	2.63	2.62
Area	2.09	2.07	2.07	2.03	2.12	2.06	2.12	2.06	2.12
Major axis	2.73	2.80	2.72	2.77	2.76	2.78	2.76	2.78	2.76
Minor axis	0.79	0.70	0.75	0.70	0.76	0.70	0.76	0.70	0.76
Maximum diameter	0.91	0.85	0.88	0.85	0.88	0.85	0.88	0.85	0.88
Circularity	2.28	2.55	2.31	2.60	2.35	2.58	2.35	2.58	2.35
Shape factor 2	0.47	0.52	0.51	0.53	0.49	0.53	0.49	0.53	0.49
Shape factor 3	0.64	0.59	0.62	0.59	0.63	0.59	0.63	0.59	0.63
Contour to area	0.02	0.01	-0.08	0.03	-0.02	0.04	-0.02	0.04	-0.02
Minor to major	0.73	0.69	0.70	0.69	0.70	0.67	0.70	0.67	0.70
Major axis angle	0.62	0.58	0.60	0.58	0.61	0.58	0.61	0.58	0.61
Diagonal angle	0.13	0.13	0.14	0.13	0.15	0.17	0.16	0.17	0.16
Occupation rate	0.80	0.80	0.84	0.69	0.77	0.78	0.77	0.78	0.77
	1.09	1.17	1.17	1.00	1.00	1.09	1.03	1.09	1.03
	0.59	0.61	0.61	0.51	0.56	0.58	0.56	0.58	0.56
	0.55	0.61	0.61	0.52	0.51	0.59	0.55	0.59	0.55
	0.60	0.65	0.65	0.54	0.60	0.60	0.62	0.60	0.62
	0.31	0.32	0.32	0.29	0.32	0.31	0.31	0.31	0.31
	0.18	0.20	0.20	0.18	0.20	0.19	0.18	0.19	0.18
	0.80	0.93	0.93	0.75	1.05	1.09	1.21	1.09	1.21
	0.18	0.19	0.19	0.18	0.18	0.22	0.19	0.22	0.19
	0.18	0.18	0.18	0.16	0.17	0.19	0.17	0.19	0.17
	0.47	0.44	0.44	0.43	0.44	0.43	0.45	0.43	0.45
	0.16	0.18	0.18	0.15	0.18	0.17	0.16	0.17	0.16
	0.13	0.14	0.14	0.13	0.15	0.14	0.16	0.14	0.15

Table 1c-1d: Average and standard deviation of 13 factors for observed images of threshold 70 and 135 with the number of galaxies identified.

T-S figure no.	Fig.13,14,15,16			Fig.17,18,19,20			Fig.21,22,23,24			Artificial						
	200	227	Std	200	138	Std	200	197	Std	70	28	Std	135	129	Avr	Std
Threshold	3.69	4.45	0.76	3.56	4.25	0.59	3.59	4.32	0.67	3.46	4.32	0.54	3.79	4.70	3.79	0.70
Number of galax.	2.61	2.09	0.59	2.51	1.98	0.44	2.56	2.01	0.51	2.52	2.05	0.50	2.77	2.17	2.77	1.05
Contour length	2.75	0.75	0.60	2.66	0.47	0.45	2.69	2.01	0.54	2.62	2.05	0.45	2.89	2.89	2.89	0.54
Area	0.88	0.75	0.31	0.77	0.28	0.28	0.78	0.27	0.27	0.94	0.19	0.19	0.76	0.76	0.76	0.26
Major axis	2.34	0.51	0.83	2.40	0.53	0.84	2.34	0.52	0.80	2.01	0.42	0.42	2.44	2.44	2.44	0.15
Minor axis	0.62	0.03	0.18	0.61	0.05	0.17	0.61	0.06	0.17	0.64	0.14	0.14	0.59	0.59	0.59	0.18
Maximum diameter	0.68	0.68	0.42	0.71	0.71	0.49	-0.06	0.47	0.47	-0.03	0.51	0.51	-0.03	-0.03	-0.03	0.43
Circularity	0.61	0.61	0.17	0.60	0.60	0.17	0.69	0.17	0.17	0.71	0.16	0.16	0.71	0.71	0.71	0.17
Shape factor 2	0.61	0.61	0.13	0.60	0.60	0.12	0.61	0.61	0.13	0.66	0.09	0.09	0.59	0.59	0.59	0.13
Shape factor 3																
Contour to area																
Minor to major																
Major axis angle																
Diagonal angle																
Occupation rate																

Table 2: Average and standard deviation of 13 factors for observed images of threshold 200 and artificial image of threshold 70 and 135.

