Topological Structure of Mind

Wasaburo Unno
Research Institute for Science and Technology, Kinki University
Higashi-Osaka-shi, Osaka 577, Japan

(Received December 17, 1991)

Abstract

Topological representation is undertaken for the function of mind. Klein's bottle so modified as to have the point cross section of infinite area at the intersection of different parts of its own surface is considered as the simplest model of elementary topological structure of mind. Brain may be the manifestation of the multiple combination of such singular elementary structures. Mind is considered to be the excitation of one combination and change of mind is then the transfer of excitation to other combination of those singular elementary structures.

Key words: Fractal structure, Paradox, Klein's bottle, Incompleteness Theorem

1 Introduction

Human mind should have a rather simple structure, if everybody feels in the same way for basic elementary psychological stimulation, since the principle must be simple for anything which is universal. In physics, for instance, a simple model to describe the universe is the distribution of positions of all the elementary particles exerting forces for any given instant of time. The properties of the elementary particles are taken to be universal. In this case, an alternative simpler description of the universe could be given by vacuum in a high dimensional asymmetric space. In fact, the present day super-string theory seems to be consistent with the above conjecture. In such cases, mathematical description is quite useful. In short, anything universal may have a simple mathematical model. The structure of mind may be also universal and, therefore, have a simple mathematical model.

Imanuel Kant wrote the following famous sentence in his "Kritik der reinen praktischen Vernunft", "Zwei Dinge erfüllen das Gemüt mit immer neuer und zunehmender Bewunderung und Ehrfurcht, je öfter und anhaltender sich das Nachdenken damit beschäftigt: der bestirnte Himmel über mir und das moralische Gesetz in mir."The beauty of this sentence seems to come from the compact description of the universality of the structure of mind. One should then be able to construct a simple topological model of mind by using this sentence as the guide line for the mathematical epistemology.

Dualism is apparent in the sentence. Therefore, we consider a bottle of which der bestirnte Himmel occupies the outside and das moralische Gesetz forms the inside. It has to be noticed, however, that at least der bestirnte Himmel and perhaps also das moralische Gesetz extend to infinity but they are together filling up das Gemüt that is finite in extent. They are glued together within it. Such a paradoxical topology may be realized by a structure known as Klein's bottle. Inside continues to outside and vice versa in the latitudinal direction along the bottle. This feature of Klein's bottle is considered to be appropriate to model the gross structure of the mind. However, another special structure of mind beyond Klein's bottle has to be mentioned here also. That is the self exciting mechanism working within the mind as expressed by "mit immer neuer und ..., je öfter

und .... damit beschäftigt".

The presence of the self exciting mechanism in the mind should be essential in the creation from chaos, a feature which is absent in the most powerful computer of the present time.

The oriental philosophy especially the Buddhism describes the basic state of everything to be "Kuu" in Japanese or "Sunya" in Sanskrit, the emptiness or zero, in which Shankara, an Indian philosopher in the 8th century saw structures. Therefore "Kuu" or the nothingness involves the self excitation mechanism, which is another paradox of the mind. We feel, together with Albert Einstein, that the most curious and surprising thing is that the human being can understand the universe. The human being is a tiny part of the universe but can map the universe on the mind almost by one-to-one correspondence from time to time, a feature which is possible only for infinite sets. Therefore, both the universe and the mind must be infinite sets. Also, the existence of the human being within the universe forms the famous paradox of the self reference, and so is the structure of the mind.

In the next section, we will introduce a primitive mathematical model which describes the structure of the mind discussed above. We will first discuss whether this is possible or not.

2 Self-identity of Absolute Contradiction

The Gödel-Henkin completeness theorem in mathematical logic (see J. N. Crossley et al., 1972) tells that there exists a model for a consistent set of predicate sentences. That means that if there exist no models, not all the sentences are valid. Therefore, to see the validity of sentences that are paradoxical, the construction of a model is crucial. Such a model does not, of course, prove the universal validity of the sentences, but opens the possibility and encourages further development of the concept.

