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# Nonscale Model of Economic Growth with Public Input

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**Abstract**　This paper presents development of a nonscale model of economic growth with public input. For this paper, it is assumed that the production function has no restriction of scale in the three inputs of labor, capital, and public input. The balanced growth rate is determined by the production elasticities and population growth rate. Therefore, government policy is effective during the short term. Results show that a quantity-oriented government is not only successful in attaining its purpose: it also attains the second best equilibrium, although a growth-oriented government fails to attain its purpose.

**Key words**　Endogenous growth; Nonscale model; Public input
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**概要**　本論文は，生産投入要素としての公共財を考慮した経済成長モデルを用いて，持続的な経済成長の条件及び公共投資の最適条件について理論的検証を行っている。主要な結果は以下のとおりである。第一に，持続的な経済成長のための必要十分条件は，生産関数が規模に関して収穫逓増であることであり，また，持続的経済成長経路において必ずしも動学的効率性が達成されないことが示された。第二に，公共投資は短期的な経済成長促進効果を持つが，長期的にみれば経済成長への貢献は，生産要素としての役割に限定され，最適な公共投資の対 GDP 比率は生産に対する公共財の弾力性に等しいことが示された。

**キーワード**　内生的経済成長，生産技術，公的投入要素

## 1. Introduction

Very important contributions to modern economic growth theory were those of Solow (1956) and Swan (1956), who assume that the production function is of the neoclassical form: constant returns to scale and diminishing returns to each input of capital and labor. A simple general equilibrium model of capital accumulation can be constructed as a combination of this production function and a constant saving rate. Their models show that the long-run economic growth rate depends on the exogenous rate of population growth. Consequently, the per-capita growth rate is zero.

Observation of the determinants of economic growth shows that the economic growth rate is strongly affected by government policy and other economic factors: Many empirical studies have examined the growth effects of fiscal policy, including effects of public investment and other government expenditures (e.g. Aschauer 1989; Devarajan et al. 1996; Kneller et al. 1999; and Shioji 2001). Numerous studies, including those, support the positive growth effects of fiscal policy.

The first model linking public investment to sustainable per-capita growth was presented by Barro (1990). For that model, it is assumed that public investment affects aggregate production and renders the long-run growth rate an endogenous variable.(1) His study promoted numerous subsequent studies of extensions of his model (e.g. Lee 1992; Greiner 1998; Piras 2001). However, endogenous growth models are limited in that the production function must include constant returns to scale in the reproducible factors of production. This is not only a strong necessary condition in the sense that it strongly restricts the production structure; it also raises the annoying problem of scale effects.(2)

To evade these limitations and re-examine the macroeconomic effects of fiscal

(1) Futagami et al. (1993) extend the Barro model by assuming that public capital has a positive effect on aggregate production.

(2) See also Solow (1994) and Yoshikawa (2000) regarding the former criticism. Regarding the latter problem, Backus et al. (1992) finds little empirical evidence of the existence of a scale effect.

policy, we construct a nonscale model of endogenous growth with public input by extension of Turnovsky (2000, Ch.14). This paper shares essential features with studies from the literature on investigation of the effects of a fiscal policy exogenous growth model (e.g. Baxter and King 1993; Chang et al. 1999). Results from our analysis, especially those specifically related to increasing returns to scale in three production factors, are summarized as follows.

First, we show that a unique balanced growth equilibrium exists. The per-capita balanced-growth rate is determined by the production elasticities and population growth rate. Its balanced-growth rate is positive if constant returns to scale in labor and capital pertain, while the per-capita growth rate in a neoclassical growth model is zero. Characterizing transitional dynamics, consumption increases over time in the economy with capital stock less than its stationary level.

Second, results demonstrate that fiscal policy does not affect the long-run per-capita growth rate because the long-run growth rate depends only on exogenous variables. Of course, fiscal policy has a positive effect on the per-capita growth rate of consumption, capital, and output in the short run. However, the growth-oriented government fails to attain higher long-run growth. On the other hand, a quantity-oriented government (e.g. which has plans to maximize consumption, national income, or per-capita income) can put its purpose into practice and attain the second best equilibrium.

Third, in the short run, a rise in population growth rate has negative impacts on per-capita growth rates of consumption and capital. In some cases, it also negatively affects the per-capita growth rate of output. In contrast, the long-run balanced growth rate is increasing in the population growth rate if there are increasing returns to scale in three production factors. However, from the viewpoint of welfare analysis, high population growth is not always desirable.

This paper is organized as follows: Section 2 presents a description of our model. Section 3 solves the model, characterizes the transitional dynamics, and investigates the dynamic effects of policy and demographic shocks. Section 4 provides welfare analysis. Finally, Section 5 concludes this paper.

## 2. The economy

We follow Turnovsky (2000, Ch.14) in terms of the details of the basic structure of our model, excluding the presence of public input.(3) Time is continuous and indexed as $t$.(4) Final good $Y(t)$ is producible using

$$Y(t) = N(t)^{\sigma_N} K(t)^{\sigma_K} G(t)^{\sigma_G}, \tag{1}$$

where $N(t)$ is the labor input, $K(t)$ the physical (private) capital input, $G(t)$ the public input, $\sigma_N > 0$, $\sigma_K > 0$, and $\sigma_G > 0$.