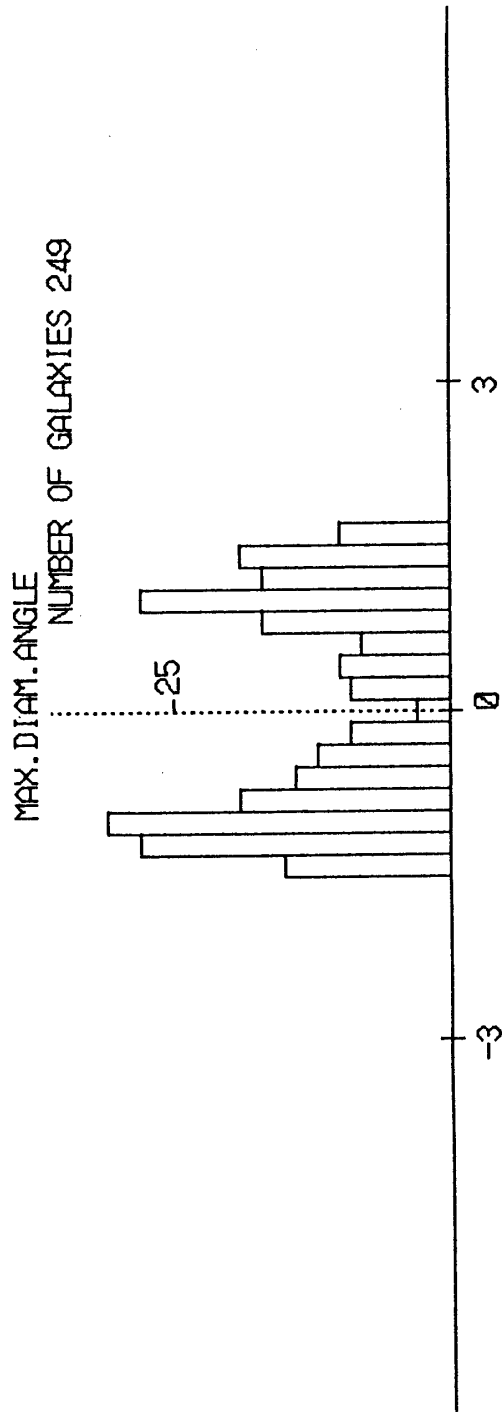


Figure 2: The frequency distribution of the slant angle of maximum diameter of threshold 70 image for T-S figure 23.

14. The slant angle of diagonal line: This is the slant angle of diagonal line of rectangle which has parallel line to sight axis and border contact to the object. The range is $0 \leq \theta \leq \pi$.

Figures 1a and 1b show the images of galaxies binarized respectively at threshold 70 and 135 for the original figure 23 of Tyson and Seitzer(1988, hereafter referred to as T-S).

Figure 2 shows the frequency distribution of the slant angle of maximum diameter for the

threshold 70 image of T-S figure 23. The observed values are standardized so that the average becomes to zero and the standard deviation becomes to 1. It is a big surprise that galaxies show a marked tendency of alignment in east-west direction in every threshold images for all T-S figures. It is not the artifact of our PIAS system, since 90° rotation of the T-S figures against our CCD Camera system does not alter the result. We suppose that some asymmetric process entered in the observation or the printing process. So, we did not apply the statistical calculation to this factor.

3 Results and Discussion

The average values and the standard deviations are given in table 1a to table 1d for each factor measured for the image binarized at threshold 70 and 135.

The average values for the first five factors in table 1a-1d are given in logarithmic scale.