Paradox looks like but differs from contradiction. A paradox in logic arises in the self reference. There are, however, true paradoxes and avoidable paradoxes in the logical paradoxes. The true predicate paradox arises in the purest form when we ask the validity of the sentence, " This sentence is not true". If the sentence is assumed to be true, the assumption contradicts with the sentence, and, if the sentence is assumed to be false, that means that the sentence is assumed to be true, the assumption again contradict with the sentence. Thus, in a true predicate paradox, the validity is undecidable.

There is a quite similar well-known paradox which says, " A Cretan said that all Cretans are liars". In this case, there is a narrow escape from being a true paradox. The solution can be found by noticing that the Cretan is referring not to himself but all Cretans and, therefore, that his position can be in a different class from others. In other word, what he said is the thing within his brain which simulates the real world only approximately, and it is quite possible that he himself has dropped out as a small error from the set constituted by all the Cretans.

The same logic of "the different class" does not seem to apply if he said "I am a liar". But, the paradox is strictly unavoidable only during the time when he was so speaking, since a liar may not always tell a lie. Within that one second only, whether he was a liar or not is uncertain. A person, unlike a sentence, is not entirely predicative, making some difference in the logic. Being a specified person and acting as a liar are within the uncertainty within one second of the speech which plays the role of the Planck constant.

"Self-identity of absolute contradiction" is a key word of the philosophy by Kitaro Nishida (1870-1945) which describes the function of mind in both the pure experience before consciousness and the insight cooperating with action (Nishida, 1953). It is related somewhat to the concept of dialectics but more to the Zen-Buddhism. A question arises whether it is a logically sound concept or not. Is it possible for anything to exist in absolute contradiction ? If it does exist, it gives a new category of paradox differing from the paradox discussed above. A particular example of the self-identity of absolute contradiction has been pro-

posed in the previous paper (Unno, 1991) in a different context. It is the straight line segment of infinite length. The construction is quite simple as described below.

A sine curve of one wave length is drawn between two points A and B, as expressed by

$$y = A\sin(2\pi x/L), (0 < x < L), \qquad (1)$$

L being the distance between A and B, and A the amplitude. Then, a sine curve having the wave length just half of that of the previous curve and with the amplitude less than but larger than half of that of the previous curve is drawn between A and B $(0 < x < L)$,

$$y = Aa\sin(2\pi 2x/L), (a > 0.5). \qquad (2)$$

The new curve (two wavelengths between A and B) is certainly longer than the previous curve. If the process is repeated infinite times, the limiting line will have zero amplitude (straight line) and infinite length. The n-th curve will be represented by

$$y = Aa^n\sin(2\pi 2^n x/L), \qquad (3)$$

having a length of $4A(2a)^n$, tending to infinity for infinite n.

The secret of the paradox lies in the fractal structure with the fractal dimension between 1 and 2. The self similarity is there (sine curve!) as in usual fractal figures, but here length ratios between successive sine curves are different in the direction parallel to AB and perpendicular to it. The involvement of these two directions gives rise to the fractal structure in the limit. The self similarity corresponds structurally to the self reference of the "this sentence" paradox. The latter paradox is so to say a one dimensional paradox. Therefore, in the "liar" paradox, the addition of other dimension, in plurarity of persons and/or in time, dissolves the paradox. Only within one second during which a Cretan was exclaiming, "I am a liar!", he was in the self identity of absolute contradiction. In that, he was exclaiming something spiritual which is beyond his statement. The new paradox may be called as the fractal structure paradox.

