

# Unsteady Near Wall Residence Times and Shear Exposure in Model Distal Arterial Bypass Grafts

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## Abstract:

Building on previous studies of unsteady flow within model distal bypass grafts we analyse the near wall residence times and shear exposure in a 45 degrees anastomosis under symmetrical and symmetry breaking geometric configurations. We define residence time as the minimum time for a particle to exit a spherical region and shear exposure as a temporal integral of the Huber-Henky-von-Mises criterion along a particle path over a fixed time interval. Decomposing the pulsatile cycle into four equal intervals we find that the interval of peak residence time in the host vessel is from mid-deceleration to peak diastole and peak diastole to mid-acceleration. The asymmetric model is shown to have a significantly lower residence time during these intervals. Considering the shear exposure prior to the residence time evaluation we determine that a higher average shear exposure exists in the asymmetric model associated with the upstream geometry modification. Analysis of the regions of high residence time and shear exposure suggests that the “toe” region and interface between the “heel” and bulk flow are more significant than the bed and heel region. Although the asymmetric model considered in this study reduces residence times in the host artery, the product of the measure of shear exposure and residence time is not found to be preferable. If shear exposure were to be considered as an important factor in particle activation, the findings imply that for junction optimisation, greater consideration needs to be given both to the local junction asymmetry and upstream influence on the shear history.

## 1. Introduction

Arterial bypass graft surgery is a procedure to relieve the symptoms of arterial occlusion. It has been reported [1] that 50 % of the grafts implanted fail within 10 years due to restenosis caused primarily by intimal hyperplasia. Although a link between the local flow field and vessel wall biology has been found [2], the exact mechanism of this interaction is not fully understood. The advent of improved computational and experimental techniques has led to detailed studies of steady and unsteady haemodynamics within both model and, more recently, anatomically correct geometries.

In the absence of a better understanding of the detailed mechanisms of the disease process, different measures, principally focused around the fluid properties at the wall, have been considered. These are wall shear stress and its spatial and temporal gradients as well as the oscillatory shear index [3]. Based upon these measures it has been suggested the regions denoted as the “toe”, “bed” and “heel” of the anastomosis (see figure 1) are likely sites for the onset of disease.

One of the significant advantages of computational modelling is the possibility to consider quantities which are very difficult if not impossible to measure experimentally such as the wall shear stress. In this paper we extend our previous analysis in model geometries by introducing a combined measure of near wall residence time and particle history characteristics in the form of shear exposure. The near wall residence time can be directly correlated to the instantaneous wall shear stress and, at higher order, to its spatial and temporal gradients for short times. Shear exposure however introduces a measure along particle paths, and may be affected by the interior flow structure, particularly in the presence of separation.

## 2. Methodology

*2.1 Computational Technique:* The computations were performed using a spectral/*hp* element algorithm [1] to solve the three-dimensional incompressible Navier-Stokes equations for unsteady flow. The model configurations shown in Fig. 1 were computationally represented using approximately 2000, body conforming, tetrahedral elements including a boundary layer of elements of thickness  $0.1D$  where  $D$  is the diameter of the vessels. We apply a polynomial expansion within the element and simulations were performed at polynomial orders of  $p = 4, 6$  and  $8$  which correspond to approximately 20 000, 70 000 and 160 000 local degrees of freedom respectively. At the inflow boundary, the exact Womersley solution was imposed using a sinusoidal waveform plus a mean component such that the time average Reynolds number,  $Re_D$ , was 250 (based on mean cross sectional velocity) and ranged over  $62.5 < Re_D < 437.5$  with a Womersley parameter of  $\alpha = 4$ . At outflow a fully developed pipe flow with constant pressure and zero normal derivatives of velocity was enforced. The computational technique is described in greater detail in [4].

*2.2 Data Analysis:* For the parameters specified above, and non-dimensionalising the time  $T$  for one cycle by the time for the mean velocity,  $U$ , to travel a distance  $D$  we obtain that  $T=24$ . It is therefore evident that the mean flow will have travelled beyond the anastomosis in one cycle and we have therefore chosen to consider the unsteady flow over four separate intervals each of a quarter of the cycle. Each interval corresponds to 6 non-dimensional time units, and comprises i) mid-acceleration to ‘‘peak systole’’ (maximum flow), ii) peak systole to mid-deceleration, iii) mid-deceleration to ‘‘peak diastole’’ (minimum flow) and iv) peak diastole to mid-acceleration. In each interval, near wall residence time and shear exposure are computed using a high order particle tracking algorithm [5] about the anastomosis in the host vessel.