The size of unit pixel corresponds to 0.01 arc-sec. The objects having area less than 20 pixels requires special statistical treatment for morphology and we are not studied in the present study. Table 2 shows the results of the same process for the image binarized at the threshold 200. The number of galaxies found for each image are not enough for statistics, so the average is calculated for nearby 4 figures. The results for artificial image is also shown in table 2. An artificial image is constructed in order to reveal the difference in morphology between distant and nearby galaxies. The average of circularity and shape factor 3 are obviously larger than that of observed images. Several differences are found between the average values of threshold 70 and the values of threshold 135. The average value of the circularity, the ratio of minor axis to major axis, the shape factor 2 and the occupation rate for image of threshold 70 are comparatively larger than that of threshold 135 for all figures. On the contrary, the average values of the ratio of contour length to area of threshold 70 image are mostly smaller than that of threshold 135 image, although homologous enlargement (distance effect) gives the opposite result. This results mean that distant galaxy tends to be morphologically more complex. The figure 3a and 3b

show the frequency distribution of the circularity of galaxies in threshold 70 and 135 images for T-S figure 23. The figure 4a and 4b show the frequency distribution of the ratio of contour length to area of galaxies in both threshold image for T-S figure 23. These figures reveal that the frequency of galaxies which have lower circularity and complicated in shape increases in lower brightness image. The principal component analysis was performed for 11 factors, contour length, area, length of major axis, length of minor axis, circularity, ratio of contour length to area, slant angle of major axis of approximating ellipse, ratio of minor axis length to major axis length, shape factor 2, slant angle of diagonal line length and ratio of occupation area.

The correlation to each element of the 1st to 6th principal component calculated for threshold 70 and 135 images of T-S figure 23 is shown in table 3. The feature of these values are similar between threshold 70 and 135. The 1st component shows positive large correlation with size factors and the negative large correlation with circularity. The 2nd component means morphological thickness which is supposed by large value of the ratio of contour line length to area and also the ratio of minor axis length to major axis length. The 3rd and 4th components indicates the slant of galaxy which is supposed by large value of slant angle factor. Figure 5a and 5b show the scattered diagram of 1st and 2nd principal component for threshold 70 and 135 image or T-S figure 23.

Table 4a- 4d show the correlation to each element of the 1st and 2nd principal component cal-

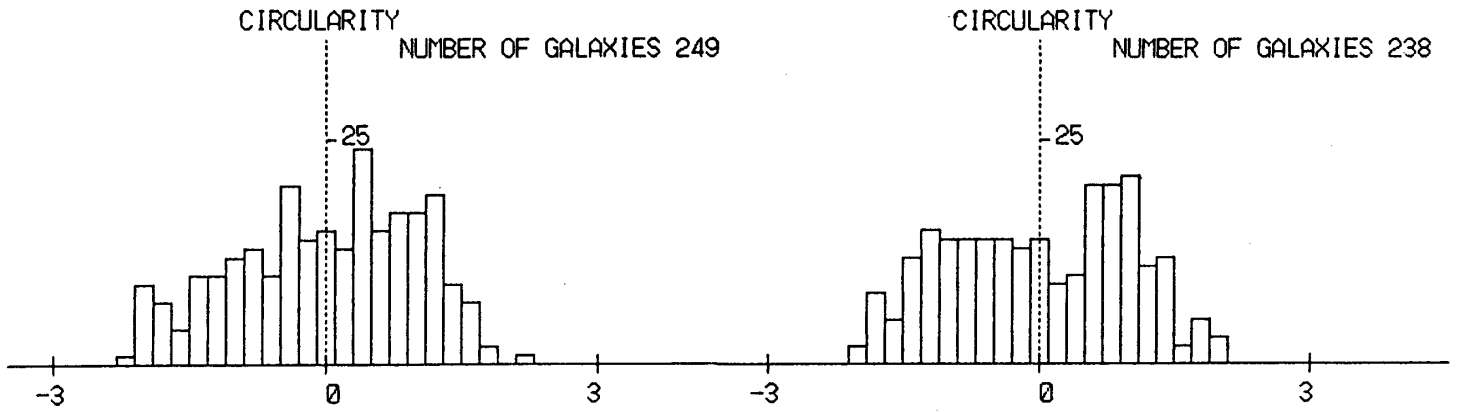


Figure 3a, 3b: Frequency distribution of circularity of images of threshold 70(3a) and 135(3b) for T-S figure 23.