The construction of the line segment of infinite length tells that there are innumerable models for the self identity of absolute contradiction, since there is no limit of dimensions of figures to start with. The model not only proves the logical validity of the concept but also implies that the concept is the result of the limiting procedure with amplification from a higher dimension. We can construct even a point of infinite length by diminishing the distance between A and B in every step by a factor larger than unity only so slightly that the total length of the sine curve still increases. In the limit, A and B coincides, but the length of the sine curve will tend to infinity. In fact, the n-th sine curve constructed as above will have the length,

$$l = 4A(2a)^n, (2a > 1), \qquad (4)$$

for the limit of large n, which is independent on L. So, one can choose L as small as possible from the beginning, which proves the existence of a point of infinite length. Quite similarly, we can start with a surface:

$$z = A\sin(2\pi x/L)\sin(2\pi y/L), (0 < x, y < L), \qquad (5)$$

and then, we can proceed to the n-th surface given by

$$z = Aa^n\sin(2\pi 2^n x/L)\sin(2\pi 2^n y/L), (0 < x, y < L) \qquad (6)$$

having the area:

$$s = 4A^2(2a)^{2n}, \quad \text{for large } n. \qquad (7)$$

Since this limiting value is independent on L, we can reduce L to zero after making n to infinity, obtaining a point of infinite surface area. A point of infinite length and a point of infinite area can be identified literally as models of the self-identity of absolute contradiction. It is actually possible, therefore, that "sunya", the zero, can have not only structure but also infinite structure, a statement beyond Shankara of the 8th century.

The self-identity of absolute contradiction is the basic concept of Nishida's epistemology which is in a sense the philosophical description of Zen's spirit. Since the point of infinite length or infinite area is the simplest (dimension 1+ or 2+) model of the self-identity of absolute contradiction which itself is a partial interpretation (model) of the mind, the simple mathematical model of the mind should be suitably constructed in part with the structure having the property of the point of infinite length or of infinite area.

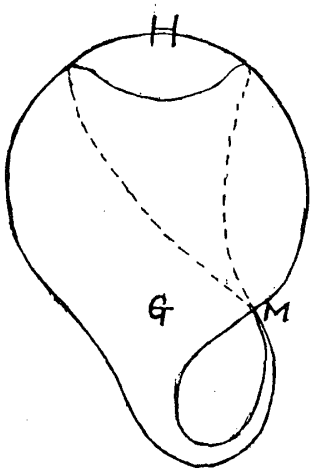

Figure 1. Kkein's bottle with point intersection, M, of infinite area. Inside: das moralisch Gesetz, G; outside: der bestirnte Himmel, H.

3 Klein's Bottle

Judgment is the action of mind which distinguishes truth/false, yes/no, in/out, der bestirnte Himmel/das moralische Gesetz, etc.. The global structure of mind is, therefore, represented by a surface in multi-dimensional phase space composed of different kinds of judgment. Since we are familiar with the three dimensional space, we will consider a two dimensional surface on which the reduced multi-dimensional phase space belongs to every point. The problem to be considered in this section is the global structure of the two dimensional surface in the three dimensional space. A partial support of this consideration is the fact that mind is the function of brain which is enclosed within finite volume in the three dimensional space. Change of mind or the transformation from one world to the other will be discussed later.

We now consider, as a simplified model, the surface of which the outside extends der bestirnte Himmel and the inside extends das moralische Gesetz. The dualism, however, has its origin in the human frame of judgment. In other words, there is membrane in the mind which separates inside from outside. Without the membrane, inside is continuous to outside. In fact, Möbius' belt has no front and back surfaces, since each surface is the continuation of its back surface. (A belt of which both ends are connected after twisting by half rotation forms Möbius' belt.) Thus, Möbius' belt can in principle reduce the dualism to the monism. Möbius' belt, however, is incomplete in a sense that the sides are cut artificially and disconnected with the opposite sides.

In this respect, Klein's bottle is much more satisfactory and natural. (Starting with an elastic spherical surface having holes on both the top and the bottom, pulling down the top hole inside the sphere with a little bending until it hits and penetrates the sphere, and then stretching the tube further down and bending it sharply towards the bottom hole, we obtain Klein's bottle by sealing up both holes with each other. The membrane is at the sealing cross-section.

