Journal of Physics: Conference Series

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Abstract. The subject of research is the longitudinal vibrations of stab knife refiners. A model of the fibrous layer between the rotor and stator is developed. It is shown that it is advisable to use the Maxwell-Thomson model for liquid friction of the rotor and stator, and the Hooke elastic model for boundary friction. The formula for determining the dynamic stiffness of the fibrous layer is obtained. Dynamic and mathematical models of the refiner in the longitudinal direction are developed. The mathematical model is a system of linear differential equations with periodically varying coefficients (Mathieu-Hill equations). A method for calculating the amplitude of oscillations of the refiner rotor and stator is developed. According to this technique, Andritz 54-60-1c stab refiner was designed. Oscillation parameters of the refiner rotor and stator are determined theoretically and experimentally. The stator oscillation amplitude is 1.6–2.3 times smaller than the rotor oscillation one. In case of boundary friction, the amplitude of oscillations of the stator and rotor increases by 2–3 times in comparison with liquid friction. The oscillation amplitude of the rotor and stator in the longitudinal direction is comparable with the gap between these elements. Therefore, it is recommended while designing refiners, to develop methods and means of vibration protection, and during operation, to prevent boundary friction between headsets. The developed calculation procedure can be used in other industries, for example, mining and metallurgy.

1. Introduction

Knife refiner is the main technological equipment for grinding fibrous materials in the pulp and paper industry. The most unreliable element of these machines is the headset [1]. The gap between the grinding sets of the rotor and stator is a fraction of a millimeter and depends on the mill operating mode, type and concentration of pulp [1-4]. During grinding, axial forces act on the rotor and stator, which consist of constant, periodic and random components [5,6].

The headset can work with fluid or boundary friction. With boundary friction, the technical resource of the headset decreases [7.8].

For an effective grinding process in knife refiners, it is necessary to ensure the stability of the gap between the rotor and stator headsets [9,10]. The purpose of the work is to study the oscillations of the refiner in the longitudinal direction.

2. Methods and materials

2.1. Model of the fibrous layer between the rotor and stator

A distinction is made between grinding with liquid friction of the rotor and stator, when the rubbing surfaces are completely separated by the fibrous layer, and boundary friction, when the rubbing surfaces are not completely separated by this layer, there is a metal contact [8, 11, 12]. For liquid friction of the rotor and stator, it is advisable to use the Maxwell – Thomson model [1, 3, 13], and for boundary friction, the elastic Hooke model (figure 1).

The elastic element C_2 with the E_r module determines the instantaneous deformation of the system. A viscoelastic element comprising C_1 spring with E_k module and a damper with viscosity b(t) connected in parallel with it characterizes a delayed elastic deformation. C_3 spring "turns on" only with boundary friction of the rotor and stator. It should be noted that the model parameters depend on factors influencing the course of grinding [14].

In the general case, the relationship between stress and deformation of the fibrous layer can be described in the form of a differential equation [15]

$$\sum_{k=0}^{m} P_k \frac{\partial^k \sigma}{\partial t^k} = \sum_{l=0}^{n} ql \frac{\partial^l \varepsilon}{\partial t^1} . \quad (1)$$

It is known that when the knives of the rotor headset pass over the stator knives, the concentration of the fibrous layer changes [1,3]. The modulus of elasticity of a mass of various concentrations remains almost constant, but its viscosity changes to a large extent [2]. Analysing the foregoing in relation to the model of the fibrous layer, it is concluded that when grinding the semi-finished products, the damping coefficient of the model changes. Moreover, the change in damping will occur with the same frequency with which the rotor and stator knives intersect - with headset frequencies ω_f [16].

Damping in the pulp occurs due to the friction of the surfaces of the fibers against each other, resistance to the movement of fibers in water, the passage of water through the pores of the fibers, the presence of adhesion forces between the fibers. The presence of damping forces causes the manifestation of nonlinear properties in the system. This complicates the study of such systems. In practice, various damping approximation methods are used. This approach allows nonlinear systems to be reduced to simpler linear ones, in particular, using the energy balance method [17, 18]. With boundary friction, the contact area of the headset changes, that is, the stiffness of the metal contact of the rotor and stator changes [13,19,20].

