

## A NONPARAMETRIC SYSTEM IDENTIFICATION BASED ON TRANSIENT ANALYSIS WITH PLANT PROCESS OF HEAT EXCHANGER AS STUDY CASE

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### Abstract

In this article, a nonparametric identification system based on transient analysis has been reviewed, by taking the case in some of the data plant process of heat exchanger. Results of the study found that the first-order transfer function without time-delay the proposed model to the data with a temperature constant value is 35.20 °C and the time constant is 7200 seconds. This model has been fit to meet the existing data proving that the results of the calculation error do not exceed 2%.

*Keywords:* nonparametric identification systems, transient analysis, process plant heat exchanger, temperature constant, time constant, calculated error

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### 1. Introduction

Introducing a mathematical model to represent the actual system is very important especially in simulation and prediction purposes, particularly for designation of digital control and dynamic systems. Basically, there are two ways of constructing mathematical models i.e. mathematical modeling and system identification [1]. A mathematical modeling is an analytical approach. For example, basic laws from physics (such as Newton's laws and balance equations) are used to describe the dynamic behavior of a phenomenon or a process. On the other hand, system identification is an experimental approach where a model is then fitted to the recorded data by assigning suitable numerical values to its parameters. In many cases such as plant processes are so complex that it is not possible to obtain reasonable models using only analytical approach (physical insight, e.g. balance equations).

In such cases one is forced to use system identification techniques. It often happens that a model that is based on physical insight contains a number of known parameters even if the structure is derived from physical laws. On the other hand system identification methods can be applied to estimate the unknown parameters. To a dynamic system, the system identification is a technique to estimate the mathematical models based on data observed from the system [2,3]. Many researchers such as [4], [5], and [6] are identifying the dynamic model of heat exchanger using system identification.

Based on [1], techniques used in system identification can be divided into two distinct

methods, i.e. parametric and nonparametric methods. The parametric method is built through a model structure determination which is described by a set of parameters while the nonparametric method is determined based on an output system response in a function or a graph forms. In general, a parametric method can be characterized as a mapping from the recorded data to the estimated parameter vector.

A typical example of the nonparametric method is transient analysis where the input can be a recorded based on step signal and step response. This response will by itself give certain characteristics (dominating time constant, damping factor, static gain, etc.) of a process. Nonparametric techniques are often sensitive to noise and do not give very accurate results. However, as they are easy to apply they often become useful means of deriving preliminary or crude models. Therefore, in order to solve the difficulties to get the dynamics model of process system, one can use system identification.

### 2. Nonparametric Identification

This section describes a nonparametric system identification based on a transient analysis. A transient is defined if an input is taken as a step or impulse, and the recorded output constitutes the model [1]. Such identification methods are characterized by the property that the resulting models form curves or functions, which are not necessarily parameterized by a finite-dimensional parameter vector. Sometimes, it is of interest to fit a simple low-order model to a step response. This is illustrated as the first- and the second-order systems,

which are described using the transfer function model in Eq. (1) where  $Y(s)$  is the Laplace transform of the output signal  $y(t)$ ,  $U(s)$  is the Laplace transform of the input signal  $u(t)$ , and  $G(s)$  is the transfer function of the system [1].

$$Y(s) = G(s)U(s) \quad (1)$$

Consider a transfer function system which represented by Eq. (2). In such a case, the system is therefore described by a first-order differential Eq. (3). Note that a time delay  $\tau$  is included in the model. The step response of such a system is illustrated in Fig. 1 that demonstrates a graphical method for determining the parameters  $K$ ,  $T$  and  $\tau$  from the step response. The gained  $K$  is given by the final value. By fitting the steepest tangent,  $T$  and  $\tau$  can be obtained. The slope of this tangent is  $K/T$ , where  $T$  is the time constant. The tangent crosses the  $t$  axis at  $t=\tau$ , which is the time delay [1].