Government provides the public input. It taxes household income and maintains the tax rates as constant over time. Consequently, the government's budget constraint is

$$G(t) = \tau Y(t). \tag{2}$$

The number of households is $N(t)$, which is assumed to grow at the constant rate of $n$ (i.e. $\dot{N}/N = n$). The lifetime utility of the representative household is defined over per-capita private consumption:

$$U(0) = \int_0^{\infty} \frac{[C(t)/N(t)]^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt, \tag{3}$$

where $\theta$ and $\rho$ respectively represent the inverse of intertemporal elasticity of substitution and the subjective discount rate. The budget constraint is

(3) Regarding the relevant literature, a paper by Eicher and Turnovsky (1999) presents development of a two-sector nonscale model of economic growth. Although their approach is a general characterization of a nonscale model of economic growth, we adopt a one-sector non-scale model of economic growth for analytical simplicity because the introduction of public capital complicates the mechanism of effects of public investment.

(4) Throughout this paper, a dot above a letter denotes the time derivative. Furthermore, the time index $t$ is not used for time-variant variables, except for those cases in which it must be called to the reader's attention (i.e. $\dot{x}(t) \equiv dx(t)/dt$ and $x$ is used as $x(t)$. It is noteworthy that $x(0)$ denotes the initial value of $x(t)$).

$$\dot{K}(t) = (1 - \tau)Y(t) - C(t), \tag{4}$$

where $\tau$ is the tax rate on income $(0 < \tau < 1)$. Solving the optimization problem of households, we obtain

$$\dot{C}(t) = \left[(1-\tau)\frac{\partial Y(t)}{\partial K(t)} - \rho + (\theta - 1)n\right]\frac{C(t)}{\theta},$$

and the transversality condition.

We now consider the properties of balanced growth equilibrium. From (1), the growth rate of aggregate output is

$$\frac{\dot{Y}}{Y} \equiv g_Y = \sigma_N \frac{\dot{N}}{N} + \sigma_K \frac{\dot{K}}{K} + \sigma_G \frac{\dot{G}}{G}.$$

Using (2), the equation shown above is rewritten as

$$g_Y = \frac{\sigma_N}{1 - \sigma_G}\frac{\dot{N}}{N} + \frac{\sigma_K}{1 - \sigma_G}\frac{\dot{K}}{K}. \tag{5}$$

The standard definition of the balanced growth equilibrium (or stationary equilibrium) is that all endogenous variables grow at a constant rate. According to the stylized facts, we assume that the ratio of output to physical capital is constant (i.e., $Y/K = const.$) in the long run.(5) Consequently, Eq. (5) is rewritten as

$$\frac{\dot{Y}}{Y} = \frac{\sigma_N}{1 - \sigma_K - \sigma_G}\frac{\dot{N}}{N}$$

Equation (6) implies that both $Y$ and $K$ grow at the weighted growth rate of population in the long run. Therefore, we define the dynamic system as the time paths of

$$c \equiv \frac{C}{N^{\sigma_N/(1-\sigma_K-\sigma_G)}} \text{ and } k \equiv \frac{K}{N^{\sigma_N/(1-\sigma_K-\sigma_G)}}. \tag{7}$$

(5) See Kaldor (1961) and Romer (1989) for stylized facts.

Derivation of the dynamic equations of $c$ and $k$ requires the growth rates of consumption, private capital, and public capital. The optimal condition for household leads to

$$\frac{\dot{C}}{C} \equiv g_C = \frac{(1-\tau)\sigma_K \tau^{\sigma_G/(1-\sigma_G)} k^{(\sigma_K+\sigma_G-1)/(1-\sigma_G)} - \rho + (\theta-1)n}{\theta}. \quad (8)$$

After some manipulation, we obtain the growth rate of physical capital as

$$\frac{\dot{K}}{K} \equiv g_K = (1-\tau)\tau^{\sigma_G/(1-\sigma_G)} k^{(\sigma_K+\sigma_G-1)/(1-\sigma_G)} - \frac{c}{k}. \quad (9)$$

## 3. Dynamic analysis

This section presents an investigation of the dynamic system of this economy, the long-run and short-run effects of policy shock, and those of demographic shock.