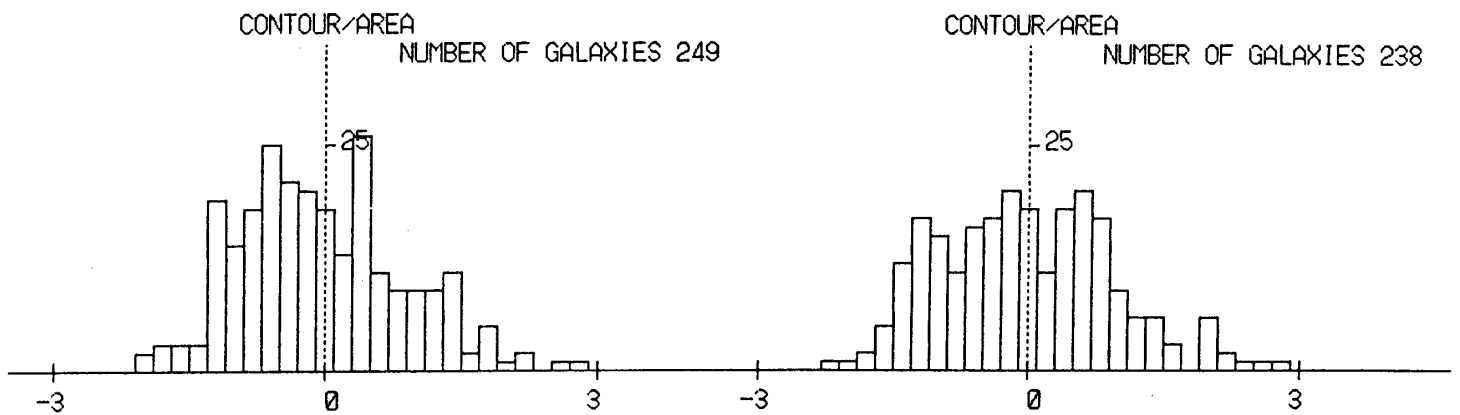


Figure 4a, 4b: Frequency distribution of ratio of contourline length to area of images of threshold 70(4a) and 135(4b) for T-S figure 23.

Principal Comp.	1ST		2ND		3RD		4TH		5TH		6TH	
	70	135	70	135	70	135	70	135	70	135	70	135
Threshold	0.41	0.40	0.04	0.06	0.00	0.03	0.00	0.00	-0.03	0.02	0.13	0.18
Contour length	0.39	0.38	0.18	0.21	-0.03	-0.07	-0.08	0.01	0.08	-0.07	0.02	0.05
Area	0.40	0.39	0.03	0.05	-0.04	-0.14	-0.12	0.03	0.25	-0.24	0.05	0.07
Major axis	0.36	0.34	0.32	0.37	-0.02	0.01	-0.03	-0.01	-0.11	0.13	-0.01	0.03
Minor axis	-0.35	-0.35	0.30	0.28	-0.04	-0.17	-0.12	0.01	0.17	-0.13	-0.27	-0.26
Circularity	-0.26	-0.23	-0.45	-0.50	0.08	0.29	0.20	-0.04	-0.33	0.26	0.31	0.31
Contour to area	0.01	-0.02	0.05	0.05	-0.75	0.24	0.64	0.97	0.14	-0.06	-0.01	0.00
Major axis angle	-0.06	-0.15	0.54	0.52	0.06	0.24	0.18	-0.04	-0.72	0.70	-0.16	-0.08
Minor to major	-0.36	-0.36	0.21	0.17	-0.06	-0.25	-0.16	0.03	0.33	-0.28	-0.31	-0.29
Shape factor 2	-0.02	-0.09	0.23	0.27	0.62	0.78	0.64	-0.24	0.36	-0.51	0.11	0.07
Diagonal angle	-0.26	-0.31	0.42	0.32	-0.15	-0.28	-0.21	0.04	0.11	-0.11	0.82	0.84
Occupation rate	5.97	6.19	2.19	2.07	1.03	1.03	0.97	0.99	0.58	0.49	0.20	0.16
Eigenvalue												

Table 3: Eigen vectors of correlation matrix for 1st to 6th principal component calculated for 70 and 135 threshold images of T-S figure 23.

