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M. Benacquista

D. M. Sedrakian

M. V. Hairapetyan

K. M. Shahabasyan

A. A. Sadoyan

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GRAVITATIONAL RADIATION FROM PULSATING WHITE DWARFS

M. BENACQUISTA,¹ D. M. SEDRAKIAN,² M. V. HAIRAPETYAN,² K. M. SHAHABASYAN,³ AND A. A. SADOYAN³

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ABSTRACT

Rotating white dwarfs undergoing quasi-radial oscillations can emit gravitational radiation in a frequency range from 0.1 to 0.3 Hz. Assuming that the energy source for the gravitational radiation comes from the oblateness of the white dwarf induced by the rotation, the strain amplitude is found to be $\sim 10^{-27}$ for a white dwarf at ~ 50 pc. The Galactic population of these sources is estimated to be $\sim 10^7$ and may produce a confusion-limited foreground for proposed advanced detectors in the frequency band between space-based and ground-based interferometers. Nearby oscillating white dwarfs may provide a clear enough signal to investigate white dwarf interiors through gravitational wave asteroseismology.

Subject headings: gravitational waves — stars: oscillations — white dwarfs

1. INTRODUCTION

There are a number of gravitational radiation detectors, planned, under construction, and operational, covering a wide frequency spectrum from $\sim 10^{-9}$ Hz all the way up to $\sim 10^4$ Hz. Expected sources of gravitational radiation include numerous astrophysical sources such as compact binaries, supermassive black holes, and binary coalescence. In addition, there is an expected cosmological background of gravitational radiation arising from the very earliest times of the universe. The coverage of the spectrum is not complete, and the gap between space-based interferometers (such as the *Laser Interferometer Space Antenna*) and ground-based interferometers (such as the Laser Interferometer Gravitational-Wave Observatory and Virgo) has been proposed as a possible “clean window,” devoid of continuous foreground sources, through which the cosmological background of gravitational radiation could be seen (Seto, Kawamura, & Nakamura 2001). Most binary white dwarf systems will coalesce before their gravitational wave frequency rises above 0.1 Hz, while more massive binaries such as double neutron star or black hole binaries will be sweeping through this band on the way to their eventual coalescence in the ground-based frequency band. However, rotating white dwarfs undergoing quasi-radial oscillations will emit gravitational radiation in this frequency band. These sources will be essentially monochromatic and long-lived. Given the number of white dwarfs in the Galaxy, it is quite possible that this population will produce a confusion-limited foreground of sources in this frequency band that will mask the cosmological background. We propose a possible energy source for these oscillations and attempt to estimate the average signal strength.

2. QUASI-RADIAL OSCILLATIONS

Quasi-radial oscillations of rotating white dwarfs were investigated in the early 1970s (Papoyan, Sedrakian, & Chubaryan 1972; Haroutyunyan, Sedrakian, & Chubaryan 1972), when the frequency spectrum of the fundamental oscillation

mode for maximally rotating white dwarfs was determined. These stars are oblate because of their rotation, and consequently, they have a nonzero quadrupole moment. The oscillations add a time dependence to the quadrupole moment (Vartanian & Hajian 1977). The oscillation is described by assigning each mass element a time-dependent coordinate given by $x_\alpha = x_\alpha^0(1 + \eta \sin \omega t)$, where $\eta \ll 1$ and a constant. Thus, the reduced quadrupole moment is given by

$$Q_{\alpha\beta} = \int \rho(x_\alpha x_\beta - \frac{1}{3}\delta_{\alpha\beta} x^2) d^3x \approx Q_{\alpha\beta}^0(1 + 2\eta \sin \omega t), \quad (1)$$

where $Q_{\alpha\beta}^0$ are the components of the quadrupole moment of the rotating oblate white dwarf in equilibrium and we have neglected terms of order η^2 . Taking the axis of rotation to lie along the z -axis, the nonzero components of the quadrupole moment obey

$$Q^0 = -Q_{zz}^0 = 2Q_{xx}^0 = 2Q_{yy}^0. \quad (2)$$

The power emitted in gravitational radiation is given by

$$J = \frac{G}{5c^5} \left| \frac{d^3}{dt^3} Q_{\alpha\beta} \right|^2, \quad (3)$$

and consequently, one obtains

$$J = \frac{6G}{5c^5} \eta^2 \omega^6 |Q^0|^2 \cos^2 \omega t' = J_0 \cos^2 \omega t', \quad (4)$$

where the retarded time is $t' = t - r/c$ for a source at distance r .