$$G(s) = \frac{K}{1 + sT} e^{-s\tau} \quad (2)$$

$$T \frac{dy(t)}{dt} + y(t) = Ku(t - \tau) \quad (3)$$

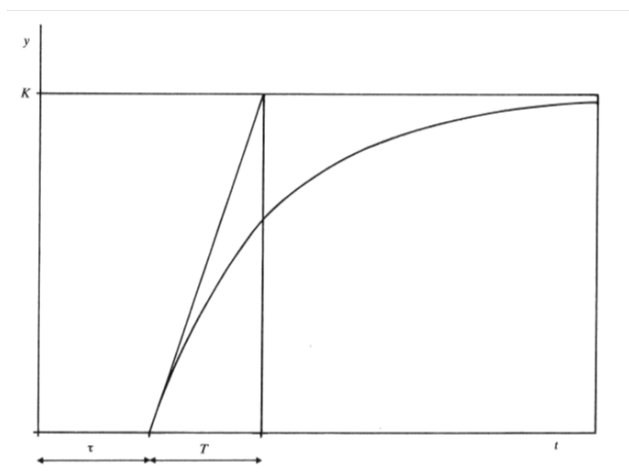


Fig. 1 Step response first order system with time delay

Physically, this equation describes a damped oscillator. After some calculations, the step response is found to be as expressed by Eq. (4). The output response,  $y(t)$ , in Eq. (4) for various  $\zeta$  is illustrated by Fig. 2 [1].

$$y(t) = K \left[ 1 - \frac{1}{\sqrt{(1-\zeta^2)}} \sin(\omega_0 \sqrt{(1-\zeta^2)}t + \tau) \right] \quad (4)$$

$\tau = \arccos \zeta$

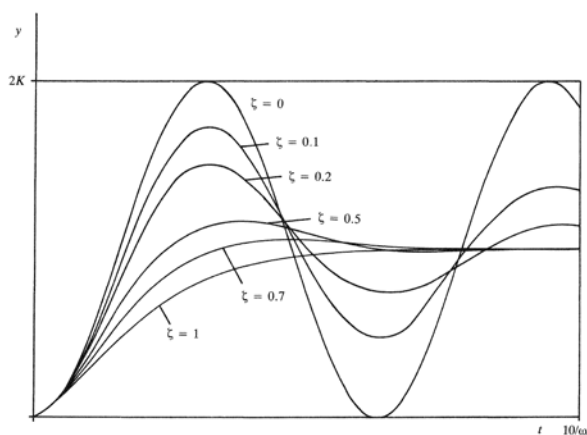


Fig. 2 Step response damped oscillator

Fig. 2 obviously shows that the relative damping  $\zeta$  influences the character of the step responses. The remaining two parameters,  $K$  and  $\omega_0$ , merely act as scale factors. The gained  $K$  scale is the amplitude axis while  $\omega_0$  scale is the time axis. The three parameters of the model in Eq. (2), namely  $K$ ,  $\zeta$  and  $\omega_0$  could be determined by comparing the measured step response with Fig. 2 and choosing the curve that is most similar to the recorded data. However, one can also proceed in a number of alternative ways. One possibility is to look at the local extreme (maxima and minima) of the step response. With some calculation it can be found from Eq. (4). At times, it occurs as given by Eq. (5). Where Eq. (6) defines time  $t_k$  and overshoots  $M$  is given by Eq. (7) [1].

$$y(t_k) = K[1 - (-1)^k M^k] \quad (5)$$

$$t_k = k \frac{\pi}{\omega_0 \sqrt{(1-\zeta^2)}} \quad k = 1, 2, \dots \quad (6)$$

$$M = \exp \left[ \frac{-\zeta \pi}{\sqrt{(1-\zeta^2)}} \right] \quad (7)$$

The relationship between the overshoot  $M$  and the relative damping is illustrated in Fig. 3. The parameters  $K$ ,  $\zeta$  and  $\omega_0$  can be determined based on values obtained in Fig. 4. The gained  $K$  is easily obtained as the final value once convergence is achieved. The overshoot  $M$  has been determined in several ways. One possibility is to use the first maximum. An alternative is to use several extreme and the fact as given by Eq. (6). The amplitude of the oscillations in Eq. (5) is reduced by a factor  $M$  for every half-period. Once  $M$  is determined;  $\zeta$  can

be derived from Eq. (7), and will produce Eq. (8) [1].