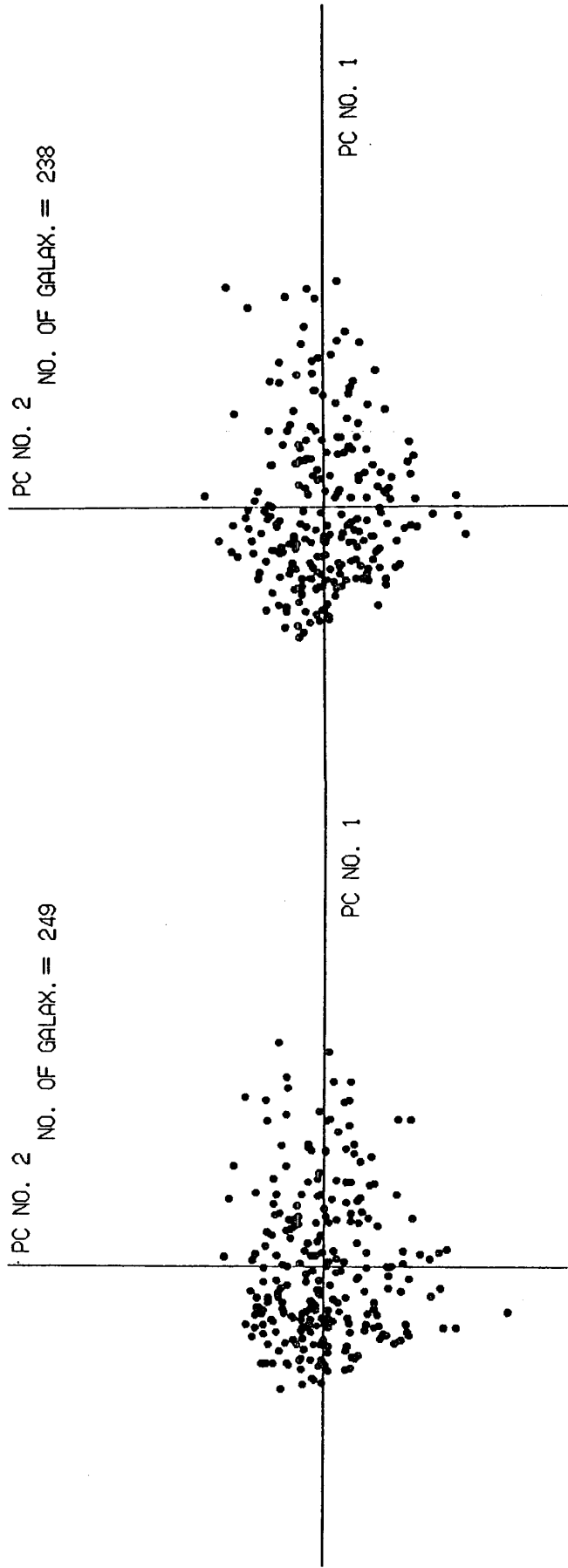


Figure 5a, 5b: Scattered diagram of 1st and 2nd principal component for threshold 70(5a) and 135(5b) images of T-S figure 23.