If the force сила *PA(t)* acts in section A of the fiber layer (figure 1), then the same force will act in sections B and C, since the model does not contain inertial elements

$$P_A(t) = P_B(t) = P_C(t) .$$

Let $y_a(t)$, $y_в(t)$, $y_c(t)$ – be *A*, *B* and *C* cross sections, then

$$P_A(t) = C_1(y_c - y_a) + b(t)(\dot{y}_c - \dot{y}_a) = P_C(t) = C_2(y_b - y_c) = P_B(t) \quad (2)$$

We believe that harmonic oscillations with ω_f frequency occur in the system.

Having substituted

$$y_a = A_1 \sin \omega_f t + B_1 \cos \omega_f t;$$
$$y_в = A_2 \sin \omega_f t + B_2 \cos \omega_f t,$$

and having eliminated y_c, we will find solution (1) in the following form

$$P_A(t) = P_A' \sin \omega_f t + P_A \cos \omega_f t;$$
$$P_B(t) = P_B' \sin \omega_f t + P_B \cos \omega_f t,$$

whence we determine the dynamic stiffness *C(t)* of the fibrous layer

$$C(t) = \frac{C_2\left[C_1^2 + b^2(t)\omega_f{}^2\right] + \left[C_1(C_1 + C_2) + b^2(t)\omega_f{}^2 + C_2 b(t)\omega_f\right]}{\left[C_1(C_1 + C_2) + b^2(t)\omega_f{}^2\right]^2 + C_2^2 b^2(t)\omega_f{}^2}; \quad (3)$$

2.2. The model of a disk refiner in the longitudinal direction

The model of a disk refiner in the longitudinal direction is a rotating shaft with a rotor disk, which interacts with the stator disk through a fibre interlayer, which is modelled by liquid friction - the Maxwell-Thomson model, and with boundary friction - by Hooke's body (figure 2) [21].

We will make the following designations: ω is the rotational speed of the rotor disk; m_p is the mass of the rotor disk with the reduced mass of the shaft; m_c is the mass of the stator disk. Stator and rotor disks are interconnected by means of elastic C_4, C_5 and inelastic b_1, b_2 resistances. The refiner body is taken absolutely rigid, and the parameters of C_4, C_5, b_1 и b_2 elements are constant in time.

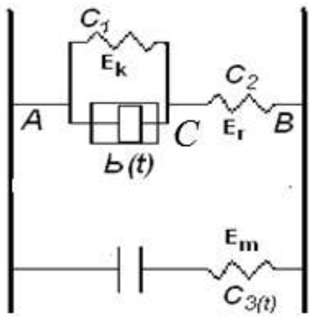


Figure 1. Model of the fibrous layer between the rotor and stator.

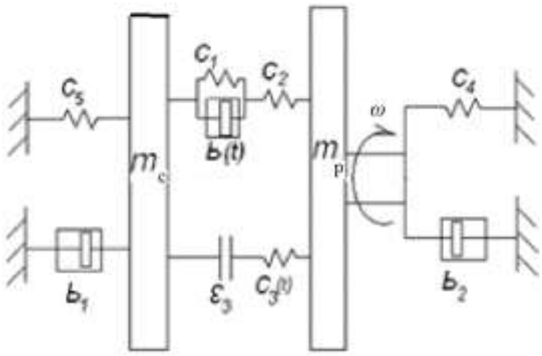


Figure 2. Model of the longitudinal refiner.

Using the D'Alembert's principle, we obtain differential equations describing the oscillations of the refiner. With fluid friction

$$m_c\ddot{y}_1 + b_1\dot{y}_1 + C_5 y_1 + C_1(t)(y_1 - y_2) = 0;$$
$$m_p\ddot{y}_2 + b_2\dot{y}_2 + C_4 y_2 + C_1(t)(y_2 - y_1) = 0, \quad (4)$$

where y_1, y_2 – are m_c, m_p mass movements.