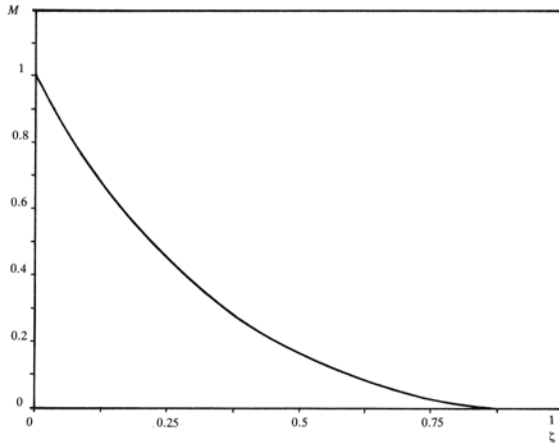


Fig. 3 Overshoot  $M$  versus relative damping  $\zeta$  damped oscillator

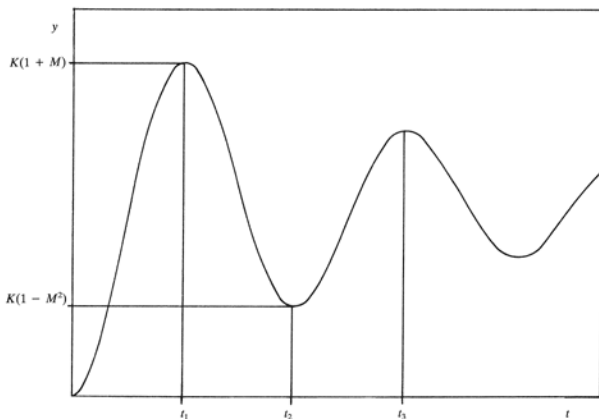


Fig. 4 Determination parameters damped oscillator from the step response

$$\zeta = \frac{-\log M}{[\pi^2 + (\log M)^2]^{1/2}} \quad (8)$$

From the step response, the period  $T$  of the oscillations has also been determined. From (6), the period  $T$  is given by Eq. (9) [1]. Then  $\omega_0$  is given by Eq. (10) [1].

$$T = \frac{2\pi}{\omega_0 \sqrt{1 - \zeta^2}} \quad (9)$$

$$\omega_0 = \frac{2\pi}{T \sqrt{1 - \zeta^2}} = \frac{2}{T} [\pi^2 + (\log M)^2]^{1/2} \quad (10)$$

### 3. Nonparametric Model

A transfer function of shell and tube heat exchanger is described as Eqs. (11) up to (17). It has complex forms and it is difficult to apply to the design of practical controllers. In some cases, it is found that the outlet responses are well approximated by using the first- or second-order system to replace the complicated forms of the transfer functions obtained directly from the transformed solutions. This transfer functions are a model form based on the Laplace transform and it is very useful in analysis and design of a linear dynamic shell and tube heat exchanger.

To obtain a nonparametric model, a step response analysis is used in this case. The dynamic model in Eq. (11) can be simplified as Eq. (12). The obtained transfer function which assumes  $T_{s,4}(s) \cong T_{s,5}(s) \cong T_{s,6}(s)$  is shown in Eq. (13) where  $H(s)$  is the transfer function,  $Y(s)$  is the output signal from the cold water temperature outlet in shell side heat exchanger,  $T_{s,6}$ , and  $U(s)$  is the input signal from the hot water flow rate inlet in tube side heat exchanger,  $F_t$ .

$$\begin{aligned} & (C_{ps}\rho_t V_t C_{pt} s F_s(s) + C_{ps} U A_s F_s(s) - (U A_s)^2) T_{s,6}(s) = \\ & [5C_{pt} C_{ps} F_s(s) T_{s,4}(s) + .5C_{pt} U A_s T_{s,4}(s) + .5C_{pt} C_{ps} \rho_s V_s s T_{s,4}(s) \\ & + 5C_{pt} C_{ps} \rho_s V_s s T_{s,4}(s) + \rho_t V_t C_{pt} s C_{ps} \rho_s V_s s T_{s,4}(s) \\ & - \rho_t V_t C_{pt} U A_s s T_{s,4}(s) - (U A_s)^2 T_{s,4}(s) \\ & 5U A_s C_{pt} T_{t,6}(s)] F_t(s) \end{aligned} \quad (11)$$

$$A(s)Y(s) = B(s)U(s) \quad (12)$$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{T_{s,6}(s)}{F_t(s)} = \frac{B(s)}{A(s)} \quad (13)$$

$$A(s) = C_{ps}\rho_t V_t C_{pt} s F_s(s) + C_{ps} U A_s F_s(s) - (U A_s)^2 \quad (14)$$