T-S figure no.	FIGURE13						FIGURE14						FIGURE15								
	1ST		2ND		1ST		2ND		1ST		2ND		1ST		2ND		1ST		2ND		
	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	
Principal Comp.	0.39	0.38	0.39	0.05	0.03	0.42	0.39	0.08	0.06	0.40	0.40	0.06	0.06	0.40	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Threshold	0.38	0.38	0.38	0.17	0.12	0.39	0.38	0.26	0.18	0.38	0.38	0.18	0.18	0.38	0.21	0.23	0.21	0.23	0.21	0.23	0.23
Contour length	0.38	0.39	-0.03	-0.04	-0.04	0.41	0.39	0.10	0.02	0.39	0.39	0.02	0.02	0.39	0.07	0.08	0.07	0.08	0.07	0.08	0.08
Area	0.33	0.36	0.37	0.37	0.28	0.31	0.34	0.40	0.36	0.34	0.34	0.36	0.36	0.34	0.35	0.37	0.35	0.37	0.35	0.37	0.37
Major axis	-0.35	-0.35	0.24	0.24	0.26	-0.36	-0.35	0.30	0.27	-0.35	-0.35	0.27	0.27	-0.35	0.28	0.31	0.28	0.31	0.28	0.31	0.31
Minor axis	-0.28	-0.27	-0.41	-0.41	-0.36	-0.20	-0.28	-0.49	-0.45	-0.23	-0.23	-0.45	-0.45	-0.23	-0.49	-0.51	-0.49	-0.51	-0.49	-0.51	-0.51
Contour to area	0.03	0.00	0.18	0.18	0.01	-0.03	-0.02	-0.03	-0.11	-0.03	-0.01	-0.11	-0.11	-0.03	0.09	0.01	0.09	0.01	0.09	0.01	0.01
Major axis angle	-0.17	-0.08	0.57	0.57	0.61	-0.20	-0.16	0.42	0.56	-0.16	-0.16	0.56	0.56	-0.16	0.49	0.48	0.49	0.48	0.49	0.48	0.48
Minor to major	-0.37	-0.37	0.10	0.10	0.10	-0.35	-0.36	0.26	0.17	-0.36	-0.36	0.17	0.17	-0.36	0.20	0.23	0.20	0.23	0.20	0.23	0.23
Shape factor 2	-0.02	0.02	0.41	0.41	0.53	0.03	-0.01	-0.10	0.28	-0.03	-0.03	0.28	0.28	-0.03	0.30	0.19	0.30	0.19	0.30	0.19	0.19
Diagonal angle	-0.30	-0.30	0.28	0.28	0.24	-0.26	-0.29	0.42	0.34	-0.30	-0.30	0.34	0.34	-0.30	0.36	0.35	0.36	0.35	0.36	0.35	0.35
Occupation rate	6.54	6.39	2.08	2.08	2.09	5.42	6.43	2.84	1.90	6.12	6.04	1.90	1.90	6.12	2.14	2.24	2.14	2.24	2.14	2.24	2.24
Eigenvalue																					

T-S figure no.	FIGURE16						FIGURE17						FIGURE18								
	1ST		2ND		1ST		2ND		1ST		2ND		1ST		2ND		1ST		2ND		
	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	70	135	
Principal Comp.	0.40	0.39	0.40	0.07	0.04	0.40	0.40	0.07	0.05	0.39	0.39	0.05	0.05	0.39	0.07	0.04	0.07	0.04	0.07	0.04	0.04
Threshold	0.38	0.39	0.24	0.24	0.17	0.38	0.39	0.23	0.20	0.38	0.38	0.20	0.20	0.38	0.21	0.15	0.21	0.15	0.21	0.15	0.15
Contour length	0.40	0.39	0.09	0.09	0.00	0.39	0.39	0.08	0.05	0.39	0.39	0.05	0.05	0.39	0.03	-0.01	0.03	-0.01	0.03	-0.01	-0.01
Area	0.33	0.34	0.38	0.38	0.33	0.33	0.34	0.39	0.34	0.33	0.36	0.34	0.34	0.33	0.37	0.30	0.37	0.30	0.37	0.30	0.30
Major axis	-0.36	-0.35	0.30	0.30	0.30	-0.35	-0.35	0.30	0.29	-0.35	-0.35	0.29	0.29	-0.35	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Minor axis	-0.22	-0.25	-0.52	-0.52	-0.44	-0.25	-0.25	-0.50	-0.47	-0.30	-0.30	-0.47	-0.47	-0.30	-0.41	-0.38	-0.41	-0.38	-0.41	-0.38	-0.38
Contour to area	-0.04	0.03	0.06	0.06	0.06	-0.01	0.00	0.04	-0.09	-0.04	-0.04	-0.09	-0.09	-0.04	0.06	0.05	-0.04	0.06	0.05	0.05	0.05
Major axis angle	-0.19	-0.10	0.43	0.43	0.54	-0.20	-0.13	0.46	0.50	-0.15	-0.15	0.50	0.50	-0.15	0.55	0.61	0.55	0.61	0.55	0.61	0.61
Minor to major	-0.36	-0.37	0.23	0.23	0.17	-0.35	-0.36	0.24	0.20	-0.36	-0.36	0.20	0.20	-0.36	0.19	0.14	0.19	0.14	0.19	0.14	0.14
Shape factor 2	0.18	0.01	0.09	0.09	0.38	-0.05	-0.02	0.12	0.29	-0.04	-0.04	0.29	0.29	-0.04	0.26	0.39	0.26	0.39	0.26	0.39	0.39
Diagonal angle	-0.29	-0.30	0.42	0.42	0.32	-0.29	-0.30	0.40	0.38	-0.27	-0.27	0.38	0.38	-0.27	0.39	0.35	0.39	0.35	0.39	0.35	0.35
Occupation rate	5.99	6.17	2.18	2.18	2.24	6.17	6.03	2.05	2.23	6.31	6.34	2.23	2.23	6.31	2.08	2.02	2.08	2.02	2.08	2.02	2.02
Eigenvalue																					

