Expected Reconstruction Error of Arrival Direction in Observation by Water Cerenkov Detector

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Abstract

Reconstruction error of emitted direction of electrons observed in water cerenkov detector is estimated assuming gaussian approximation for multiple scattering process and exponential decrease with increasing mean square angle for detecting efficiency of cerenkov photon. After correcting geometrical limitation by Mott formula we got reasonable result explaining experimental data of wide energy range. Similar estimation by Kamioka group seems unable to explain those data because of inadequate cut of large angle scattering used in EGS simulation code.

1 Introduction

Estimation of ambiguity of arrival direction of cosmic ray particles measured in many instruments seems very important nowadays because particle astrophysics is a central theme in non-accelerator physics. So we estimated reconstruction error for emergent direction of electrons in water cerenkov detector.

Relativistic electrons traversing in water radiate cerenkov photon towards a constant angle from the direction of motion, depending on its energy. So we can get emitted direction of electrons in water cerenkov detector from observed photons. But the reconstructed direction have certain ambiguity because electron changes its direction successively due to the multiple scattering process. We evaluated the ambiguity by square root of mean deflection angle weighted by cerenkov photon count.

2 Estimation of Reconstruction Error by Gaussian Approximation

We get mean square deflection angle of electrons in multiple scattering process under the gaussian approximation, according to Fermi formulation.[1] We assume continuous energy loss, thus we get differential equation

$$\frac{d\langle\theta^2\rangle}{dt} = \frac{E_s^2}{p^2v^2} \quad , -\frac{dE}{dt} = \epsilon. \tag{1}$$

The solution considering rest energy of electron becomes

$$\langle\theta^2\rangle = \frac{E_s^2}{2\epsilon}\left[\frac{E}{p^2c^2} - \frac{E_0}{p_0^2c^2} - \frac{1}{2mc^2}\ln\frac{\frac{E-mc^2}{E_0-mc^2}}{\frac{E+mc^2}{E_0+mc^2}}\right]. \tag{2}$$

On the other hand, cerenkov photon count decreases with energy as

$$dN \propto \left(1 - \frac{1}{n^2\beta^2}\right)dt$$

$$\propto \frac{\beta^4}{(1-\beta^2)}\left(1 - \frac{1}{n^2\beta^2}\right)d\langle\theta^2\rangle. \tag{3}$$

Photon count against mean square angle is plotted in Fig.1, for various incident energy. We see photon count decreases exponentially with mean square deflection angle.

Thus if we assume detecting efficiency of cerenkov photon decreases exponentially with increase of mean square angle, we get expectation value of mean square deflection angle weighted by cerenkov photon count as

$$E[\langle\theta^2\rangle] \simeq -\left[\frac{d}{d\langle\theta^2\rangle}\ln\left\{\frac{\beta^4}{1-\beta^2}\left(1-\frac{1}{n^2\beta^2}\right)\right\}\right]^{-1}$$

$$(\mathrm{at}\langle\theta^2\rangle = 0)$$

$$= \frac{1 - 1/(n^2\beta^2)}{2 - \beta^2 - 1/(n^2\beta^2)}\frac{E_s^2}{2\epsilon pv}. \tag{4}$$

Square root of expected mean square deflection angle is plotted in Fig.2 (broken line).

3 Difference from The Monte Carlo Result Using EGS Simulation Code

Similar estimation is made by Kamioka group[2] using EGS simulation code[3] for reconstruction error of emitted direction of electron in water cerenkov detector. Their results are plotted in Fig.2 (dot broken line), as well as experimental data of Kamioka[4] group (dot point data) and IMB[5] group (open circle data).

Our estimation and Kamioka estimation crosses at about 20 MeV of electron incident energy. This difference seems to come from cut of large angle scattering. As well known, in gaussian approximation large angle scattering is cut at λ/d, considering interference by nucleus, where λ is the De Broglie wave length of incident electron divided by 2π and d is the nuclear radius.[1] On the other hand in EGS simulation code large angle scattering is cut as $\sqrt{\sin\theta/\theta}$,[3] independent of electron energy. Thus Kamioka group seems to estimate higher reconstruction error for electron energy greater than 20 MeV and lower error at electron energy lower than 20 MeV.