To determine the waveform and the angular distribution of the radiation, we rotate to coordinates in which the wavevector lies along the z -axis and use the transverse-traceless gauge. Consequently,

$$h_+ = \frac{1}{2}(h_{xx} - h_{yy}) = \frac{3GQ^0\eta\omega^2}{c^4 r} \sin^2 \theta \sin \omega t', \quad (5)$$

$$h_\times = h_{xy} = 0, \quad (6)$$

where θ is the angle between the wavevector and the axis of rotation of the white dwarf. We can express the strain amplitude

¹ Montana State University at Billings, 1500 University Drive, Billings, MT 59101; benacquista@msubillings.edu.

² Department of General Physics, Yerevan State University, Alek Manoogian Street 1, Yerevan 375025, Armenia; dsdrak@www.physdep.r.am, mhayr@www.physdep.r.am.

³ Department of Theoretical Physics, Yerevan State University, Alek Manoogian Street 1, Yerevan 375025, Armenia; kshabas@www.physdep.r.am, asadoyan@www.physdep.r.am.

TABLE 1
STRUCTURAL PARAMETERS OF MAXIMALLY ROTATING WHITE DWARFS

$\rho_{c(6)}$ (10^6 g cm^{-3})	$N_{(57)}$ (10^{57} particles)	M/M_\odot	Ω_{max}	$I_{(48)}$ ($10^{48} \text{ g cm}^{-2}$)	$Q_{(48)}^0$ ($10^{48} \text{ g cm}^{-2}$)	$R_{(8)}$ (10^8 cm)	$\Delta Mc^2_{(49)}$ (10^{49} ergs)	$W_{r(49)}$ (10^{49} ergs)	$W_{g(49)}$ (10^{49} ergs)	ω
2.403	0.4997	0.5945	0.196	128.0	20.48	10.93	4.799	0.246	4.55	0.758
19.38	0.8398	0.9993	0.476	88.6	14.27	7.342	8.066	1.00	7.06	0.794
157.7	1.0695	1.2731	1.063	39.5	4.766	4.625	10.27	2.23	8.05	1.513
866.1	1.1340	1.3502	2.042	15.9	1.554	3.044	10.90	3.32	7.58	1.985
2586	1.1261	1.3412	3.105	8.17	0.673	2.287	10.82	3.94	6.89	0.967

in terms of the power by combining equation (4) with equation (5) to obtain

$$h_+ = \sqrt{\frac{15GJ_0}{2c^3}} \frac{1}{r\omega} \sin^2 \theta \sin \omega t'. \quad (7)$$

If an energy source can be found to drive the pulsations, the rate at which power is put into the vibrations can be combined with the lifetime of the energy source to estimate the strain amplitude from an individual white dwarf and thus from the Galactic population as a whole. We discuss a possible mechanism in the next section.

3. ENERGY SOURCE

If there is no permanent source of energy to feed the gravitational radiation, the oscillation energy will quickly radiate away in about 1000 yr (Vartanian & Hajian 1977). Since the ultimate source of the gravitational radiation from the white dwarf is the oblateness arising from the rotation, we propose that the deformation energy of the white dwarf provides the energy to drive the oscillations. In this scenario, as the white dwarf spins down, it will transition from oblate to spherical. This transition will trigger starquakes that will feed the oscillations that drive the gravitational radiation. We assume that a part of the deformation energy will be converted into gravitational radiation while the remaining part will dissipate through thermal, electromagnetic, and other channels. We use the technique of Sahakian, Sedrakian, & Chubarian (1972) to calculate the deformation energy.

