11-2010

Battese-coelli estimator with endogenous regressors

Levent Kutlu
The University of Texas Rio Grande Valley, levent.kutlu@utrgv.edu

Battese-Coelli Estimator with Endogenous Regressors

Levent Kutlu*

Keywords: Battese-Coelli Estimator; Endogeneity; Efficiency.
JEL Classification: C13.

Abstract

We provide a framework for dealing with the endogeneity problem in the Battese-Coelli estimator for productive efficiency measurement.

1 Introduction

The Battese-Coelli (1992) (BC) estimator is a very widely used estimator in stochastic frontier analysis literature. This is mostly because of its simplicity and availability. Unfortunately, in the presence of endogenous regressors this estimator gives inconsistent parameter estimates. Following Kim and Kim (2007)[1], we deal with the endogeneity problem for the BC estimator. This is accomplished by decomposing the irregular term into two parts: one correlated with the regressors and the other not. After the decomposition, one can use a slightly modified version of the BC estimator in order to estimate the parameters model and the technical efficiencies of the firms.

Section 2 gives the model specification and derivations of our estimator. Section 3 provides Monte Carlo experiments to investigate the performance of our estimator. Section 4 gives summary and conclusion.

2 Estimation Procedure

Consider the following production model with endogenous explanatory variables:

$$
\begin{aligned}
y_{it} &= x_{it}'\beta + \varepsilon_{it} - u_{it} && (1)\\
x_{it} &= Z_{it}'\delta + v_{it} && (2)\\
\begin{bmatrix} \tilde{v}_{it} \\ \varepsilon_{it} \end{bmatrix} &\equiv \begin{bmatrix} \Sigma_v^{-1/2} v_{it} \\ \varepsilon_{it} \end{bmatrix} \sim \mathbf{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} I_m & \rho\sigma_\varepsilon \\ \rho'\sigma_\varepsilon & \sigma_\varepsilon^2 \end{bmatrix}) && (3)
\end{aligned}
$$

where y_{it} is the dependent variable; x_{it} is a $m \times 1$ vector of regressors; $Z_{it} = I_m \otimes z_{it}$ where z_{it} is a $l \times 1$ (with $l \geq m$) vector of exogenous variables.

*Department of Economics, Rice University, MS 22, P.O. Box 1892, Houston, TX 77251, USA. Contact: e-mail:levent.kutlu@rice.edu, Tel: +1 713 348 2304

[1] See also Kim (2006), Kim and Nelson (2006), and Kutlu and Sickles (2010).

For the sake of avoiding unnecessary details, we assume that all regressors are endogenous. It is assumed that the irregular term ε_{it} is correlated with the regressors though independent of the inefficiency term $u_{it} = \eta_t u_i$, where u_i is a non-negative truncation of the $\mathbf{N}(\mu, \sigma_u^2)$ distribution and $\eta_t = \exp(-\eta(t-T))$. Moreover, regressors are independent with the inefficiency term.

By a Cholesky decomposition of the variance-covariance matrix of $\begin{bmatrix} \tilde{v}_{it}' & \varepsilon_{it} \end{bmatrix}'$, we can represent $\begin{bmatrix} \tilde{v}_{it}' & \varepsilon_{it} \end{bmatrix}'$ as follows:

$$\begin{bmatrix} \tilde{v}_{it} \\ \varepsilon_{it} \end{bmatrix} = \begin{bmatrix} I_m & 0 \\ \sigma_\varepsilon \rho' & \sigma_\varepsilon \sqrt{1-\rho'\rho} \end{bmatrix} \begin{bmatrix} \tilde{v}_{it} \\ w_{it} \end{bmatrix} \tag{4}$$

where $\tilde{v}_{it}$ and $w_{it} \sim \mathbf{N}(0, (1-\rho'\rho)\sigma_\varepsilon^2)$ are independent.

Hence, we can write the production equation as follows:

$$\begin{aligned} y_{it} &= x_{it}'\beta + \sigma_\varepsilon \rho' \tilde{v}_{it} + w_{it} - u_{it} \\ &= x_{it}'\beta + \sigma_\varepsilon \rho' \Sigma_v^{-1/2}(x_{it} - Z_{it}'\delta\ ) + w_{it} - u_{it} \end{aligned} \tag{5}$$