$$Y(s) = T_{s,6}(s) \quad (15)$$

$$\begin{aligned} B(s) = & .5C_{pt} C_{ps} F_s(s) T_{s,4}(s) + .5C_{pt} U A_s T_{s,4}(s) \\ & + .5C_{pt} C_{ps} \rho_s V_s s T_{s,4}(s) + \rho_t V_t C_{pt} s C_{ps} \rho_s V_s s T_{s,4}(s) \\ & + U A_s C_{ps} \rho_s V_s s T_{s,4}(s) - \rho_t V_t C_{pt} U A_s s T_{s,4}(s) \\ & - (U A_s)^2 T_{s,4}(s) - .5U A_s C_{pt} T_{t,6}(s) \end{aligned} \quad (16)$$

$$U(s) = F_t(s) \quad (17)$$

A first order transfer function without time delay is given by Eq. (18), where  $K$  is the gain and  $T$  is the time constant. A step response of such a first

order transfer function in Eq. (18) has the following characteristics, which are demonstrated by Fig. 5 [7].

$$H(s) = \frac{K}{Ts + 1} \quad (18)$$

The first order transfer function with time delay is given by Eq. (19), where  $K$  is the gain,  $T$  is the time constant and  $\tau$  is time delay.

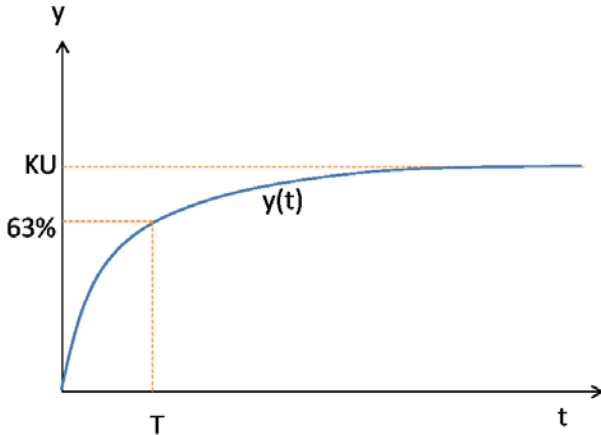


Fig. 5 Step response first order transfer function without time delay

A step response of such a transfer function has the following characteristics, which are demonstrated by Fig. 6 [7].

$$H(s) = \frac{K}{Ts + 1} e^{-\tau s} \quad (19)$$

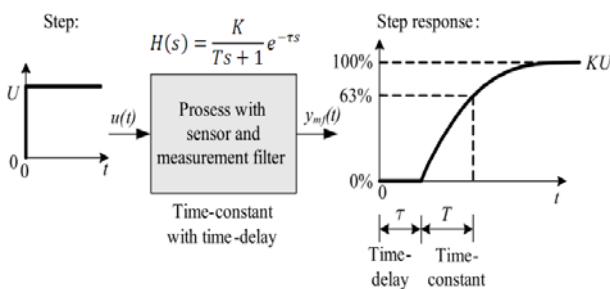


Fig. 6 Step response first order transfer function with time delay

A transfer function of a time delay which is given by Eq. (20) is irrational transfer function (it has no s-polynomial in the numerator and in the denominator). In some situations as in frequency response based analysis of control systems containing a time delay, it is necessary to substitute  $e^{-\tau s}$  with an approximation in the form of a rational transfer function. The most common approximation

is the Padé-approximation as given in Eq. (21), where  $n$  is the order of the approximation and this order must be chosen [7]. The coefficients  $k_i$  are functions of  $n$ .

$$H(s) = e^{-\tau s} \quad (20)$$

$$e^{-\tau s} \approx \frac{1 - k_1 s + k_2 s^2 + \dots \pm k_n s^n}{1 + k_1 s + k_2 s^2 + \dots + k_n s^n} \quad (21)$$

The Padé-approximations are based on a minimization of the truncation errors in a finite series expansion of  $e^{-\tau s}$ . Table 1 shows, as an illustration, the  $k$ -values for the orders  $n = 1$  and  $n = 2$  [7].