Asymmetry of the configuration may not be a defect, since universe starts from asymmetry. But, the hole on the spherical surface penetrated by the tube surface must be remedied. The remedy can be achieved by making the penetrating tube thickness on the sphere surface (M in Fig.1) to be infinitesimal. But, then no information will transmit from inside to outside and from outside to inside of the sphere. At this point, we may ask help to the point of infinite area to secure large flux of information from H to G (Fig.1) and vice versa. Thus, the area of the penetrating tube of infinitesimal cross section can be infinite in the neighborhood of M. Also, no artificial membrane is needed to separate inside from outside. We consider that the point of infinite area at M forms the core of the mind and its infinite area acts like infinite number of infinitely thin optical fibers trans-

mitting infinitely large flux of information through the infinitesimal pin hole from H to G and from G to H.

4 Creation

Klein's bottle with point intersection of infinite area can be a model of mind at one time. However, mind changes from time to time. Since the change of mind is one of the general characteristics of mind, it should be represented most properly as the trajectory in multi-dimensional space. An alternative way may be to provide several (more than three) Klein's bottle structures discussed above and to make them interact nonlinearly with each other to form multi-dimensional chaos. This is essentially the simulation of the brain which is the multiple combinations of different parts governing different functions. In the latter case, the construction of new space depends on the analog dynamical system which has been developed through the evolution of the life. The system seems to be driven by curiosity, empathy, and logic. We will start with discussing the logic in what follows.

The Gödel-Henkin completeness theorem tells that the universally valid formulae in the predicate calculus are provable and vice versa and that their models can always be constructed. But, at the same time, Gödel's incompleteness theorem tells that there are formulae in the predicate calculus that are neither provable nor disprovable (see Crossley,1972).

Since the predicate calculus and the logic are essentially the same, we may understand that all the truth we know are provable in logic but there are always problems that cannot be logically judged with our present knowledge. In other words, the ideal computer, the universal Tüling machine cannot create new ideas. In order to study essentially unknown problems, therefore, we must work on higher dimensional world by introducing new experience either from the outside world or from the inside (cf. Hofstadter 1979, for similar arguments). The natural history in science is to hear the story (histoire) told by nature and therefore is the mother of science! The voice from inside should not be through logic but through intuition and body (e.g. Zen) which carries knowledge on universe through the evolution of the life.

The point of infinite area situated in the core of mind has a fractal dimension of $2+\alpha$. So, it has more dimensions than the predicate calculus may have. This may be the origin of the intuition on which the philosophy and the natural philosophy depend. If it is true, the inclusion of chaos dynamics in the computer may provide possibility to develop a bio-computer which creates.

On the other hand, curiosity is basic for advanced animals. Amphibia may have developed curiosity, when they began to land on the shore. Now rain flogs climb up perpendicular wall into the upper floors in house, perhaps with curiosity in spite of danger of death by dryness. Flogs detect newness by comparing the image of the moving object with the model in their memory (Hokkyo,1990).

Empathy is basic for advanced mammals. Human being developed it with the help of language which describes the situation and transfer information. Thus, curiosity, empathy, and logic are brothers, cooperating to create new ideas.

I thank Dr. M. Kiguchi for calling attention to Hofstadter (1979).

References

[1] Crossley, J. N., 1972, "What is Mathematical Logic ?", Oxford University Press: republished in 1990, Dover.

[2] Hofstadter, D. R., 1979, Gödel, Escher, Bach", Basic Books Inc., New york.

[3] Hokkyo, N., 1990, "Advanced Front of Sciences" Tetugaku Pub.Co.,Kyoto.

[4] Nishida, K., 1953, "Lecture on Philosophy", in Japanese, Iwanami Publ.Co.,Tokyo.

[5] Unno, W., 1990, Science and Technology, No.2, 1.