Let $y_i = (y_{i1}, y_{i2}, ..., y_{iT})'$, $x_i = (x_{i1}', x_{i2}', ..., x_{iT}')'$, $y = (y_1', y_2', ..., y_N')'$, and $x = (x_1', x_2', ..., x_N')'$. The joint density function of y and x is given by:

$$f(y, x) = \prod_{i=1}^{N} f(y_i|x_i) f(x_i) \tag{6}$$

Defining $e_{it} = w_{it} - u_{it}$, $e_i = (e_{i1}, e_{i2}, ..., e_{iT})'$, and $v_i = (v_{i1}', v_{i2}', ..., v_{iT}')'$, the conditional density function of u_i given e_i and v_i is given by:

$$f_{u_i|e_i,v_i}(u_i) = \frac{\exp(-\frac{1}{2}[(u_i - \mu_i^*)/\sigma^*]^2)}{(2\pi)^{1/2}\sigma^*[1 - \Phi(-\mu_i^*/\sigma^*)]},\ u_i \geq 0 \tag{7}$$

where

$$\begin{aligned} \mu_i^* &= \frac{\mu(1-\rho'\rho)\sigma_\varepsilon^2 - \eta' e_i \sigma_u^2}{(1-\rho'\rho)\sigma_\varepsilon^2 + \eta'\eta\sigma_u^2} \\ \sigma^{*2} &= \frac{(1-\rho'\rho)\sigma_\varepsilon^2 \sigma_u^2}{(1-\rho'\rho)\sigma_\varepsilon^2 + \eta'\eta\sigma_u^2} \end{aligned}$$

The conditional expectation of technical efficiency, $TE_{it} = \exp(-u_{it})$, is:

$$E[\exp(-u_{it})|e_i, v_i] = \frac{1 - \Phi(\eta_t\sigma^* - (\mu_i^*/\sigma^*))}{1 - \Phi(-\mu_i^*/\sigma^*)} \exp(-\eta_t\mu_i^* + \frac{1}{2}\eta_t^2\sigma^{*2}) \tag{8}$$

The log-likelihood function for the sample observations, (y, x), is:

$$\ln L = \ln f(y|x) + \ln f(x) \tag{9}$$

where

$$\begin{aligned}
\ln f(y|x) &= -\frac{NT}{2}\ln(2\pi) - \frac{N(T-1)}{2}\ln((1-\rho'\rho)\sigma_\varepsilon^2) - \frac{N}{2}\ln((1-\rho'\rho)\sigma_\varepsilon^2 + \eta'\eta\sigma_u^2) \\
&\quad -N\ln(1-\Phi(-\mu/\sigma_u)) + \textstyle\sum_{i=1}^N \ln(1-\Phi(-\mu_i^*/\sigma^*)) - \frac{1}{2}N(\frac{\mu}{\sigma_u})^2 \\
&\quad -\frac{1}{2}\textstyle\sum_{i=1}^N\sum_{t=1}^T \frac{(y_{it} - x_{it}'\beta + \rho'\sigma_\varepsilon\Sigma_v^{-1/2}(x_{it} - Z_{it}'\delta\ ))^2}{(1-\rho'\rho)\sigma_\varepsilon^2} + \frac{1}{2}\textstyle\sum_{i=1}^N(\frac{\mu_i^*}{\sigma^*})^2 \\
\ln f(x) &= -\frac{lNT}{2}\ln(2\pi) - \frac{NT}{2}\ln(|\Sigma_v|) - \frac{1}{2}\textstyle\sum_{i=1}^N\sum_{t=1}^T v_{it}'\Sigma_v^{-1}v_{it}
\end{aligned}$$

Using the parametrization of the model, where $(1-\rho'\rho)\sigma_\varepsilon^2 + \sigma_u^2 = \sigma_S^2$ and $\gamma = \frac{\sigma_u^2}{\sigma_s^2}$, the conditional log-likelihood function, $f(y|x)$, is expressed by[2]:

$$\begin{aligned}
\ln f(y|x) &= -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma_s^2) - \frac{N(T-1)}{2}\ln(1-\gamma) \\
&\quad -\frac{N}{2}\ln(1+(\eta'\eta-1)\gamma) - N\ln(1-\Phi(-z)) - \frac{1}{2}Nz^2 + \\
&\quad \frac{1}{2}\textstyle\sum_{i=1}^N \ln(1-\Phi(-z_i^*)) - \frac{1}{2}N(\frac{\mu}{\sigma_u})^2 + \frac{1}{2}\textstyle\sum_{i=1}^N z_i^{*2} \\
&\quad -\frac{1}{2}\textstyle\sum_{i=1}^N \frac{A_i'A_i}{(1-\gamma)\sigma_s^2}
\end{aligned} \tag{10}$$