From numerical results, the dependence of the mass of the rotating and nonrotating configurations is found to be linear with the baryon number, so $M = kN$ and $M_0 = k_0N$, where the subscript ‘‘0’’ indicates the nonrotating configuration. The mass difference between the rotating and nonrotating configurations with the same baryon number is a result of the additional energy of rotation [$W_r(\Omega) = \frac{1}{2}I\Omega^2$] as well as the potential energy due to crustal deformation [$W_g(\Omega)$]; thus,

$$W_g(\Omega) = \Delta Mc^2 - W_r(\Omega), \quad (8)$$

where Ω is the angular velocity of the white dwarf, I is its moment of inertia, and the mass difference in grams is given by

$$\Delta M = (k - k_0)N = 8.96 \times 10^{-29}N. \quad (9)$$

The appropriate parameters for maximally rotating white dwarfs were calculated in Haroutyunian, Sedrakian, & Chubarian (1971) and Sahakian et al. (1972). These parameters are presented in Table 1. To obtain the deformation energy for rotation rates less than Ω_{max} , we use the fact that the results of Haroutyunian et al. (1971) were obtained using a linear ex-

pansion in the small dimensionless parameter $\Omega^2/8\pi G\rho_c$, where ρ_c is the nonrotating central density. Consequently, we can write

$$W_g(\Omega) = \left(\frac{\Omega}{\Omega_{\text{max}}}\right)^2 W_g(\Omega_{\text{max}}). \quad (10)$$

It now remains to determine the timescale, τ , for a spin-down mechanism so that we can relate the power in gravitational radiation to the decrease in deformation energy by

$$J_0 = \beta \frac{W_g}{\tau}, \quad (11)$$

where $\beta \ll 1$ is a ‘‘branching ratio’’ that quantifies the fraction of deformation energy that goes into gravitational radiation from the fundamental mode.

One possible mechanism for the spin-down of a rotating white dwarf is magnetodipole radiation torque, which occurs if the magnetic field is oblique (Schmidt et al. 2001). Observational data for 65 isolated white dwarfs indicates the magnetic field strength on the surface of these stars lies in the range $\sim 3 \times 10^4$ to $\sim 10^9$ G (Wickramasinghe & Ferrario 2000). If we define α to be the angle between the magnetic and rotation axes, the spin-down rate of the white dwarf is given by

$$\dot{\Omega} = -\frac{2\mu^2\Omega^3}{3Ic^3} \sin^2 \alpha, \quad (12)$$

where $\mu = BR^3$ is the magnetic moment, B is the magnetic field strength, and R is the radius of the white dwarf. The characteristic timescale is then

$$\tau = \frac{\Omega}{2|\dot{\Omega}|}. \quad (13)$$

We neglect the influence of gravitational radiation on the spin evolution because the ratio of the energy lost through gravitational radiation to that of the energy lost through the magnetic-dipole spin-down mechanism is about 3.6β . With this information, we can now estimate the gravitational radiation luminosity and strain amplitude for several white dwarfs.

4. STRAIN AMPLITUDES AND LUMINOSITIES

The rotation rates for white dwarfs are difficult to measure, since these objects have little or no surface blemishes and gravitational broadening of their spectral lines overwhelms the expected rotational broadening (Kawaler 2003). Fortunately, some magnetic white dwarfs show time variability in their magnetic features that allows for their rotation rates to be inferred (Wickramasinghe & Ferrario 2000; Kawaler 2003). Magnetic white dwarfs are thought to make up roughly 1% of the

TABLE 2
ISOLATED MAGNETIC WHITE DWARFS

Name	r (pc)	B (MG)	P_{rot}	M/M_{\odot}
PG 1031+234	142 ^a	500 ^b	3.4 hr	0.93 ^c
EUVE J0317–855	35 ^a	450	725 s	1.35
PG 1015+015	66 ^a	90	99 minutes	1.15 ^c
Feige 7	49	35	2.2 hr	0.6
G99-47	8 ^c	25	1 hr?	0.71 ^c
KPD 0253+5052	81 ^d	17	3.79 hr	...
PG 1312+098	10	5.43 hr	...
G217-037	11 ^c	≤0.2	2–20? hr ^e	0.89

^a Heyl 2000.

^b B ranges from 500 to 1000.

^c Liebert, Bergeron, & Holberg 2003.

^d Downes 1986.

^e $P_{\text{rot}} = 2$ hr was used for the calculation.

white dwarf population, and isolated magnetic white dwarfs tend to have an average mass ($\geq 0.95 M_{\odot}$) that is well above that of nonmagnetic white dwarfs ($\sim 0.57 M_{\odot}$). There is evidence that the population of magnetic white dwarfs is bimodal with high-mass stars (near the Chandrasekhar limit) having rotational periods between ~ 700 s and several hours (Wickramasinghe & Ferrario 2000). This may be evidence that such objects are the result of double-degenerate mergers (Wickramasinghe & Ferrario 2000; Dupuis, Vennes, & Chayer 2002), as has been proposed for EUVE J0317–855 (Ferrario et al. 1997).