### 3.1. Dynamic system and transitional dynamics

First we derive the dynamic system of the economy. Equations (8), (89), and $\dot{N}/N = n$ engender

$$\dot{c} = \left[\frac{(1-\tau)\sigma_K \tau^{\sigma_G/(1-\sigma_G)}}{k^{(1-\sigma_K-\sigma_G)/(1-\sigma_G)}} - \rho + (\theta-1)n - \frac{\sigma_N \theta n}{1-\sigma_K-\sigma_G}\right]\frac{c}{\theta}, \quad (10)$$

$$\dot{k} = (1-\tau)\tau^{\sigma_G/(1-\sigma_G)} k^{(\sigma_K+\sigma_G-1)/(1-\sigma_G)} - c - \frac{\sigma_N n k}{1-\sigma_K-\sigma_G}. \quad (11)$$

We define the balanced-growth equilibrium as that which satisfies $\dot{c}(t) = \dot{k}(t) = 0$. Then, three endogenous variables, $C(t)$, $K(t)$, and $Y(t)$, grow at the same rate:(6)

$$g^* \equiv g_C = g_K = g_Y = \frac{\sigma_N n}{1-\sigma_K-\sigma_G}. \quad (12)$$

(6) Superscript“ * ”denotes the stationary value of the endogenous variable.

We assume for $g^* > 0$ that there are decreasing returns to scale in $K$ and $G$ (i.e., $\sigma_K + \sigma_G < 1$). Using (12), the per-capita growth rate is

$$g^* - n = \frac{(\sigma_K + \sigma_G + \sigma_N - 1)n}{1 - \sigma_K - \sigma_G}$$

The cases of constant and decreasing returns to scale in all factors are trivial. We specifically examine the case of increasing returns to scale in all factors (that is $\sigma_K + \sigma_G + \sigma_N > 1$).

Regarding the existence, uniqueness, and stability of the stationary equilibrium, the following proposition is established (See Appendix for a proof of Proposition 1).

**Proposition 1.** *A unique balanced-growth equilibrium exists, which is stable in the saddle-point sense. Then, the balanced-growth equilibrium is dynamically efficient, a golden rule path, or inefficient, according to*

$$(1 - \sigma_K - \sigma_G)\rho + [(\theta - 1)(\sigma_K + \sigma_G + \sigma_N - 1) + \sigma_G \sigma_N]n \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$$

*respectively.*

Equation (14) is positive if $\theta > 1$, although it might be negative if $\theta < 1$.[7] Because $\theta \geq 1$ is reported by many empirical studies, we might safely say that the balanced growth equilibrium is dynamically efficient for given exogenous parameters.

The phase diagram of the dynamic system is portrayed in Fig. 1 (dynamically efficient case). A unique stable branch $SS$ has a positive slope, so that $c(t)$ and $k(t)$ is increasing (decreasing) over time for $t < \infty$ when $k(0) < k^*$ (>). Formally, we have the following dynamic equation:

$$c(t) = c^* + \alpha \cdot (k(0) - k^*) \exp(\chi_1 t), \tag{15}$$

(7) When $\sigma_K + \sigma_G = 1$, Eq. (14) is positive.

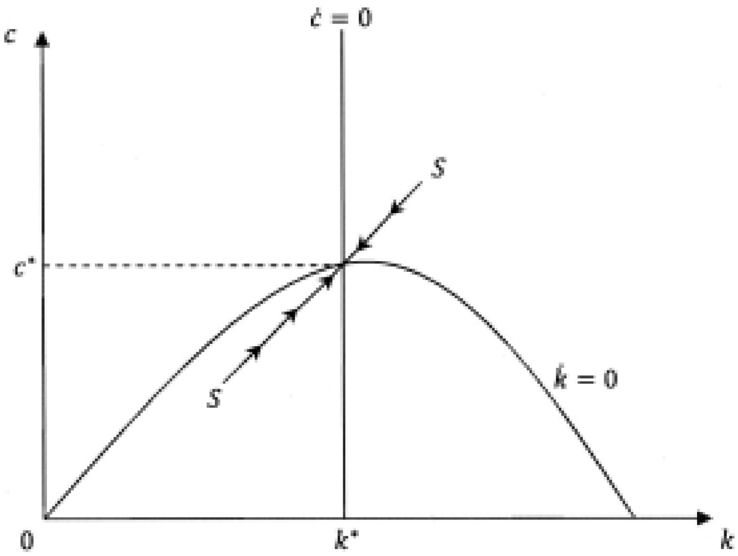


**Figure 1.　Phase diagram**

$$k(t) = k^* + (k(0) - k^*)\exp(\chi_1 t), \tag{16}$$

where $\alpha > 0$.[8]　Applying (15) and (16) to (5), (8), and (9), we can derive the time path of $g_C$, $g_K$, and $g_Y$ as

$$\frac{dg_C}{dt} < 0,\ \frac{dg_K}{dt} \gtreqless 0,\ \text{and}\ \frac{dg_Y}{dt} \gtreqless 0\ \text{for}\ k(t) < k^*$$

Starting from $k(0) < k^*$, $g_C$ is decreasing over time for $t < \infty$.　However, the time paths of $g_K$ and of $g_Y$ are ambiguous.

### 3.2.　Dynamic effects of policy shock

We next examine the long-run effects of an increase in the tax rate.　Through total differentiation of a stationary dynamic system, we obtain the long run effect of a policy shock on $k$:

$$\frac{\tau}{k^*}\frac{dk^*}{d\tau} = \frac{\sigma_G - \tau}{(1-\tau)(1-\sigma_K - \sigma_G)} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr \sigma_G. \tag{17}$$

(8)　See Appendix for derivation of (15) and (16).　Also, $\alpha$ denotes the slope of stable arm.