Table 1: Coefficients of Pade-approximations of order  $n = 1$  and  $n = 2$

Order-1	Order-2
$n = 1$	$n = 2$
$k_1 = \frac{\tau}{2}$	$k_1 = \frac{\tau}{2}$
$k_2 = 0$	$k_2 = \frac{\tau^2}{12}$
	$k_3 = 0$

Table 1 shows  $k_1$  and  $k_2$  which are given by Eqs. (22) and (23). Therefore, the Padé-approximation transfer functions for first and second orders are given by Eqs. (24) and (25) [7].

$$k_1 = \frac{\tau}{2} \quad (22)$$

$$k_2 = \frac{\tau^2}{12} \quad (23)$$

$$H(s) = \frac{1 - k_1 s}{1 + k_1 s} \quad (24)$$

$$H(s) = \frac{1 - k_1 s + k_2 s^2}{1 + k_1 s + k_2 s^2} \quad (25)$$

Substituting Eqs. (24) and (25) into (18) will produce Eqs. (26) and (27).

$$H(s) = \frac{1 - k_1 s}{1 + k_1 s} \frac{K}{Ts + 1} \quad (26)$$

$$H(s) = \frac{1 - k_1 s + k_2 s^2}{1 + k_1 s + k_2 s^2} \frac{K}{Ts + 1} \quad (27)$$

The step and phase responses of the first order Padé-approximation is shown in Fig. 7. Fig. 8 shows the step and phase responses of the second order Padé-approximation. A second order transfer functions is given by Eq. (28). Where  $K, \zeta$  and  $\omega_0$  refer to system gain, relative damping factor and undamped resonance frequency [1,2]. Table 2 shows type of step response  $y(t)$  for various values of  $\zeta$ . When  $\zeta > 0$  and the poles are real and distinct, it is given by Eq. (29).

$$H(s) = \frac{K\omega_0^2}{s^2 + 2\zeta\omega_0s + \omega_0^2} = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\frac{s}{\omega_0} + 1} \quad (28)$$

$$H(s) = \frac{K}{(T_1s + 1)(T_2s + 1)} \quad (29)$$

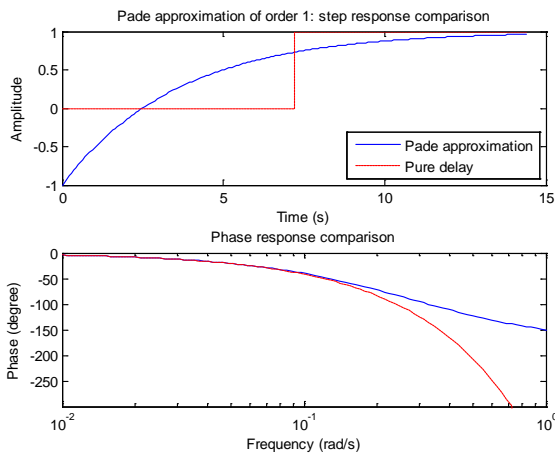


Fig. 7 Step and phase's responses order-1 Padé-approximation

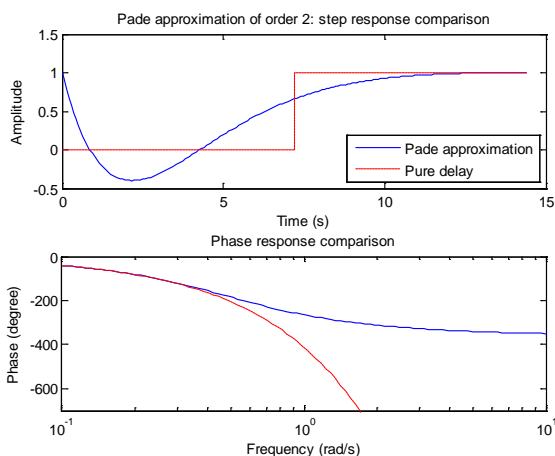
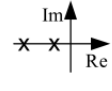
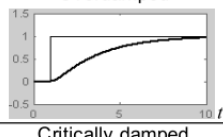
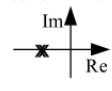
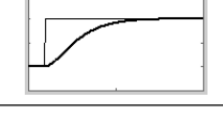
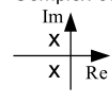
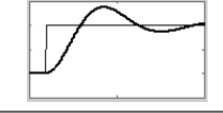

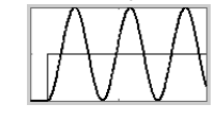
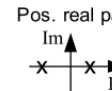
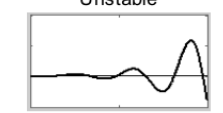