Table 4a-4b: Eigen vectors of correlation matrix for 1st and 2nd principal component calculated for 70 and 135 threshold images of 12 T-S figures.

T-S figure no.	FIGURE19			FIGURE20			FIGURE21		
	1ST 70	1ST 135	2ND 70	1ST 70	1ST 135	2ND 70	1ST 70	1ST 135	2ND 70
Principal Comp.	0.40	0.39	0.06	0.05	0.39	0.05	0.04	0.40	0.07
Threshold	0.39	0.38	0.20	0.18	0.38	0.20	0.16	0.38	0.22
Contour length	0.40	0.38	0.05	0.02	0.39	0.05	0.01	0.39	0.07
Area	0.35	0.35	0.34	0.33	0.34	0.36	0.30	0.34	0.36
Major axis	-0.35	-0.35	0.30	0.28	-0.35	0.27	0.27	-0.34	0.32
Minor axis	-0.25	-0.27	-0.46	-0.48	-0.26	-0.49	-0.43	-0.27	-0.47
Circularity	0.02	-0.03	0.08	0.01	0.00	0.16	0.15	-0.02	0.05
Contour to area	-0.11	-0.12	0.52	0.60	-0.17	0.50	0.58	-0.17	0.49
Major axis angle	-0.37	-0.36	0.19	0.17	-0.35	0.20	0.14	-0.35	0.24
Minor to major	-0.05	-0.01	0.25	0.22	-0.01	0.25	0.36	0.04	0.17
Shape factor 2	-0.27	-0.31	0.41	0.34	-0.29	0.37	0.34	-0.28	0.41
Diagonal angle	5.98	6.49	2.40	1.71	6.37	1.73	2.01	6.16	2.03
Occupation rate									
Eigenvalue									

T-S figure no.	FIGURE22			FIGURE23			FIGURE24		
	1ST 70	1ST 135	2ND 70	1ST 70	1ST 135	2ND 70	1ST 70	1ST 135	2ND 70
Principal Comp.	0.40	0.40	0.06	0.04	0.41	0.04	0.06	0.40	0.05
Threshold	0.39	0.39	0.22	0.16	0.39	0.18	0.21	0.39	0.19
Contour length	0.39	0.39	0.06	-0.07	0.40	0.03	0.05	0.39	0.04
Area	0.34	0.35	0.37	0.31	0.36	0.32	0.37	0.35	0.35
Major axis	-0.35	-0.35	0.32	0.30	-0.35	0.30	0.28	-0.35	0.29
Minor axis	-0.26	-0.27	-0.48	-0.43	-0.26	-0.45	-0.50	-0.24	-0.47
Circularity	-0.03	0.00	0.08	0.00	0.01	0.05	0.05	-0.02	0.02
Contour to area	-0.15	-0.07	0.49	0.57	-0.06	0.54	0.52	-0.13	0.53
Major axis angle	-0.35	-0.37	0.24	0.15	-0.36	0.21	0.17	-0.36	0.19
Minor to major	0.00	0.02	0.17	0.34	-0.02	0.23	0.27	0.00	0.29
Shape factor 2	-0.28	-0.28	0.39	0.38	-0.26	0.42	0.32	-0.29	0.36
Diagonal angle	6.02	6.17	2.08	2.17	5.97	2.19	2.07	6.09	2.09
Occupation rate									
Eigenvalue									