We define normalised residence time as

$$R(\mathbf{x}, t_0, \delta) = \frac{\bar{U}}{D} \inf \{t : |\mathbf{x}(t) - \mathbf{x}(t_0)| > \delta\}$$

where  $\bar{U}$  is the temporally averaged mean velocity. This measure can be physically interpreted as the time for a fluid particle initially at position  $\mathbf{x}(t_0)$  to exit a spherical region of radius  $\delta$ . Although the fluid particle may re-enter the spherical region, we denote the first exit time as the residence time. As  $\delta \rightarrow 0$ , the residence time  $R$  depends solely on the instantaneous local velocity; hence in the near wall region  $R$  is related at first order to the reciprocal of the wall shear stress.

There are alternative measures for shear accumulation or exposure of a particle [6], and we use the Huber-Henky-von-Mises criterion as described by Bludzuweit [7]. This criterion takes into account the interaction between direct and shear stresses and defines an equivalent stress as

$$\sigma_e = \frac{1}{\sqrt{2}} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{11} - \sigma_{33})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2) \right]^{1/2} \quad \text{where} \quad \sigma_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

and  $(u_1, u_2, u_3)$  is the velocity vector. The normalised shear exposure  $E(\mathbf{x}, t_0, T)$ , over a time interval  $T$  at a point  $\mathbf{x}(t_0)$  is then defined as

$$E(\mathbf{x}, t_0, T) = \frac{1}{T \bar{\tau}_w} \int_{t_0-T}^{t_0} \sigma_e(\mathbf{x}(t)) . dt$$

where the integral is evaluated along the particle path and  $\bar{\tau}_w$  is the temporally averaged wall shear stress in an equivalent straight pipe. If a particle exits the computational domain during the integration, the ex-

posure was extrapolated assuming a steady Poiseuille shear exposure at the exit radius and a mean Reynolds number. The particle residence time and shear exposure were evaluated over four equal time intervals over the cycle on an equi-spaced cylindrical grid of  $60 \times 52$  points over the host region of  $-1.5D < x < 2D$  at a radius of  $0.49D$ . A value of  $\delta = 0.01D$  was used for  $R(x_o, t, \delta)$ , and the shear exposure was evaluated over a time interval of  $T=6$ . Note that the toe of the bifurcation is located at  $x = 0.25D$  where  $x=0$  represents the intersection point of the centrelines of the vessels.

### 3. Results and discussion

For each of the four intervals defined in section 2.2 the residence time,  $R$ , and shear exposure,  $E$ , of both the symmetric and asymmetric models were evaluated. As would be expected from the flow waveform, in both models the regions of highest spatially averaged  $R$  occur during mid-deceleration to peak diastole and peak diastole to mid-acceleration. We have therefore focused our discussion on these time intervals. The leftmost plots of figures 2 and 3 present spatial maps of  $R$  for the symmetric and asymmetric geometries, for these two time intervals. The location of the bed, toe and heel regions of the host vessel are also identified (compare fig. 1).

For both time intervals, the regions of highest  $R$  are in the vicinity of the heel in both models. We note that the only other areas of significant  $R$  are in the vicinity of the stagnation point, just distal to the toe location, where a separation zone exists in both the unsteady symmetric and asymmetric models. The mean value of  $R$  over the whole map for the symmetric model was 0.90 during the mid-deceleration to peak systole and 1.62 from peak systole to mid acceleration. For the asymmetric model the mean value of  $R$  was 0.45 during the mid-deceleration to peak systole and 1.28 during peak systole to mid-acceleration. The increase in residence time between the two time intervals is located primarily at the heel and toe regions. The residence time of the asymmetric model is significantly less in the first time interval than the symmetric model, however there is a reasonable increase in residence time in the second time interval. In the present study,  $R$  has only been measured in four discrete time intervals. However, to determine the fixed time interval with the largest spatially averaged mean requires a greater study of more starting times which was beyond the scope of this investigation.