Then, the long-run effect of policy shock on $c$ is

$$\frac{dc^*}{d\tau} = \frac{(\sigma_G - \tau)\tau^{\sigma_G/(1-\sigma_G)}(k^*)^{\sigma_K/(1-\sigma_G)}}{(1-\sigma_G)\tau} + \frac{\partial c^*}{\partial k^*}\frac{dk^*}{d\tau} \gtreqless 0 \Leftrightarrow \tau \lesseqgtr \sigma_G. \tag{18}$$

It is important to note that $\partial c^*/\partial k^* > 0$ when the balanced-growth equilibrium is dynamically efficient.

Differentiation of (15) and (16) with respect to $\tau$ yields

$$\frac{dc(t)}{d\tau} = \frac{dc^*}{d\tau} - \alpha\frac{dk^*}{d\tau}\exp(\chi_1 t) \gtreqless 0, \tag{19}$$

$$\frac{dk(t)}{d\tau} = [1-\exp(\chi_1 t)]\frac{dk^*}{d\tau} \gtreqless 0. \tag{20}$$

For $t = 0$, we have $dk(0)/dt = 0$ and

$$\frac{dc(0)}{d\tau} = \left[\frac{dc^*}{dk^*} - \alpha\right]\frac{dk^*}{d\tau} \gtreqless 0 \Leftrightarrow \tau \gtreqless \sigma_G,$$

where $dc^*/dk^* < \alpha$.[9]

The interpretation of the effects of policy shock is explained geometrically as follows: Suppose that the economy is initially in the balanced-growth equilibrium at the point $P$ on the stable arm $SS$ in Figure 2, and that there is a permanent increase in the tax rate $\tau$ $(\tau < \sigma_G)$. The stationary equilibrium point $P$ shifts to the new point $P'$; the stable arm $SS$ also shifts to the new locus $S'S'$. Figure 2 portrays that $c$ decreases initially, increases gradually (for $0 < t < \infty$), and finally converges to new equilibrium point $P'$. On the other hand, $k$ is monotonically increasing in time $(0 < t < \infty)$; it converges to a new equilibrium point $P'$.

A policy shock has no impact on the long-run per-capita growth rate because it depends on exogenous variables $\sigma_N$, $\sigma_K$, $\sigma_G$, and $n$. However, a policy shock has a short-run effect on the per-capita growth rate through a transitional process toward a new balanced-growth equilibrium. Presuming that the initial econ-

(9) $dc^*/dk^*$ stands for the slope of $\dot{k} = 0$. It therefore must hold that $dc^*/dk^* < \alpha$ geometrically.

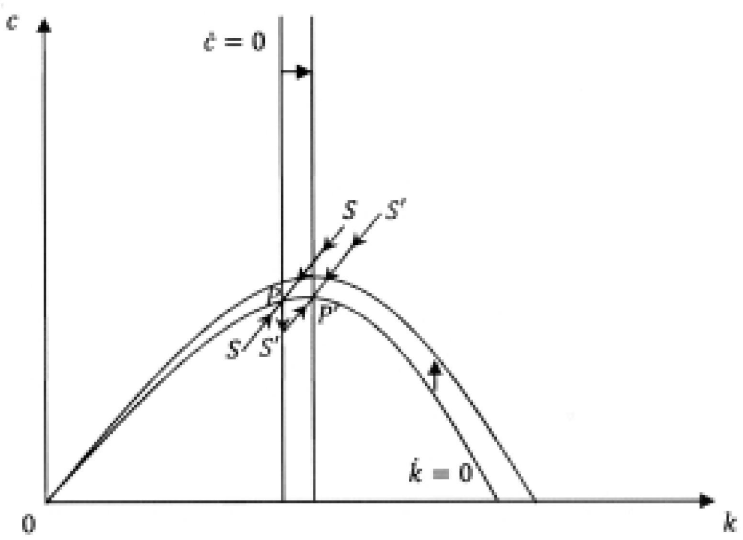


**Figure 2. Dynamic effects of policy shock on $c$ and $k$ ($\tau < \sigma_G$)**

omy is in the balanced-growth equilibrium with $\tau < \sigma_G$, then using (5), (8), (9), (19), and (20), we can see the short-run effects of a policy shock on the growth rate of per-capita endogenous variables at time 0 as follows.

$$\frac{d\,(g_C - n)}{d\tau} > 0, \frac{d\,(g_K - n)}{d\tau} > 0, \text{ and } \frac{d\,(g_Y - n)}{d\tau} > 0 \text{ for } t = 0.$$

The short-run effects of policy shock on per-capita growth rates are all positive. In light of these effects, the short-run effect on the per-capita growth rate of consumption is as depicted in Fig. 3.

Thereby, the implications in this subsection are summarized as follows.