The population of merged magnetic white dwarfs may be the most promising source of gravitational radiation from the mechanism of quasi-radial oscillations. To estimate the expected strain amplitude from both individual sources and the population as a whole, we identify eight candidates from the 65 known isolated magnetic white dwarfs from Wickramasinghe & Ferrario (2000) as those with rotational periods less than 1 day. The properties of the candidates are listed in Table 2. We estimate the local space density of merged magnetic white dwarfs by taking the local space density of white dwarfs to be 0.003 pc^{-3} (Liebert, Dahn, & Monet 1988), the fraction of isolated magnetic white dwarfs to be 5.1% (Wickramasinghe & Ferrario 2000), and the fraction of isolated magnetic white dwarfs that are remnants of mergers to be 8/65. This results in a local space density of $\rho_s = 1.9 \times 10^{-5} \text{ pc}^{-3}$. The expected distance to the nearest source is then found by

$$r = 2 \left(\frac{3}{4\pi\rho_s} \right)^{1/3} \approx 46 \text{ pc}. \quad (14)$$

Assuming a Galactic distribution of white dwarfs to follow the disk population, we assign a density distribution of

$$\rho = \rho_0 e^{-r/R} e^{-z/h} \quad (15)$$

in Galactocentric cylindrical coordinates, with $R = 2.5$ kpc and $h = 200$ pc. Taking the solar location as $r_s = 8.5$ kpc and $z_s = 0$, we obtain $\rho_0 = 5.5 \times 10^{-4} \text{ pc}^{-3}$ and a total number of $N = 8.6 \times 10^6$ in the Galaxy.

Expected strain amplitudes, τ , and η for the eight isolated magnetic white dwarfs are given in Table 3. These are calculated by first determining τ from equations (12) and (13), where we have chosen $\sin^2 \alpha = \frac{1}{2}$. This is a reasonable assumption, since a distribution in α that is either uniform or spiked at $\alpha = \pi/4$ is supported by observation (Schmidt & Norsworthy 1991). Choosing a value for β with which to calculate the luminosity is somewhat problematic because of the dearth of observational data for pulsations in this frequency range. Most observations of white dwarf pulsations are in the millihertz range, and consequently, any pulsations in the decihertz range will be averaged out in an observation. We choose an upper bound to β by requiring that the largest Doppler broadening of spectral lines due to pulsations be less than thermal Doppler broadening. The most stringent constraint comes from PG 1031+234 and yields $\beta = 10^{-4}$. We note that the resulting pulsational Doppler broadening in the remaining seven white dwarfs is at least an order of magnitude below the thermal Doppler broadening. Thus, the luminosity is calculated from equation (11) and used with equations (14) and (7) to determine the strain amplitude where we have averaged over all orientations and taken $\beta = 10^{-4}$. We have used the average mass of $0.95 M_{\odot}$ whenever the mass was undetermined, and we have used the average distance of 46 pc whenever the distance was undetermined. The expected energy flux on Earth (in units of $\text{ergs s}^{-1} \text{ cm}^{-2}$) for a population made entirely of each type of white dwarf is also shown in Table 3. The flux, F , is calculated using

$$F = \frac{4\pi\rho_s h f(h) J_0}{4\pi(3 \times 10^{18})^2}, \quad (16)$$

where $h = 200$ pc and $f(h) = 6.15$ is calculated using Appendix A of Hils, Bender, & Webbink (1990). Finally, we note that a simple average of the strain amplitudes in Table 3 gives $h_+ = 5.9 \times 10^{-28}$ and an average flux of $F = 9.21 \times 10^{-16} \text{ ergs s}^{-1} \text{ cm}^{-2}$. The flux is spread out over a frequency band of $\nu_1 = 0.12$ to $\nu_2 = 0.32$ Hz, and we can estimate an average strain amplitude for the Galactic population of pulsating white dwarfs by using the angle and polarization averaged expression

