Statistical Tests as Spherical Data for Angular Distributions of Neutrinos Burst from SN 1987A

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Abstract

Using some statistical methods, many investigators have claimed on the polar angle distribution of neutrinos emitted from SN1987A that the IMB data seems to be non-isotropic distribution, whereas the Kamiokande data shows isotropic distribution. It seems unable to be explained by only $\bar{\nu}_e$+p $\rightarrow$ n + e^+ (capture). They, however, haven't used the statistical test method of spherical distributions in which both the polar and azimuthal angles must be utilized in polar co-ordinates. Also, the azimuthal angle distribution of the IMB burst has been affected by lacks of tracks owing to failed supplying high-voltage power to a part of the PMT at the SN explosion time. Taking into account the effect of lacks of tracks and using the Rayleigh test on spherical data, we find a significance probability of 12% for a uniform distribution. Accordingly, it is concluded all events of the IMB and Kamiokande observations are not in contrast to the assumption that all events are due to $\bar{\nu}_e$+p $\rightarrow$ n + e^+ in accordance with standard models of supernova explosion.
Key Word: SN1987A, Statistical Test on Spherical Data, Polar and Azimuthal Angle Distribution,

1 Introduction

Following the discovery of the supernova in the Large Magellanic Cloud, now called SN1987A, four groups have reported neutrino events, being eleven (removing No. 6) from the Kamiokande experiment,[1),2)] eight from the IMB experiment,[3),4)] six from the Monte Blanc experiment[5)] and three from the Baksan experiment.[6)] Within them, only water Cerenkov detectors of the Kamiokande and IMB groups can measure the incident angles and energies of neutrino signals. The revised data from Kamiokande and IMB groups are cited in Table 1 and Table 2.

About the angular distribution of Kamiokande events, Arafune and Fukugita[7)] asserted a probability of the two events, No. 1 and No. 2, emitting such a forward cone is $\sim 1/2000$ for isotropic angular distribution. Burrow and Lattimer,[8)] but, indicated the angular distribution is not inconsistent with isotropic hypothesis using the Kolmogorov-Smirnov statistical test. Kolb et al[9)] also found that for all Kamiokande events the Kolmogorov-Smirnov significance level is 0.08, with which non isotropic hypothesis cannot be established. Kitamura discussed[10)] using the other methods for

Table 1: The twelve electron events from SN 1987A detected in Kamiokande which are cited from ref. 1). Event 6 did not satisfy their criterion $N_{hit} > 20$, so was removed. Also event 2 is corrected at ref. 2) as shown in the lowest column.

Event	Time (sec)	Number of PMT's	Electron energy (MeV)	Electron angle (degree)
1	0.	58	20.0±2.9	18±18
2	0.107	36	13.5±3.2	15±27
3	0.303	25	7.5±2.0	108±32
4	0.324	26	9.2±2.7	70±30
5	0.507	39	12.8±2.9	135±23
6	0.686	16	6.3±1.7	68±77
7	1.541	83	35.4±8.0	32±16
8	1.728	54	21.0±4.2	30±18
9	1.915	51	19.8±3.2	38±22
10	9.219	21	8.6±2.7	122±30
11	10.433	37	13.0±2.6	49±26
12	12.439	24	8.9±1.9	91±39
2	0.107	36	13.5±3.2	40±27

Table 2a: The eight events from SN1987A detected in IMB which are cited from ref. 3).
a) Error in energy determination is ±25% (systematic plus statistical).
b) Individual track reconstruction uncertainty is 15°.

Event no.	Time (UT)	Number of PMT's	Electron energy (MeV)	Electron angle (degree)
1	7:35:41.37	47	38	74
2	7:35:41.79	61	37	52
3	7:35:42.02	49	40	56
4	7:35:42.52	60	35	63
5	7:35:42.94	52	29	40
6	7:35:44.06	61	37	52
7	7:35:46.38	44	20	39
8	7:35:46.96	45	24	102

Table 2b. Same as Table 2a, except for cited from ref. 4).
a) Relative time are accurate to the nearest millisecond.
b) Additional systematic error in energy scale estimated to be ±10%.