**Proposition 2.** *If and only if $\tau = \sigma_G$, the competitive economy attains the maximum of the long-run per-capita consumption. In the short run, an increase in the tax rate has a positive effect on per-capita growth rates of consumption, capital, and output.*

### 3.3. Dynamic effects of demographic shock

In the long run, a rise in the population growth rate has a positive effect on the per-capita growth rate if and only if there are increasing returns to scale in

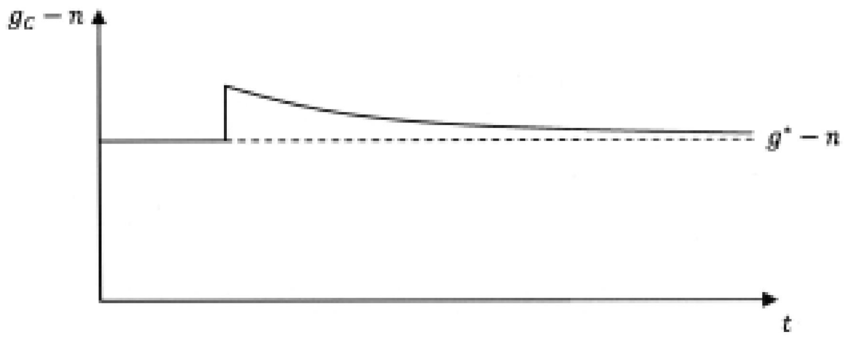


**Figure 3. The short-run positive effect of policy shock on $g_C - n$ ($\tau < \sigma_G$)**

three production factors (i.e., $\sigma_K + \sigma_G + \sigma_N > 1$). However, the short-run effect of the demographic shock is not so simple. In this subsection, we investigate the short-run effect of demographic shocks on per-capita growth. Without loss of generality, we assume that the government sets the income tax rate to $\tau = \sigma_G$.

Comparative statics reveal the following:

$$\frac{n}{k^*}\frac{dk^*}{dn} = -\frac{(1-\sigma_K-\sigma_G)n + (\sigma_K+\sigma_G+\sigma_N-1)\theta n}{(1-\sigma_K-\sigma_G)(\rho+n) + (\sigma_K+\sigma_G+\sigma_N-1)\theta n}\frac{1-\sigma_G}{1-\sigma_K-\sigma_G} < 0. \quad (21)$$

The long-run effect on $c$ is

$$\frac{dc^*}{dn} = -\frac{\sigma_N k^*}{1-\sigma_K-\sigma_G} + \frac{\partial c^*}{\partial k^*}\frac{dk^*}{dn}. \quad (22)$$

Finally, we examine the short-run effects of demographic shock. Differentiation of (15) and (16) with respect to $n$ yield

$$\frac{dc(t)}{dn} = \frac{dc^*}{dn} - \alpha\frac{dk^*}{dn}\exp(\chi_1 t) \gtreqless 0, \quad (23)$$

$$\frac{dk(t)}{dn} = [1-\exp(\chi_1 t)]\frac{dk^*}{dn} \leq 0. \quad (24)$$

For $t = 0$, we have $dk(0)/d\tau = 0$ and

$$\frac{dc(0)}{dn} = \left[\frac{dc^*}{dk^*} - \alpha\right]\frac{dk^*}{dn} < 0.$$

The interpretation of the short-run and long-run effects of demographic shock on $c$ and $k$ is as follows: Suppose that the economy is initially in the balanced-growth equilibrium at the point $Q$ on the stable branch $XX$ in Figure 4, and that there is a permanent increase in the fertility rate $n$. The stationary equilibrium point $Q$ shifts to the new point $Q'$; the stable arm $XX$ also shifts to the new locus $X'X'$. Figure 4 shows that $c$ increases initially, gradually decreases (for $0 < t < \infty$), and finally converges to a new equilibrium point $Q'$. On the other hand, $k$ is decreasing over time for $0 < t < \infty$. It then converges monotonically to new equilibrium point $Q'$.

Equations (5), (8), (9), (23), and (24), engender short-run effects of demographic shock on per-capita growth rates at time 0:

$$\frac{d(g_C - n)}{dn} < 0, \frac{d(g_K - n)}{dn} < 0, and\ \frac{d(g_Y - n)}{dn} \gtreqless 0 \text{ for } t = 0.$$

The short-run effect of demographic shock on the per-capita growth rate of output is generally ambiguous, although that on per-capita growth rates of consump-