Fig. 8 Step and phase's responses order-2 Padé-approximation

Table 3: Type of step response  $y(t)$  for various value of zeta  $\zeta$

Value of $\zeta$	Poles $p_1$ and $p_2$	Type of step response $y(t)$
$\zeta > 1$	Real and distinct 	Overdamped 
$\zeta = 1$	Real and multiple 	Critically damped 
$0 < \zeta < 1$	Complex conj. 	Underdamped 
$\zeta = 0$	Imaginary 	Undamped 
$\zeta < 0$	Pos. real part 	Unstable 

#### 4. Case Study

The candidate of nonparametric model has been described. When the experimental data obtained are observed, the data are apparently similar to the step response of the first order transfer function. This model is candidate model and has been expressed by Eq. (30). Therefore, the candidate model chosen is the first order transfer function. The error of this model is calculated after offset, where the time of offset is determined according to time constant that occurred. The error is given by Eq. (31), where  $E(t)$  is the error,  $N$  is the number of the data and  $y$  is the output signal value with  $y_{data}$  for the data, and  $y_{cal}$  for calculated value from the model. The percentage of this error is given by Eq. (32), where  $\Delta y_{data}$  is within the range of the  $y_{data}$ .

$$H(s) = K_0 + \frac{K}{Ts + 1} = \frac{K_0(Ts + 1) + K}{Ts + 1} \quad (30)$$

$$E(t) = \sum \frac{|y_{data} - y_{cal}|}{N} \quad (31)$$

$$E(\%) = \frac{E(t)}{\Delta y_{data}} \times 100\% \quad (32)$$

The step response of the resulted nonparametric model for dataexp1 is shown in Eq. (33).

$$H(s) = 30 + \frac{35.2}{7200s+1} = \frac{216000s+65.2}{7200s+1} \quad (33)$$

The comparison of the model output and the measured dataexp1 is shown in Fig. 9.

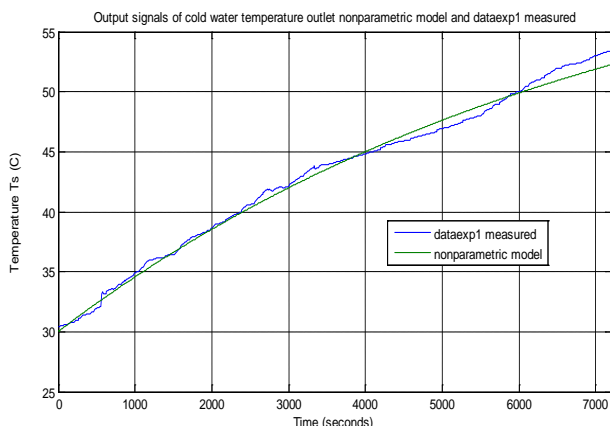


Fig. 9 Nonparametric model output and data measured dataexp1

From Fig. 9, it is obvious that the trend of the model output which is given in Eq. (33) is similar to the measured dataexp1. The data and the model start with the same values, i.e. **30°C.**, and the difference of both temperatures reaches the smallest figure. where is fluctuating between zero to one. These facts are presented in Table 3.

Table 3. Error calculation nonparametric model and data measured dataexp1

Time, t (s)	y <sub>data</sub> (°C)	y <sub>cal</sub> (°C)	y <sub>data</sub> - y <sub>cal</sub>   (°C)
0	30.0	30.0	0.0
1000	35.0	34.5	0.5
2000	37.0	37.0	0.0
3000	42.5	42.5	0.0
4000	45.0	45.0	0.0
5000	47.0	48.0	1.0
6000	50.0	50.0	0.0
7000	54.0	53.0	1.0
<b>E(t) = 0.31°C dan E(%) = 1.3%</b>			

Fig. 10 shows these facts based on the error values in Table 3. The error has been calculated using Eqs. (31) and (32) which takes 8 samples time between 0 and 7000 s. From Table 3, it is obvious that the error is **0.31°C** and the percentage of the error is **1.3%**. According to these results, therefore, the model which is shown in Eq. (33) is an excellent

model because the error is smaller than 2%. In other words, it should be accepted.