Event no.	Time (UT)	Electron energy (MeV)	Electron angle (degree)
1	7:35:41.374	38 ± 7	80 ± 10
2	7:35:41:786	37 ± 7	44 ± 15
3	7:35:42.024	28 ± 6	56 ± 20
4	7:35:42.515	39 ± 7	65 ± 20
5	7:35:42.936	36 ± 9	33 ± 15
6	7:35:44.058	36 ± 6	52 ± 10
7	7:35:46.384	19 ± 5	42 ± 20
8	7:35:46.956	22 ± 5	104 ± 20

circular statistical tests the isotropic angular distribution cannot be excluded with Kamiokande data including or excluding No. 1 and No. 2 in the significance level of 0.1 or little more. For IMB events, however, the significance level was 0.02 or less for all tests, in special, the Rayleigh test gave 0.007. In the ninth workshop on grand unification, on the other hand, Perkins[11] has indicated by combing the Kamiokande and IMB data that the binomial probability of 15 events forward is 4.14% for an isotropic distribution and thus he asserted the angular distribution of the SN1987A neutrino events is not understood. Moreover, the IMB group has noted that the 4 events above 20 MeV of Kamiokande data (No. 1, No. 7, No. 8 and No. 9) have polar angles in the range 15°-50°,[4] the angular distribution of which may be inconsistent with a isotropic distribution.

In these treatments of statistical tests, only polar angle data, θ-values, have been used by the reason that the azimuthal angles should have an isotropic distribution. The neutrino events, however, are emitted in three-dimensional space, and then it must be asked for as spherical distributions. In this paper, at first, we shall evaluate effects to the angular distribution owing to the one-quarter of inoperative PMT's in the IMB operations using Monte Carlo simulation, and using the affected result, we shall apply the statistical test[12] as spherical data for the observed polar angle and azimuthal angle distributions of IMB events.

2 Monte Carlo simulation for effects of inoperative PMT' in IMB operation

In the IMB detector, the total number of 2048 PMT's is divided by 32 patches, one of which includes 64 PMT's (8×8). At the SN explosion time, the 8 patches of them were failed with supplying high-voltage power, one patch being in the bottom surface, 3 patches in the top surface and 4 patches in the south surface. The polar angle, θ, is measured with respect to an axis in the direction away from the supernova (which was 42° below the horizon and 28° west of south at the time of the neutrino burst). For evaluating effects of the inoperative phototubes on the incident angular distribution, we made a simulated calculation with many samples of the burst with 8 events which corresponds to IMB data. The one burst event consists of 8 incident electrons with an isotropic distribution, i.e. its polar angle, θ, and its azimuthal angle, ϕ, are independently uniformly distributed. The simulated polar angle distributions and azimuthal angle distributions are calculated for different cases of $\leq$ 50%, $\leq$40% and $\leq$ 30% lacks of tracks owing to the inoperative PMT's, and their distributions are shown in Fig. 1a, 1b and 1c,respectively. These simulated distributions endorse that the polar angle distribution of the IMB data are not affected significantly by the inoperative PMT's operations, whereas the azimuthal angular distributions are affected as a dip appeared around $-130°$. All these behaviors of the polar angle and azimuthal angle distributions are not contradict with the IMB's calculation for the dependence of the trigger efficiency on polar angle at 20 MeV and 30 MeV electron energies[4], and both the angular distributions in the case for $\leq$ 30%, in special, coincide best with the IMB's distributions. We heard the IMB group has adopted the events with $\leq$ 50% for their data analysis. This difference may be attributed to neglecting of the effects of sensitivity falling off near the edges of rectangular water tank in our calculations (because of our unknown the details). Henceforth, we shall use both the distributions for the present statistical tests because those distributions for $\leq$ 30% coincide very nearly with the IMB' ones.