With boundary friction

$$m_c\ddot{y}_1 + b_1\dot{y}_1 + C_5 y_1 + C_3(t)(y_1 - y_2) = 0;$$
$$m_p\ddot{y}_2 + b_2\dot{y}_2 + C_4 y_2 + C_3(t)(y_2 - y_1) = 0, \quad (5)$$

where $C_1(t)$, $C_3(t)$ - stiffness of the fibrous interlayer during fluid and boundary friction.

We will denote:

$$\frac{C_5}{m_c} = \lambda_c^2; \quad \frac{C_4}{m_p} = \lambda_p^2; \quad \frac{b_1}{m_c} = 2\varepsilon_1'; \quad \frac{b_2}{m_p} = 2\varepsilon_2', \tag{6}$$

where λ_c, λ_p - partial frequencies of free oscillations of the stator and rotor in the longitudinal direction; ε_1' ε_2' - partial damping coefficients of the stator and rotor.

Putting (6) into (4) and (5) we will get with liquid friction

$$\begin{aligned} &\ddot{y}_1 + 2\varepsilon_1'\dot{y}_1 + \lambda_c^2 y_1 + \frac{C_1(t)}{m_c}(y_1 - y_2) = 0; \\ &\ddot{y}_2 + 2\varepsilon_2'\dot{y}_2 + \lambda_p^2 y_2 + \frac{C_1(t)}{m_p}(y_2 - y_1) = 0, \end{aligned} \tag{7}$$

with boundary friction

$$\begin{aligned} &\ddot{y}_1 + 2\varepsilon_1'\dot{y}_1 + \lambda_c^2 y_1 + \frac{C_3(t)}{m_c}(y_1 - y_2) = 0; \\ &\ddot{y}_2 + 2\varepsilon_2'\dot{y}_2 + \lambda_p^2 y_2 + \frac{C_3(t)}{m_p}(y_2 - y_1) = 0. \end{aligned} \tag{8}$$

$C_1(t)$ and *$C_3(t)$* parameters can be arranged in a Fourier series

$$\begin{aligned} &C_1(t) = C_0 + \textstyle\sum_{i=1}^{n} C_i \cos(i\omega_{fi} + \beta_i); \\ &C_3(t) = C'_0 + \textstyle\sum_{i=1}^{n} C'_0 \cos(i\omega_{fi} + \beta'_i), \end{aligned} \tag{9}$$

where C_o, C'_o - permanent stiffeners; C_i, C'_i – amplitude of *i*-th harmonic component; β_i, β'_i - shear angle of phases of *i*-th harmonic component.

Equations (7, 8) are a system of linear differential equations with *$C_1(t)$* and *$C_3(t)$* periodically varying coefficients (Mathieu-Hill equations). To consider the stability of their solutions, we will substitute expressions (9) into (7), and without taking into account the dispersion of vibrational energy, we will obtain

$$\begin{aligned} &\ddot{y}_1 + \lambda_c^2(1 + \alpha_1 cos\omega_{fi}t)y_1 = 0 \\ &\ddot{y}_2 + \lambda_p^2(1 + \alpha_2 cos\omega_{fi}t)y_2 = 0 \end{aligned} \tag{10}$$

where $\alpha_1 = \dfrac{1 - \frac{y_2}{y_1}}{m_c\lambda_c^2}$; $\alpha_2 = \dfrac{1 - \frac{y_1}{y_2}}{m_p\lambda_p^2}$.

We will introduce $\tau = W_0(t)$ dimensionless time, where $dt = d\tau / W_0$, then

$$\begin{aligned} &\frac{d^2 y_1}{d\tau^2} + (W_1 + \mu_1 \cos\tau) y_1 = 0; \\ &\frac{d^2 y_2}{d\tau^2} + (W_2 + \mu_2 \cos\tau) y_2 = 0, \end{aligned} \tag{11}$$

where $W_1 = \dfrac{1}{n_{y1}^2}$; $W_2 = \dfrac{1}{n_{y2}^2}$; $\mu_1 = \dfrac{\alpha_1}{n_{y1}^2}$; $\mu_2 = \dfrac{\alpha_2}{n_{y2}^2}$.