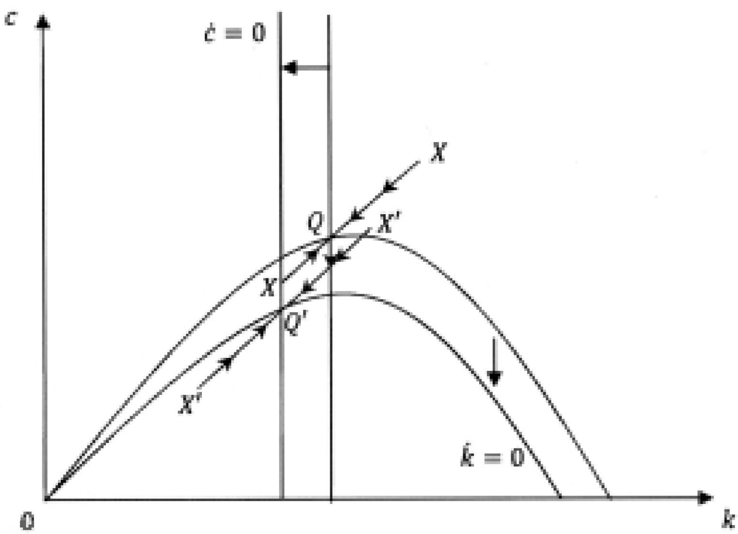


**Figure 4. Dynamic effects of demographic shock on c and k**

tion and capital are negative (see Fig. 5). The short-run effect of demographic shock on the per-capita growth rate of output is negative if there are decreasing returns to scale in labor and public input $(\sigma_G + \sigma_N < 1)$.

The implications in this subsection are summarized as the following proposition.

**Proposition 3.** *In the short run, a rise in the population growth rate has a negative effect on per-capita growth rates of consumption and capital, and an ambiguous effect on per-capita growth rate of output, although it raises the per-capita balanced-growth rate.*

## 4. Welfare analysis of fiscal policy

Considering the social planner's optimization problem, the optimal condition for providing public input is $\partial Y/\partial G = 1$, the so-called Kaizuka condition.(10) Under (1), this optimal condition gives the optimal size of government. Indeed, we obtain the optimal size of government as $(G/Y)^{\star} = \tau^{\star} = \sigma_G$ for all $t$. In the decentralized economy, the maximization of long-run per-capita consumption yields the same size of government. The decentralized equilibrium with $\tau = \sigma_G$ will give less benefit from growth of per-capita consumption during the transitional process because the income tax is a distortionary tax. However, once the

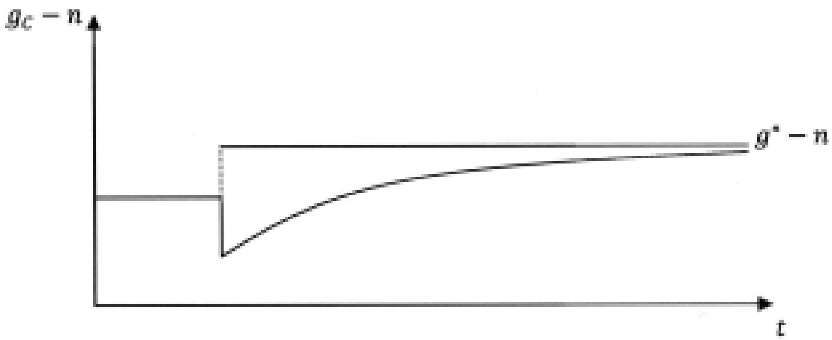


**Figure 5. The short-run negative effect of demographic shock on $g_c - n$**

(10) See Kaizuka (1965) and Sandmo (1972).

competitive economy arrives at the balanced-growth equilibrium, the per-capita growth rate is constant over time and is equal to the socially optimal growth rate.

Alternatively, the introduction of a subsidy for saving (or equivalently tax deduction for saving) financed by a lump-sum tax makes it possible for the competitive equilibrium to attain the social optimum. The basic structure of the model is unchanged, excluding for (2) and (4). Eqs. (2) and (4) can be rewritten as

$$\begin{aligned} G + Z &= \tau Y + T, \\ \dot{K} &= (1-\tau)Y + \zeta\dot{K} - C - T, \end{aligned} \tag{25}$$

where $Z$ is the expenditure for saving subsidy, $T$ is the lump-sum tax, and $\zeta$ is the subsidy rate for saving. To simplify the analysis, we assume that the saving subsidy is financed only by the lump-sum tax:

$$Z = \zeta\dot{K} = T \Leftrightarrow G = \tau Y.$$

The government arbitrarily sets the income tax rate as $\tau = \sigma_G$. Using (25) and $\tau = \sigma_G$, the budget constraint for a household is

$$\dot{K} = \frac{(1-\sigma_G)Y - C - T}{1-\zeta}. \tag{26}$$

The representative household maximizes utility (3) subject to (26). The socially optimal growth rate of consumption $g_C^\star$ and the competitive economy's growth rate of consumption $g_C$ are represented respectively as

$$g_C^\star = \left[\frac{\partial Y}{\partial K} - \rho - (\theta-1)n\right]\theta^{-1}, \tag{27}$$

$$g_C = \left[\frac{1-\sigma_G}{1-\zeta}\frac{\partial Y}{\partial K} - \rho - (\theta-1)n\right]\theta^{-1}. \tag{28}$$

By comparison between (27) and (28), if and only if $\zeta = \sigma_G$ is $g_C$ equal to $g_C^\star$. Therefore, the competitive economy with saving subsidy financed by lump-sum tax can attain the social optimum if and only if $\tau = \zeta = \sigma_G$.[11]