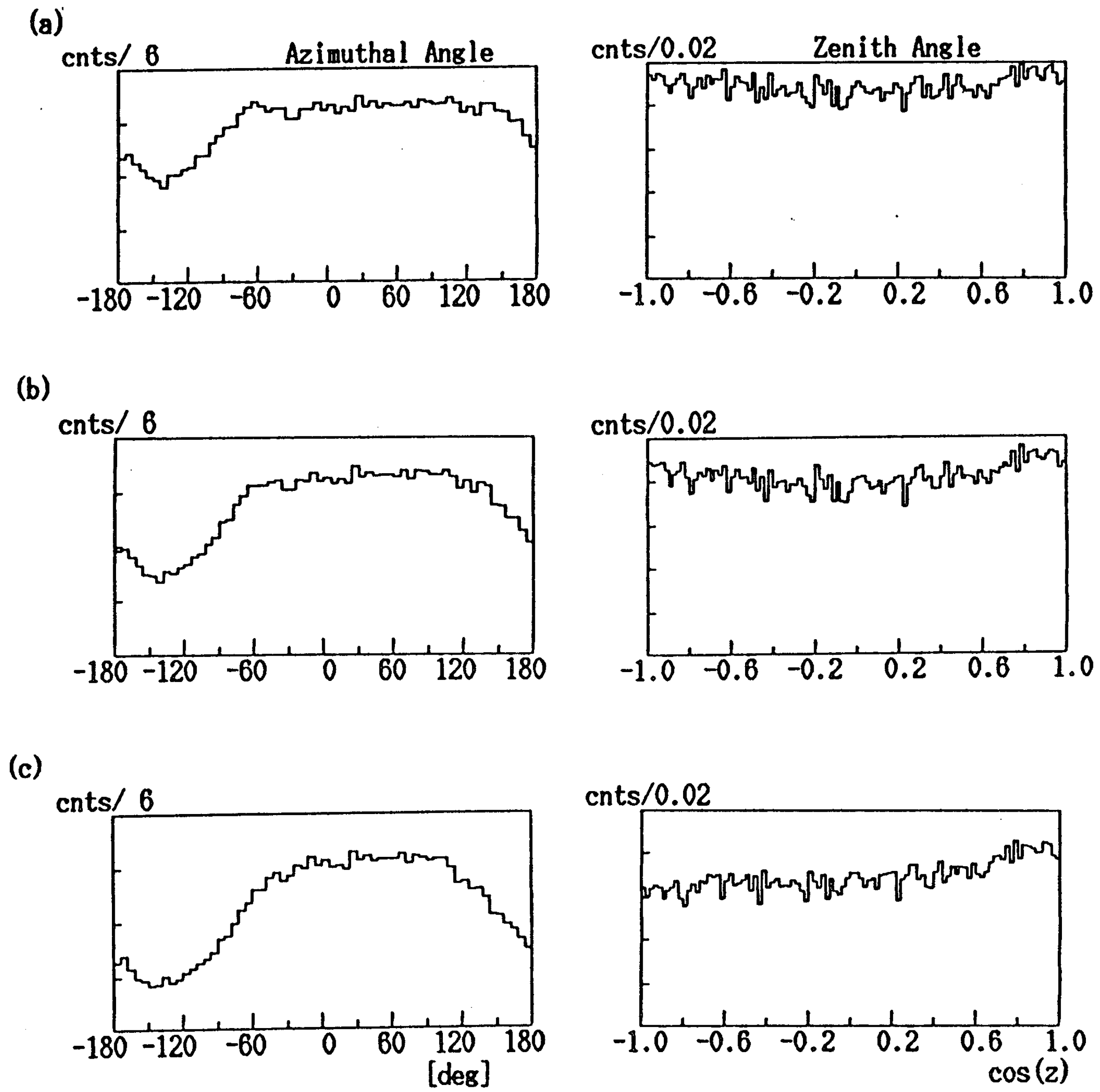

Fig. 1. Simulation results for evaluating effects of lacks of tracks owing to the inoperative PMT's on the pole angle distribution and the azimuthal angle distribution. For many samples of the burst of 8 electron events in which polar and azimuthal angles are independently uniformly distributed.

a) ≤50% lacks of tracks.

b) ≤40% lacks of tracks.

c) ≤30% lacks of tracks.