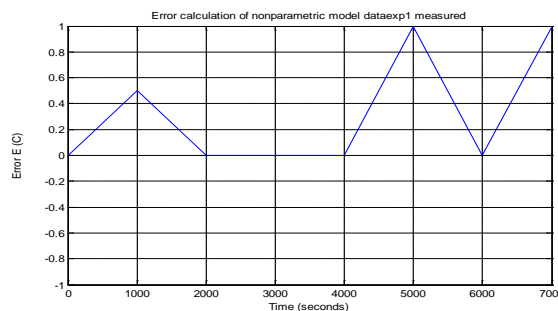


Fig. 10 Error calculation nonparametric model dataexp1

Similar way has been shown for dataexp1 and dataexp2. The step response of the resulted nonparametric model for dataexp2 is shown in Eq. (34). The comparison of the model output and the measured data is shown in Fig. 11. The error calculation to validate this model is presented in Table 4. Fig. 11 shows the obtained error. From Fig. 11, it is obvious that the trend of the model output which is given by Eq. (34) is similar to the measured dataexp2. The data starts from 29 °C and the model also starts from 29 °C. From time (t) of 0s until to the final time (t) of 7000 s, the difference of the both temperatures reaches the smallest value, i.e. between zeros to one. These facts are presented in Table 4.

$$H(s) = 29 + \frac{35.2}{7200s+1} = \frac{208800s+64.2}{7200s+1} \quad (34)$$

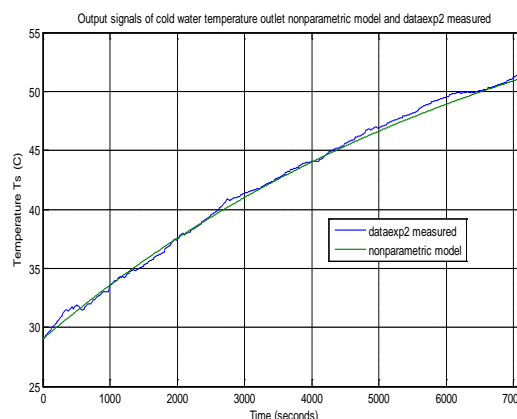


Fig. 11 Nonparametric model output and data measured of dataexp2

Table 4. Error calculation nonparametric model and data measured dataexp2

Time,t (s)	Y <sub>data</sub> (°C)	Y <sub>cal</sub> (°C)	Y <sub>data</sub> - Y <sub>cal</sub> (°C)
0	29.0	29.0	0.0
1000	34.0	34.0	0.0
2000	37.5	37.5	0.0
3000	41.5	41.0	0.5
4000	44.0	44.0	0.0
5000	47.0	46.5	0.5
6000	49.5	48.5	1.0
7000	52.0	52.0	0.0

$E(t) = 0.25^{\circ}\text{C}$  dan  $E(\%) = 1.1\%$

Fig. 12 shows the facts based on the values of the error in Table 4. The error has been calculated using Eqs. (31) and (32) which takes 8 sample times between 0 and 7000 s.

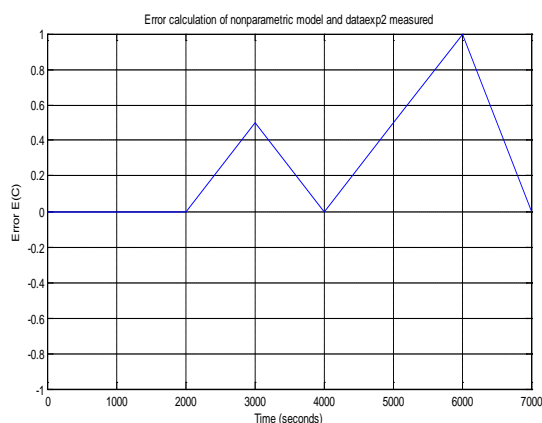


Fig. 12 Error calculation nonparametric model dataexp2

From Table 4, it is apparent that the error is **0.25°C** and the percentage of the error is **1.1%**. Based on these results, therefore, the model which has been given by Eq. (34) is an excellent model because of the error is smaller than **2%**. Therefore, it should be accepted.