TABLE 3
STRAIN AMPLITUDES AND FLUXES FOR ISOLATED WHITE DWARFS WITH $\beta = 10^{-4}$

Name	h_+	F	τ (Gyr)	η
PG 1031+234	6.0×10^{-29}	6.1×10^{-17}	11	1.0×10^{-2}
EUVE J0317–855	1.0×10^{-27}	6.7×10^{-15}	1.7	4.0×10^{-3}
PG 1015+015	9.3×10^{-30}	1.1×10^{-18}	571	7.1×10^{-4}
Feige 7	1.6×10^{-28}	4.9×10^{-17}	125	5.2×10^{-4}
G99-47	3.5×10^{-27}	5.9×10^{-16}	50.6	3.7×10^{-4}
KPD 0253+5052	2.9×10^{-30}	4.6×10^{-20}	11845	3.5×10^{-4}
PG 1312+098	1.5×10^{-30}	3.8×10^{-21}	70266	2.0×10^{-4}
G217-037	9.0×10^{-31}	8.2×10^{-23}	2.4×10^7	4.1×10^{-6}

of Douglas & Braginsky (1979) and averaging over the frequency range $\Delta\nu = \nu_2 - \nu_1$ to obtain

$$h_{+\text{ave}} = \frac{\ln \nu_2/\nu_1}{\Delta\nu} \sqrt{\frac{4GF}{\pi c^3}}, \quad (17)$$

which gives $h_{+\text{ave}} = 8.35 \times 10^{-27}$.

5. CONCLUSIONS

We have shown that the Galactic population of white dwarfs can produce a background of gravitational radiation in the frequency range of 0.12–0.32 Hz through quasi-radial pulsations. The source of energy to drive these pulsations is found in the deformation energy of the white dwarf because of its rotation. This energy can be extracted from the white dwarf as it spins down. We have proposed that a population of isolated magnetic

white dwarfs that are the remnants of merged double-degenerate binaries can be sources of this gravitational radiation. Estimates of the signal strength over the frequency band of interest indicate that this population may be comparable in strength to the level of the stochastic cosmological background of gravitational radiation predicted by standard inflationary models. In addition, some white dwarfs may be near enough that their signals will stand above the background and their individual parameters can be measured. If the frequency and amplitude of the gravitational radiation from one of these nearby single white dwarfs can be measured, then it will provide an opportunity to test models of white dwarf interiors and give a unique opportunity to check the main parameters for white dwarfs and to understand their dissipation mechanisms.

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REFERENCES

- Douglas, D. H., & Braginsky, V. G. 1979, in *General Relativity: An Einstein Centenary Survey*, ed. S. W. Hawking & W. Israel (Cambridge: Cambridge Univ. Press), 30
- Downes, R. A. 1986, *ApJS*, 61, 569
- Dupuis, J., Vennes, S., & Chayer, P. 2002, *ApJ*, 580, 1091
- Ferrario, L., Vennes, S., Wickramasinghe, D. T., Bailey, J. A., & Christian, D. J. 1997, *MNRAS*, 292, 205
- Haroutyunian, G. G., Sedrakian, D. M., & Chubaryan, E. V. 1971, *Soviet Astrophys.*, 7, 467
- . 1972, *AZh*, 49, 1216
- Heyl, J. S. 2000, *MNRAS*, 317, 310
- Hils, D., Bender, P. L., & Webbink, R. F. 1990, *ApJ*, 360, 75
- Kawaler, S. D. 2003, in *IAU Symp. 215, Stellar Rotation*, ed. A. Maeder & P. Eenens (San Francisco: ASP), in press (astro-ph/0301539)
- Liebert, J., Bergeron, P., & Holberg, J. B. 2003, *AJ*, 125, 348
- Liebert, J., Dahn, C. C., & Monet, D. G. 1988, *ApJ*, 332, 891
- Papoyan, V. V., Sedrakian, D. M., & Chubaryan, E. V. 1972, *AZh*, 49, 750
- Sahakian, G. S., Sedrakian, D. M., & Chubarian, E. V. 1972, *Soviet Astron.*, 8, 541
- Schmidt, G. D., & Norsworthy, J. E. 1991, *ApJ*, 366, 270
- Schmidt, G. D., Vennes, S., Wickramasinghe, D. T., & Ferrario, L. 2001, *MNRAS*, 328, 203
- Seto, N., Kawamura, S., & Nakamura, T. 2001, *Phys. Rev. Lett.*, 87, 221103
- Vartanian, Yu. L., & Hajian, G. S. 1977, *AZh*, 54, 1047
- Wickramasinghe, D. T., & Ferrario, L. 2000, *PASP*, 112, 873