3 Statistical tests as spherical data for polar angle distributions

The neutrino burst events are emitted in three-dimensional space. For statistical tests, then, it must be asked for as spherical distributions by the directional cosines;

$$l = \sin\theta\cos\phi, m = \sin\theta\sin\phi, n = \cos\theta,$$

$$\text{with } 0 < \theta < \pi, 0 < \phi < 2\pi.$$

Following to Mardia's book[12], the mean length of the resultant of spherical mean direction is given by

$$\bar{R} = 1/n(\sum l_i)^2 + (\sum m_i)^2 + (\sum n_i)^2)^{1/2}.$$

$\bar{R}$ concerns with a degree of concentration about the mean direction if it exists. Of course, when *theta* and ϕ are independently distributed and o is uniformly distributed on (0, 2), i.e. the directions are uniformly distributed on the surface of a sphere, the probability density function (p.d.f.) can be integrated by with 0 to 2 , and reduced to

$$\text{p.d.f.} = g(\theta, \phi) = f(\theta)/(2\pi),$$

$$0 < \theta < \pi, 0 < \phi < 2\pi.$$

which is only a function of polar angle. For making the integration possible, it is necessary to have a lot of numbers of events. Both the cases of SN burst of IMB and Kamiokande are too few number of events to make the integration possible. Accordingly, we must carry out the test by calculations of using the directional cosines l, m and n. To make sure, we calculate the mean length $\bar{R}$ values of three cases of the simulated samples of the burst which consists of incident 20, 8 and 4 electrons. Their incident angles of the electrons are decided by picking up from both distributions of Fig. 1. The figure gives that the polar angle distribution shows a uniform distribution and the azimuthal angular distribution appears a dip around -130° with the effect of $\leq$ 30% lacks of tracks in the PMT inoperations. By means of calculations using each number of about 20,000 samples for the cases of incident 20, 8 and 4 electrons, their respective $\bar{R}$ distributions are given in Fig. 2 by their respective curves. From the figure, one can see the $\bar{R}$ distributions of 8 and 4 incident electrons are broadly extended to the limit of $\bar{R} = 1$. The case of $\bar{R} = 1$ shows complete concentrations to a direction (non-uniform distribution), on the other hand, the case of $\bar{R} = 0$ a completely uniformed distribution. According to these behaviors, a polar angle distribution of the event with a small number around 10 or less appears more frequently with having a large $\bar{R}$. When getting a value of R_0 for a value of n events, we can calculate the probability for $\bar{R} > R_0$, Prob($\bar{R} > R_0$), which is obtained from an area under the $\bar{R}$ distribution with $\bar{R} > R_0$ as shown by α in Fig. 2. Namely, Prob($\bar{R} > R_0$) = α is defined. This is the familiar significance probability. If Prob($\bar{R} > R_0$) for a data of n events is larger the α, the data cannot establish a non-isotropic distribution of the polar angles in significance probability of α. If Prob($\bar{R} > R_0$) is smaller than α, the data can establish a non-isotropic distribution at the significance probability of α. We have made certain of that the values of R_0 and α for $n = 8$ and 20 decided from $\bar{R}$ distributions which are calculated without taking into account the effect of PMT inoperations (i.e., uniform distributions for polar and azimuthal angles) are in good agreement with the table for the Rayleigh test of uniformity with R_0 and in Appendix 3.5 of Mardia's book[12]. This agreements can ensure our calculations on the $\bar{R}$ distributions in Fig. 2 to be correct.