The step response of the nonparametric model resulted from dataexp3 is shown in Eq. (35). The comparison of the model output and the measured data is given in Fig. 13. The error calculation to validate this model is presented in Table 5. Fig. 14 shows the obtained error.

$$H(s) = 26 + \frac{35.2}{7200s + 1} = \frac{187200s + 61.2}{7200s + 1} \quad (35)$$

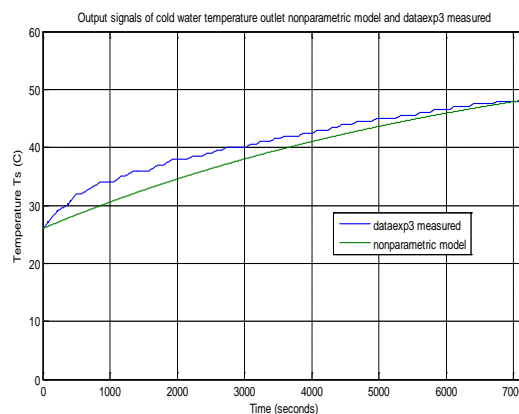


Fig. 13 Nonparametric model output and data measured of dataexp3

Table 5. Error calculation nonparametric model and data measured dataexp3

Time,t (s)	Y <sub>data</sub> (°C)	Y <sub>cal</sub> (°C)	Y <sub>data</sub> - Y <sub>cal</sub> (°C)
0	26.0	26.0	0.0
1000	33.0	31.0	2.0
2000	38.0	35.0	3.0
3000	40.0	38.0	2.0
4000	42.0	41.0	1.0
5000	45.0	44.0	1.0
6000	47.0	46.0	1.0
7000	48.0	48.0	0.0

$E(t) = 1.5^{\circ}\text{C}$  and  $E(\%) = 6.8\%$

From Fig. 13, it is apparent that the trend of the model output which is given by Eq. (35) is similar to the measured dataexp3. The data and the model start with the same values, i.e. **26°C**. From the start time of 0s until to the final time of 7000s, the difference of both temperatures reaches the smallest value that it is presented in Table 5. Fig. 14 shows these facts based on the error values in Table 5. The error has been calculated using Eqs. (31) and (32) with 8 samples of time between 0 and 7000 s. From Table 5, it is obvious that the error is **1.5°C** and the percentage of the error is **6.8%**. Referring to these results, therefore, the model which is given by Eq. (35) is a sufficiently good model because the error is less than **10%**. Therefore, it may be accepted.

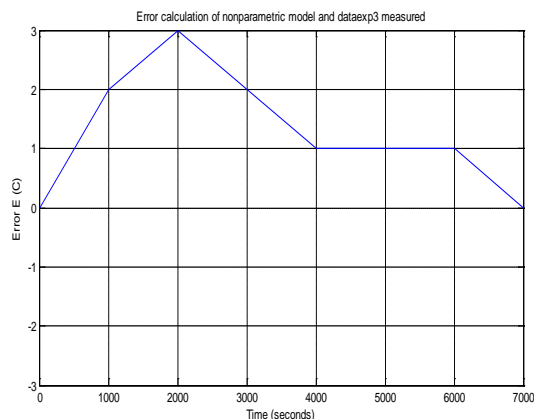


Fig. 14 Error calculation nonparametric model dataexp3

The comparison results of the nonparametric models are presented in Table 6. It is shown that all time constant or offset and the gain of the all experimental data is the same, i.e., 7200 s and 35.2 °C while the initial temperature for each experimental data is different. They are 30 °C for dataexp1, 29 °C for dataexp2 and 26 °C for dataexp3. The percentage of the error for each experimental data is different too. The dataexp1 has the error percentage of 1.3%, dataexp2 is 1.1% and dataexp3 is 6.8%. Based on these results, it concludes that the first order without time delay could be taken as nonparametric model with the gain and time constant have same values, i.e. 35.2 °C and 7200 s, while the initial temperature is different depend to state of the experimental began.

Table 7. The comparison results of nonparametric models

dataexp	Gain $K_0$ (°C)	Gain $K$ (°C)	Time Constant $T$ (s)	$E(t)$ (°C)	$E(\%)$
dataexp1	30	35.2	7200	0.31	1.3
dataexp2	29	35.2	7200	0.25	1.1
dataexp3	26	35.2	7200	1.5	6.8

## 5. Conclusion

The nonparametric model equation is generated from the dynamic model equation through step response analysis being used. This equation is represented in a transfer function form. The output signal is the cold water temperature outlet in shell side heat exchanger, whilst the input signal is the hot water flow rate inlet in tube side heat exchanger. A first order without time delay transfer function has been selected as candidate model chosen since it has a response which is similar to the three set of the experimental data. It can be observed that the all-time constant and the gain of the all experimental data is the same although the starting of the

temperature each the data is different. Based on these results, it can be concluded that the first order without time delay could be taken as nonparametric model with the gain is 35.2 °C, and the time constant of 7200 s.

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