4 Correction of Our Result Considering Mott Formula

Cut λ/d of large angle scattering loses its meaning geometrically for lower electron energy than 20 MeV, because λ/d exceeds π. Thus we tried correction of our preceding estimation. Derivatives of mean square deflection angle is decided by

$$\frac{d}{dt}\langle\theta^2\rangle = \int\int \vec{\theta}^2\sigma(\vec{\theta})d\theta = \mu_2(E), \tag{5}$$

where $\mu_2(E)$ is the second moment of the single scattering. So we made correction by the ratio of the second moments among the single scattering formulae, that used in gaussian approximation and Mott formula,

$$\sigma_G(\theta)d\omega = N\frac{4Z^2r_0^2m_e^2c^2}{Ap^2\beta^2}\theta^{-4}2\pi\theta d\theta \tag{6}$$

and

$$\sigma_M(\theta)d\omega = N\frac{Z^2r_0^2m_e^2c^2}{4Ap^2\beta^2}\{1 - \beta^2\sin^2(\theta/2)\}$$

$$\times \sin^{-4}(\theta/2)2\pi\sin\theta d\theta. \tag{7}$$

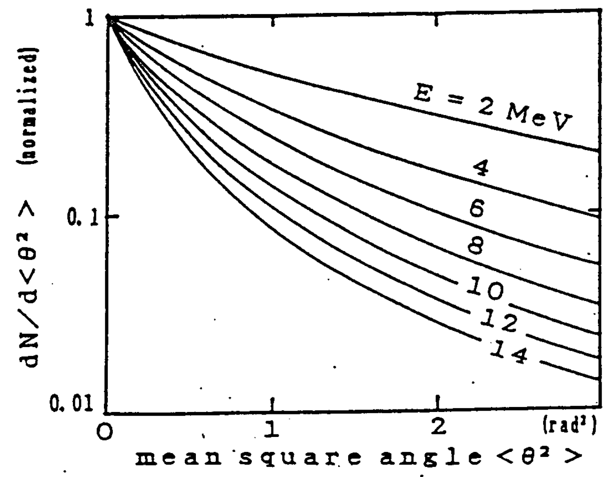

Fig. 1: Decrease of photon count vs increasing mean square deflection angle.

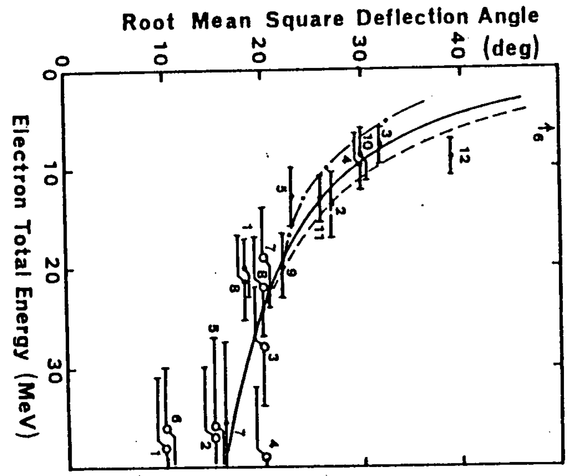

Fig. 2: Estimated reconstruction eroor against incident energy of electron. Gaussian approximation (broken line), corrected result by Mott formula (solid line), estimation by Kamioka group (dot broken line), Kamioka data (dot point), and IMB data (open circle).

Lower cut of the scattering angle is λ/a for both formulae, where a denotes atomic radius.

Our corrected results explain experimental data well for wide energy range as shown in Fig.2 (solid line).

5 Conclusion and Discussion

Reconstruction error of emitted direction of electrons in water cerenkov detector is well evaluated for wide energy range by root mean square deflection angle of electron weighted by cerenkov photon count, if we use gaussian approximation for multiple scattering process, assume exponential decrease with mean square deflection angle, and make correction by Mott formula.

Disagreement of estimation of Kamioka group with experimental data seems to come from ill application of cut of large angle scattering used in EGS simulation code.

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