An important question which arises from the study of residence times is how do we identify which regions of  $R$  are most significant to the potential onset of disease? If all suspensions are activated, possibly by a chemical reaction, when entering the junction then high residence time would probably be a sufficient measure. Alternatively we could envisage an alternative scenario where the particle path along which a given suspension arrives at the wall is more significant. Computational modelling allows us to follow this hypothesis by considering the shear exposure of all particles arriving to the vicinity of the near wall. The maps shown in the middle of figures 2 and 3 indicate the normalised shear exposure  $E$  of near wall particles that arrive at cylindrical coordinates  $(x, \theta, r = 0.49D)$  during the quarter cycle immediately preceding the residence time calculations. We recall that  $E$  has been normalised by the mean Poiseuille wall shear stress multiplied by the time period  $T$ . In both figures the particle shear exposure is higher during the interval from peak systole to mid-deceleration. In this interval the mean shear exposure is 2.32 in the symmetric geometry and 2.80 in the asymmetric geometry. The increase in shear exposure in the asymmetric geometry appears predominantly in the region distal to the bed and can be attributed to the bulk flow distortion in the curved bypass vessel as compared to the straight bypass vessel in the symmetric geometry. This trend is also evident in the second time interval considered where the mean value of  $E$  was 1.40 in the symmetric model and 1.52 in the asymmetric model. Therefore we observe that even though the asymmetric geometry has a lower mean residence time,  $R$ , compared to the symmetric geometry, there is an associated increase in the shear exposure,  $E$ , of the near wall particles. We also note that the regions of high  $E$  and  $R$  are relatively spatially orthogonal, indicating that particles with a high shear exposure,  $E$ , typically lie in regions of low residence time,  $R$ .

In order to identify spatial regions where particles have a high shear exposure when they arrive at the wall and subsequently reside a reasonable length of time at the wall we have considered product of  $R(x, \theta)$  and  $E(x, \theta)$  as shown on the right-hand plots of figures 2 and 3. We should stress that this product is proposed simply for the purpose of data analysis and we note that other products could equally well have been considered. However it does permit us to consider a combination of the two previous measures. From these plots we observe that during the interval from peak systole to peak diastole (top right plots in figures 2 and 3) the maps of product of  $R(x, \theta)$  and  $E(x, \theta)$  have similar mean values of 0.28 in the symmetric model and 0.29 in the asymmetric model. We also observe that the peak regions of the product of  $E$  and  $R$  typically occur in the vicinity of the toe. The bed region would appear to be relatively benign, especially away from the stagnation point. However, the interface of the heel region and the bed is also an area of relatively high product. The product in this region also increased during the second time interval considered in figures 2 and 3. We recall that this interval measured the shear exposure from mid-deceleration to peak diastole and residence time from peak diastole to mid-acceleration. Although the asymmetric model has a lower  $R$  the mean of the product of  $E$  and  $R$  during the second interval is higher having a value of 0.64 in the asymmetric model as opposed to 0.57 in the symmetric model.

#### 4. Conclusion

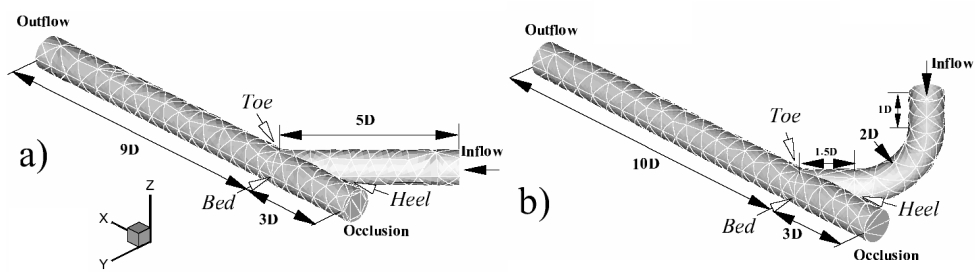
We have considered the residence time and shear exposure over consecutive quarter cycles of a non-reversing sinusoidal unsteady flow through a symmetric and asymmetric model distal anastomosis. We observed maximum average residence times during the quarter cycles from mid-deceleration to peak diastole and peak diastole to mid-acceleration. In the asymmetric model there was a notable decrease in residence time as compared to the symmetric model. However the shear exposure of near wall particles increases in the asymmetric model due to the upstream graft geometry. This observation would imply that if shear exposure is important in activation of particle suspensions then an optimal junction design would need to take into account local bifurcation asymmetry as well as the upstream influence on shear exposure.