The IMB and Kamiokande SN bursts are only 8 events and 11 events, respectively. Accordingly, the statistical test for polar angle distributions must be performed by calculation of the directional cosines l, m and n. The IMB data have been published[3] with not only values of polar angles but also of azimuthal angles, whereas the Kamiokande data with polar angle values alone. Using the polar angles and the azimuthal angles of IMB 8 events, even not taking account of angular uncertainties, we get $\bar{R} = 0.552$. Account being taken of a behavior of the $\bar{R}$ distribution in Fig. 2, the significance probability is 0.12 for a uniform distribution. Namely, a non-isotropic distribution can not establish in the polar angle distribution of the IMB data. The Kamiokande data haven't contained the azimuthal values. So, we made the test by means of that azimuthal angles are picked

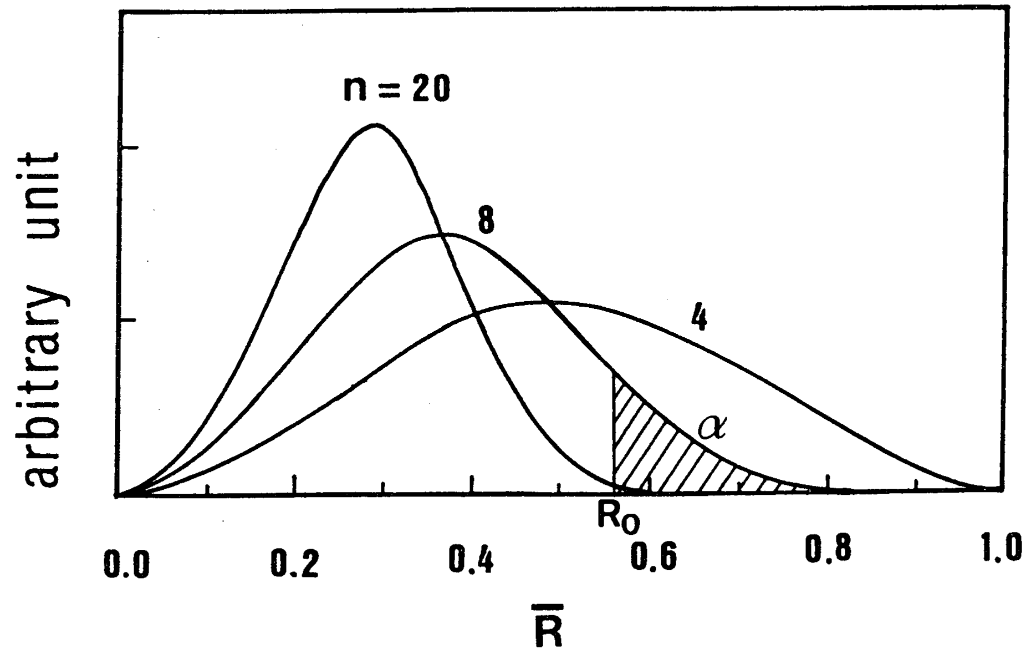

Fig. 2. Calculated $\bar{R}$ distributions for simulated samples of the burst which consists of incident 20, 8 and 4 electrons. Their incident angles of electrons are picked up from both the polar and azimuthal angle distributions in Fig. 1c. A shaded part means an area of the $\bar{R}$ distribution with $\bar{R} > R_0$.

up from an isotropic distribution. Of course, we found for a uniform distribution the significance probability of 19.8%. It is concluded that, accordingly, both the polar angle distributions of emitted from SN1987A observed by the IMB and Kamiokande detectors are nearly isotropic, and are not contrast to the assumption that all the events are due to $\bar{\nu}_e$+p → n + e^+ in accordance with standard models of supernova explosion.

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