where $z = \frac{\mu}{(\gamma\sigma_s^2)^{1/2}}$, $\lambda = -(\frac{(1-\gamma)\sigma_s^2}{(1-\rho'\rho)})^{1/2}\Sigma_v^{-1/2}\rho$, $X_i = \begin{bmatrix} x_{i1} & x_{i2} & \dots & x_{iT}\end{bmatrix}$, $\hat{x}_{it}(\delta) = Z_{it}'\delta$, $\hat{X}_i(\delta) = \begin{bmatrix} \hat{x}_{i1}(\delta) & \hat{x}_{i2}(\delta) & \dots & \hat{x}_{iT}(\delta)\end{bmatrix}$, $A_i = y_i - X_i'(\beta-\lambda) - \hat{X}_i(\delta)'\lambda$, and $z_i^* = \frac{\mu(1-\gamma)-\gamma\eta'A_i}{(\gamma(1-\gamma)\sigma_s^2[1+(\eta'\eta-1)\gamma])^{1/2}}$.

We consider a similar two-stage procedure in our Monte Carlo experiments. In the first stage the econometrician predicts $\tilde{v}_{it}$ via OLS using (2), and then maximizes the following log-likelihood function:

$$\begin{aligned}
\ln L &= -\frac{NT}{2}\ln(2\pi) - \frac{NT}{2}\ln(\sigma_s^2) - \frac{N(T-1)}{2}\ln(1-\gamma) \\
&\quad -\frac{N}{2}\ln(1+(\eta'\eta-1)\gamma) - N\ln(1-\Phi(-z)) - \frac{1}{2}Nz^2 + \\
&\quad \frac{1}{2}\textstyle\sum_{i=1}^N \ln(1-\Phi(-z_i^*)) - \frac{1}{2}N(\frac{\mu}{\sigma_u})^2 + \frac{1}{2}\textstyle\sum_{i=1}^N z_i^{*2} \\
&\quad -\frac{1}{2}\textstyle\sum_{i=1}^N \frac{A_i'A_i}{(1-\gamma)\sigma_s^2}
\end{aligned} \tag{11}$$

where $\widehat{\tilde{v}}_{it}$ is the estimate of $\tilde{v}_{it}$ from OLS, $z = \frac{\mu}{(\gamma\sigma_s^2)^{1/2}}$, $\lambda = (\frac{(1-\gamma)\sigma_s^2}{(1-\rho'\rho)})^{1/2}\rho$, $X_i = \begin{bmatrix} x_{i1} & x_{i2} & \dots & x_{iT}\end{bmatrix}$, $\widehat{\tilde{v}}_i = \begin{bmatrix} \widehat{\tilde{v}}_{i1} & \widehat{\tilde{v}}_{i2} & \dots & \widehat{\tilde{v}}_{iT}\end{bmatrix}$, $A_i = y_i - X_i'\beta - \widehat{\tilde{v}}_i'\lambda$, and $z_i^* = \frac{\mu(1-\gamma)-\gamma\eta'A_i}{(\gamma(1-\gamma)\sigma_s^2[1+(\eta'\eta-1)\gamma])^{1/2}}$.

[2]This parametrization is a variation of the parametrization suggested by Battese and Corra (1977).

Unfortunately, the standard errors from this two-stage method are inconsistent. The problem is that the estimates are conditional on estimated standardized error terms from the first stage. Hence, a proper bootstrapping procedure should be implemented in order to get the correct standard errors.

3 Monte Carlo Simulations

In this section we implement Monte Carlo experiments to examine the small sample performance of our estimator. For this purpose we considered the following data generating process:

$$Y_{it} = x_{1it}\beta_1 + x_{2it}\beta_2 + \varepsilon_{it} - u_{it}, \quad \varepsilon_{it} \sim \mathbf{N}(0, \sigma_\varepsilon^2) \tag{12}$$

$$x_{2it} = z_{it}\delta + v_{it}, \quad v_{it} \sim \mathbf{N}(0, \sigma_v^2) \tag{13}$$

$$\begin{bmatrix} x_{1it} \\ z_{it} \end{bmatrix} = R \begin{bmatrix} x_{1i,t-1} \\ z_{i,t-1} \end{bmatrix} + \xi_{it}, \quad \xi_{it} \sim \mathbf{N}(0, I_2) \tag{14}$$

$$\begin{bmatrix} v_{it} \\ \varepsilon_{it} \end{bmatrix} \sim \mathbf{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_v^2 & \rho\sigma_\varepsilon\sigma_v \\ \rho\sigma_\varepsilon\sigma_v & \sigma_\varepsilon^2 \end{bmatrix}) \tag{15}$$