However considerations of flow conditioning of the endothelium may suggest a non-linear weighting of the residence time. Also a more detailed analysis at different initialisation times during the pulsatile cycle is necessary to ensure that the most detrimental segment of the cycle is identified for each model. Nevertheless the combination of history effects such as shear exposure combined with the near wall characteristics such as residence time provides an insight into identifying likely sites of disease progression. The current study has suggested a preference for the toe region and the interface between the heel and the bed region of the distal anastomosis. Extending the analysis of the shear exposure over a full pulsatile cycle has demonstrated similar qualitative results.

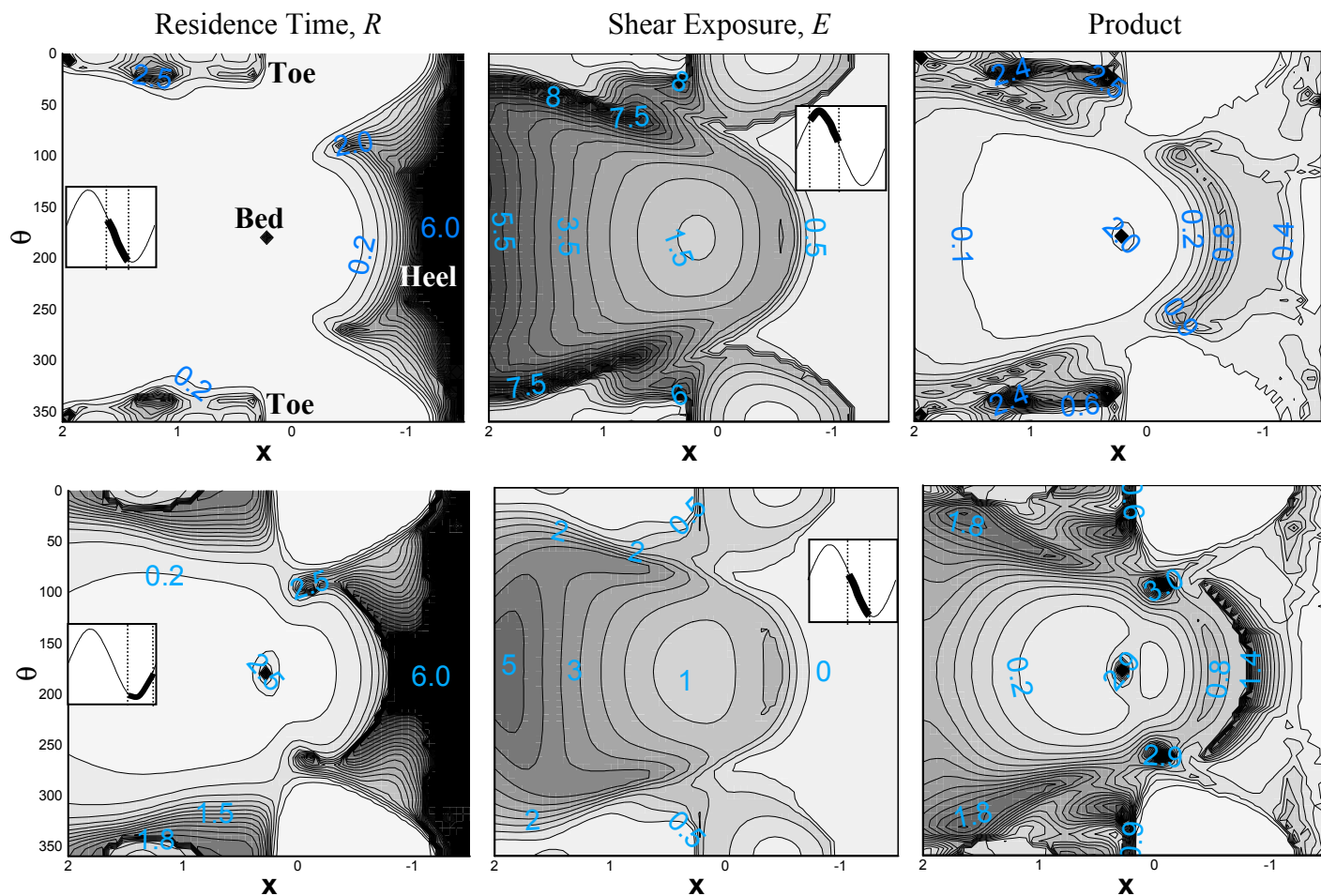