Table 4c-4d: Eigen vectors of correlation matrix for 1st and 2nd principal component calculated for 70 and 135 threshold images of 12 T-S figures.

T-S figure no. Principal Comp. Threshold	FIG.13,14,15,16		FIG.17,18,19,20		FIG.21,22,23,24		Artificial			
	1ST	2ND	1ST	2ND	1ST	2ND	1ST	135	2ND	
		200		200		200	70	135	70	135
Contour length	0.41	0.03	0.42	0.02	0.41	0.02	0.40	0.45	0.07	0.07
Area	0.39	0.17	0.39	0.19	0.40	0.17	0.39	0.38	0.18	0.20
Major axis	0.40	0.02	0.40	0.01	0.40	0.00	0.40	0.39	0.06	0.02
Minor axis	0.35	0.30	0.34	0.35	0.36	0.31	0.36	0.33	0.29	0.36
Circularity	-0.34	0.31	-0.35	0.31	-0.33	0.34	-0.32	-0.35	0.37	0.29
Contour to area	-0.27	-0.43	-0.24	-0.47	-0.28	-0.42	-0.35	-0.27	-0.31	-0.43
Major axis angle	0.02	0.04	0.04	0.14	-0.01	0.07	-0.04	0.01	-0.28	0.18
Minor to major	-0.09	0.51	-0.07	0.52	-0.05	0.53	-0.18	-0.13	0.40	0.52
Shape factor 2	-0.36	0.21	-0.37	0.22	-0.36	0.22	-0.32	-0.37	0.28	0.15
Diagonal angle	0.04	0.40	-0.01	0.19	0.06	0.27	-0.12	-0.07	-0.04	0.26
Occupation rate	-0.27	0.36	-0.27	0.38	-0.24	0.41	-0.11	-0.26	0.57	0.40
Eigenvalue	5.99	2.42	5.73	2.47	5.83	2.49	6.10	6.13	1.98	2.29

Table 5: Eigen vectors of correlation matrix for 1st and 2nd principal component calculated for 200 threshold images of 12 T-S figures and 70 and 135 threshold images of artificial image.

culated for threshold 70 and 135 images of 12 T-S figures. In every figures and thresholds, the 1st component shows large size factor and circularity factor as mentioned above. This situation is the same with other threshold and figure. This component concerns to both size and morphological complexity. And the 2nd component shows large thickness factors as ratio of contour line length to area or ratio of minor to major axis length or occupation rate. This component concerns to the thickness of galaxy. In 2nd component the eigen values of slant angle factors are found to be different between thresholds. For instance, the eigen values of diagonal angle for threshold 135 image are larger than that for threshold 70 image in every figures. This means that the slant angle factor contribute more to 2nd principal component as lower as the brightness of image. Table 5 shows

the correlation to each element of the 1st and 2nd principal component calculated for threshold 200 images of 12 T-S figures and for artificial image binarized at threshold 70 and 135. The contribution of major axis angle to 2nd principal component for artificial image is larger than that of observed images. This may come from the slant superimposing copy of the cluster image of nearby galaxy. The first principal component is related to the size of galaxy and therefore essentially to the parallax. But, at the same time ,it relates to the slenderness of the shape. It seems to be contradictory. But it could be comprehended that in low brightness image the outer envelope is observed in large sized galaxies more elongated than the core. The distant galaxies have more complicated outer envelope than the nearer galaxies.

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