We next consider the welfare-maximizing policy as the second best policy. We start our analysis by derivation of the indirect utility. Using (3) and (8),

$$U = \left(\frac{C(0)}{N(0)}\right)^{1-\theta} \int_0^\infty \frac{\exp[\{(1-\theta)(g_C - n) - \rho\}t]}{1-\theta} dt - \frac{1}{(1-\theta)\rho},$$

where $(1-\theta)(g_C - n) < \rho$. Differentiating the indirect utility function with respect to $\tau$, we obtain

$$\frac{dU}{d\tau} = \left(\frac{C(0)}{N(0)}\right)^{1-\theta} \left[C(0)^{-1} \frac{dC(0)}{d\tau} \int_0^\infty \exp[\{(1-\theta)(g_C - n) - \rho\}t] dt + \int_0^\infty \frac{dg_C}{d\tau} \cdot t \cdot \exp[\{(1-\theta)(g_C - n) - \rho\}t] dt\right]. \tag{29}$$

Therein, $\mathrm{sign}(dC(0)/d\tau) = \mathrm{sign}(dc(0)/d\tau)$. The welfare effect of fiscal policy comprises the initial effect on consumption and the effect on growth rate of consumption. Using (8), (17), (19), (20), and (29), the welfare-maximizing condition is $\tau = \sigma_G$.

Furthermore, we can derive the welfare effect of a demographic shock as

$$\frac{dU}{dn} = \left(\frac{C(0)}{N(0)}\right)^{1-\theta} \left[C(0)^{-1} \frac{dC(0)}{dn} \int_0^\infty \exp[\{(1-\theta)(g_C - n) - \rho\}t] dt + \int_0^\infty \frac{d(g_C - n)}{dn} \cdot t \cdot \exp[\{(1-\theta)(g_C - n) - \rho\}t] dt\right]. \tag{30}$$

The welfare effect of fiscal policy is decomposed into the initial effect on consumption and the effect on the per-capita growth rate of consumption. As described in the previous section, the initial effect of the demographic shock on consumption is positive, and the effect on per-capita growth rate of consumption is negative in

(11) Using a similar approach, Tamai (2008) examines the optimal tax policy in an endogenous growth model with public capital.

the short run, but positive in the long run. Consequently, there might exist a welfare-maximizing rate of population growth. A rise in fertility boosts economic growth, but it might not be desirable from a welfare perspective.

## 5. Conclusion

This paper presented a nonscale model of economic growth with public input. The balanced-growth rate depends on the various production elasticities and the population growth rate; therefore, it is not affected by the tax rate. In contrast to the neoclassical growth model, the per-capita growth rate is positive even if there are constant returns to scale in physical capital and labor.

Results show that it is possible for a government, using fiscal policy, to maximize per-capita consumption, the national income, or per-capita income in the long-run, although no fiscal policy can enhance long-run economic growth. Furthermore, fiscal policy can promote economic growth through its short-run effects; this result is consistent with existing empirical studies by Aschauer (1989), Devarajan et al. (1996), and others.

Results also show that the per-capita consumption-maximizing size of government, which is the same as the elasticity of public input to output, is equal to the welfare-maximizing size of government. Despite some limitations, a benevolent government can plan welfare-maximization using a simple condition. We provide another policy implication from this result. The growth-oriented government's attempt to enhance economic growth succeeds temporarily through the short-run effect of fiscal policy, but ends in failure eventually because a temporarily increased growth rate finally converges to the balanced-growth rate, which is unaffected by fiscal policy.

Finally, we consider the direction of future research. The per-capita growth rate is determined by the various production elasticities and the population growth rate. Therefore, it will be interesting to investigate the endogenous determination of fertility or the production elasticities. Many studies have investigated endogenous determination of fertility. For that reason, it is possible to extend various approaches.(12) These topics will be addressed in future investigations.

## Appendix

### *A.1. Proof of Proposition 1*

In the balanced-growth equilibrium, we have $\dot{c}=\dot{k}=0$. Equation (10) is

$$0=\frac{(1-\tau)\sigma_K\tau^{\sigma_G/(1-\sigma_G)}}{k^{(1-\sigma_K-\sigma_G)/(1-\sigma_G)}}-\rho+(\theta-1)n-\frac{\sigma_N\theta n}{1-\sigma_K-\sigma_G}.$$

Solving the equation shown above with respect to $k$,

$$k^*=\left[\frac{(1-\sigma_K-\sigma_G)(1-\tau)\tau^{\sigma_G/(1-\sigma_G)}\sigma_K}{(1-\sigma_K-\sigma_G)(\rho+n)+(\sigma_K+\sigma_G+\sigma_N-1)\theta n}\right]^{(1-\sigma_G)/(1-\sigma_K-\sigma_G)}$$

Equation (11) in the balanced-growth equilibrium is

$$0=(1-\tau)\tau^{\sigma_G/(1-\sigma_G)}k^{(\sigma_K+\sigma_G-1)/(1-\sigma_G)}-c-\frac{\sigma_N nk}{1-\sigma_K-\sigma_G}.$$