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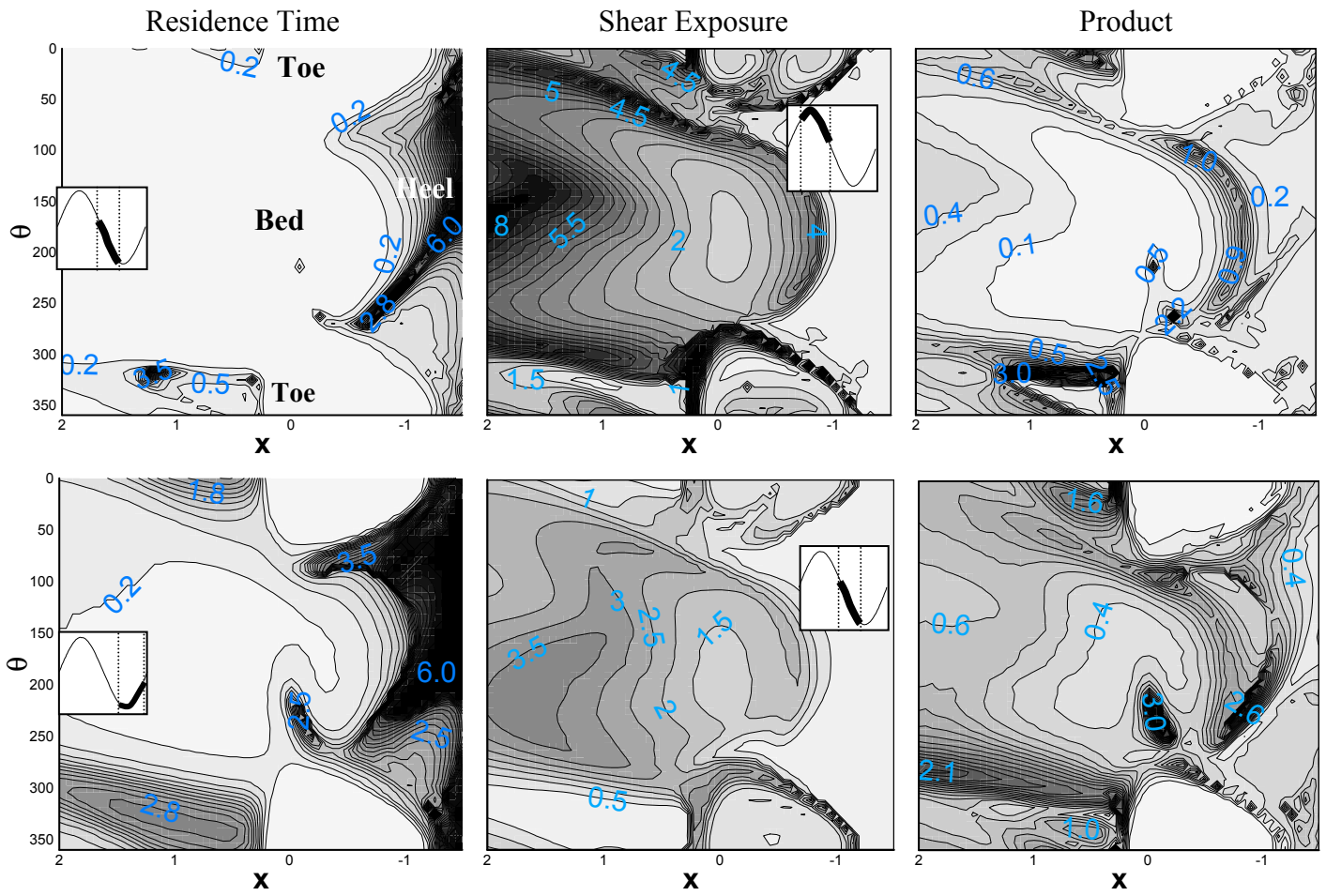
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**Figure 1.** Model geometries of the distal part of a 45 degrees end-to-side anastomosis: (a) symmetric and (b) asymmetric models. CFD mesh marked on vessel surfaces.



**Figure 2.** Maps of residence time, shear exposure and their product for the symmetric model over peak systolic to peak diastolic (top) and mid deceleration to mid acceleration (bottom). The inset figures represent the time interval over which each quantity was evaluated.



**Figure 3.** Maps of residence time, shear exposure and their product for the asymmetric model over peak systolic to peak diastolic (top) and mid deceleration to mid acceleration (bottom). The inset figures represent the time interval over which each quantity was evaluated