where $\beta_1 = 0.5$, $\beta_2 = 0.5$, $\sigma_v^2 = 1$, $\sigma_\varepsilon^2 = 1$, $\rho = 0.8$, $\delta = 1$, $u_i \sim \mathbf{N}^+(0,1) = |\mathbf{N}(0,1)|$, $u_{it} = \exp(-0.05(t-T))u_i$, $R = \begin{bmatrix} 0.4 & 0.05 \\ 0.05 & 0.4 \end{bmatrix}$, and $\begin{bmatrix} x_{1i1} \\ z_{i1} \end{bmatrix} \sim \mathbf{N}(0, (I_2 - R^2)^{-1})$.

Then the generated values for x_1 and z are shifted around three different means to obtain three balanced groups of firms. We chose $\mu_1 = (5,5)'$, $\mu_2 = (7.5, 7.5)'$, and $\mu_3 = (10,10)'$ as the group means.[3] Although, v is determined by only z, we used the constant, x_1, and z to estimate v. Simulation experiments were repeated 10000 times. Simulation results for coefficient estimates are given in Table 1 and Table 2.

N=50 T=30 $\rho = 0$	*BC*	*BCIV*	*BCIV2*
Coef. MSE	0.0006	0.0009	0.0009
Coef. Var1	0.0004	0.0004	0.0004
Coef. Var2	0.0003	0.0004	0.0004
Coef. Bias1	0.0008	0.0008	0.0008
Coef. Bias2	0.0007	0.0008	0.0008
Eff. MSE	0.0214	0.0214	0.0214
Ave. Rho	–	0.0003	0.0003

Table 1: Performances for exogenous case

[3]When generating regressors we followed Park, Sickles, and Simar (2003 and 2007).

N=50 T=30 $\rho = 0.8$	BC	BCIV	BCIV2
Coef. MSE	0.1154	0.0003	0.0003
Coef. Var1	0.0004	0.0002	0.0002
Coef. Var2	0.0002	0.0002	0.0002
Coef. Bias1	0.1284	0.0001	0.0001
Coef. Bias2	-0.3135	0.0003	0.0002
Eff. MSE	0.0809	0.0087	0.0088
Ave. Rho	$-$	0.8002	0.8001

Table 2: Performances for endogenous case

Simulations show that when there is no correlation both the joint and the two-stage estimators perform almost as good as the BC estimator. But if there is correlation, then the BC estimator is severely biased and our method fixes this bias.

4 Summary and Conclusion

This paper paved the way for estimating time-varying technical efficiency via a modified version of the Battese-Coelli estimator in the presence of endogenous regressors. This is done by decomposing the irregular term into two parts: one correlated with the regressors and the other not. The correlated part is used as a bias correction term and the other part remained an irregular term. Unfortunately, the standard errors for the two-stage procedure are inconsistent and should be corrected via a bootstrapping procedure. As a conclusion, our Monte Carlo experiments show that the proposed method works fine in a small sample.

Acknowledgements

I would like to thank Burcu Cigerli, Robin Sickles, David Splinter, and an anonymous referee for their valuable comments.

References

Battese, G.E., Coelli, T.J. (1992), Frontier Production Functions, Technical Efficiency and Panel Data with Application to Paddy Farmers in India, *Journal of Productivity Analysis*, 3, 153-169.

Battese, G.E., Corra, G.S. (1977), Estimation of a Production Frontier Model: With a Generalized Frontier Production Function and Panel Data, *Australian Journal of Agricultural Economics*, 21, 169-179.

Kim, C.J. (2006), Time-Varying Parameter Models with Endogenous Regressors, *Economics Letters*, 91, 21–26.

Kim, C.J. and Nelson, C. (2006), Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model using Ex-Post Data, *Journal of Monetary Economics*, 53, 1949-66.

Kim, Y. and Kim C.J. (2007), Dealing with Endogeneity in a Time-Varying-Parameter Model: Joint Estimation and a Two-Step Estimation Procedures, Working Paper.

Kutlu, L. and Sickles, C.R. (2010), Estimation of Market Power in the Presence of Firm Level Inefficiencies, Unpublished Manuscript.

Park, B.U., and Sickles, R.C., and Simar, L. (2003), Semiparametric Efficient estimation of AR(1) panel data models, *Journal of Econometrics*, 117, 279-309.

Park, B.U., and Sickles, R.C., and Simar, L. (2007), Semiparametric Efficient Estimation of Dynamic Panel Data Models, *Journal of Econometrics*, 136, 281-301.