Substituting $k^*$ for $k$ in the equation shown above, we obtain

$$c^*=(1-\tau)\tau^{\sigma_G/(1-\sigma_G)}(k^*)^{(\sigma_K+\sigma_G-1)/(1-\sigma_G)}-\frac{\sigma_N nk^*}{1-\sigma_K-\sigma_G}.$$

We assume implicitly that $c^*>0$. The golden rule condition is

$$\frac{dc^*}{dk^*}=0\Leftrightarrow k^*=\left[\frac{(1-\sigma_K-\sigma_G)(1-\tau)\tau^{\sigma_G/(1-\sigma_G)}\sigma_K}{(1-\sigma_G)\sigma_N n}\right]^{(1-\sigma_G)/(1-\sigma_K-\sigma_G)}\equiv\hat{k}.$$

Comparison of $k^*$ to $\hat{k}$ yields

(12) See Barro and Sala-i-Martin (1995, Ch.10) for basic continuous-time model of this literature.

$$k^* \lesseqgtr \hat{k} \Leftrightarrow (1-\sigma_K-\sigma_G)\rho + [(\theta-1)(\sigma_K+\sigma_G+\sigma_N-1)+\sigma_G\sigma_N]n \gtreqless 0$$

If $k^* > \hat{k}$, an increase in consumption makes it possible to improve welfare because it can increase both consumption during the transitional process and long-run consumption. Therefore, if $k^* > \hat{k}$, the balanced-growth equilibrium is dynamically inefficient. However, if $k^* \leq \hat{k}$, an increase in consumption cannot improve welfare, so that its balanced-growth equilibrium is dynamically efficient.

We next consider the stability of the dynamic system around the balanced-growth equilibrium. Linearizing the dynamic system of $\dot{c}$ and $\dot{k}$ around the balanced-growth equilibrium, the linearized system is given as

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} 0 & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \begin{pmatrix} c-c^* \\ k-k^* \end{pmatrix}, \tag{31}$$

where

$$J_{12} = \left.\frac{\partial \dot{c}}{\partial k}\right|_{(c,k)=(c^*,k^*)} < 0, J_{21} = \left.\frac{\partial \dot{k}}{\partial c}\right|_{(c,k)=(c^*,k^*)} = -1, \text{ and } J_{22} = \left.\frac{\partial \dot{k}}{\partial k}\right|_{(c,k)=(c^*,k^*)} \gtreqless 0.$$

The determinant of the Jacobian is $\det J = -J_{12}J_{21} < 0$. Therefore, the balanced-growth equilibrium is stable in the saddle-point sense.

### *A.2. Derivation of* (15) *and* (16)

General solutions of the linearized system are

$$c(t) - c^* = A_{11}\exp(\chi_1 t) + A_{12}\exp(\chi_2 t), \tag{32}$$
$$k(t) - k^* = A_{21}\exp(\chi_1 t) + A_{22}\exp(\chi_2 t). \tag{33}$$

In the equations shown above, $A_{ij}$ is the vector for arbitrary constants $(i,j=1,2)$, $\chi_1$ the negative eigenvalue, and $\chi_2$ the positive eigenvalue. The negative root $\chi_1$ and $k(0)$ is not jumpable. Therefore, we have $A_{22}=0$. Inserting $A_{22}=0$ into Eq. (33) and differentiating Eq. (33) with respect to time yields

$$\dot{k}(t) = \chi_1 A_{21} \exp(\chi_1 t). \tag{34}$$

Using $A_{22} = 0$, Eqs. (31), (32), and (33), we obtain

$$\dot{k}(t) = J_{21}[A_{11} \exp(\chi_1 t) + A_{12} \exp(\chi_2 t)] + J_{22} A_{21} \exp(\chi_1 t). \tag{35}$$

Combining Eq. (34) with Eq. (35), the vector $A_{ij}$ is expected to satisfy the following conditions:

$$A_{12} = A_{22} = 0, \tag{36}$$

$$A_{21}(\chi_1 - J_{22}) = A_{11} J_{21}. \tag{37}$$

Under Eq. (36), Eqs. (31) and (32) give

$$J_{12} A_{21} = \chi_1 A_{11}. \tag{38}$$

At time $t = 0$, Eqs. (33) and (36) engender $A_{21} = (k(0) - k^*)$. Substituting $(k(0) - k^*)$ for $A_{21}$ in (33), we obtain

$$k(t) - k^* = (k(0) - k^*) \exp(\chi_1 t). \tag{39}$$

Inserting $A_{21} = (k(0) - k^*)$ into Eq. (37), we have

$$A_{11} = \frac{(k(0) - k^*) J_{21}}{\chi_1}. \tag{40}$$

Using Eqs. (32), (36), (38) and (40), we arrive at

$$c(t) - c^* = \frac{(k(0) - k^*) J_{21}}{\chi_1} \exp(\chi_1 t) = \alpha \cdot (k(0) - k^*) \exp(\chi_1 t). \tag{41}$$

where $\alpha \equiv J_{21}/\chi_1 > 0$.

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