Equations (11) are Mathieu ones, whose stability regions are determined by the Ains-Strett diagram. For α_1, α_2 << l, according to the diagram, instability will occur in narrow frequency ranges at n_{y1} = 0.5, 2/3, 1.0 и n_{y2} = 0.5, 2/3, 1.0.

The solution of the equations in the field of stable states is a periodic function in the form of a Fourier series

$$y_1 = \sum_{i=1}^{n}[A_{1i}\cos(i\omega_{fi}t) + B_{1i}\sin(i\omega_{fi}t)]$$
$$y_2 = \sum_{i=1}^{n}[A_{2i}\cos(i\omega_{fi}t) + B_{2i}\sin(i\omega_{fi}t)] \quad (12)$$

Substituting (12) into (10), we will obtain a system of algebraic equations from which $A_{1i,2i}$, $B_{1i,2i}$, coefficients are found, and from them the amplitudes and angles of phase shift are determined

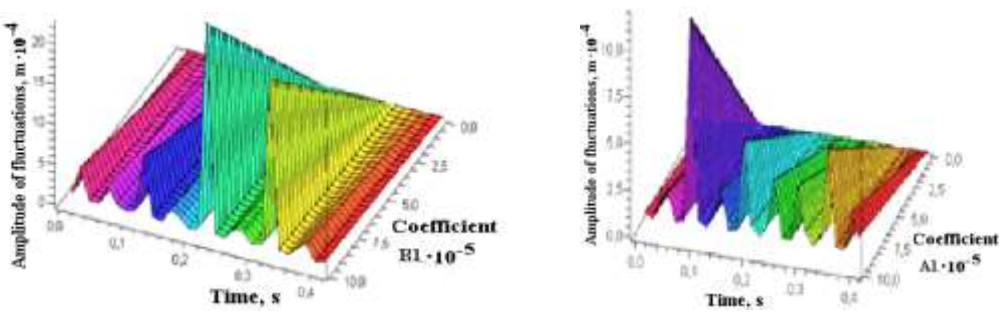
$$S_{a1} = \sqrt{\sum_{i=1}^{n} A_{1i}^2 + \sum_{i=1}^{n} B_{1i}^2};$$
$$S_{a2} = \sqrt{\sum_{i=1}^{n} A_{2i}^2 + \sum_{i=1}^{n} B_{2i}^2}, \quad (13)$$

$$tg\gamma_i = -\frac{A_{1i}}{B_{1i}}; \quad tg\gamma_i' = \frac{A_{2i}}{B_{2i}}. \quad (14)$$

For engineering calculations, it suffices to restrict ourselves to the first two numbers of the series *i*=1,2.

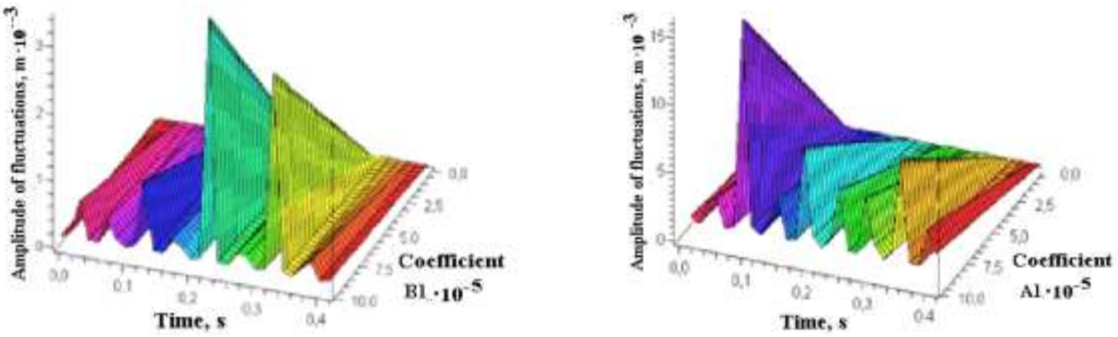
3. Results and discussion

A study of oscillations of 54-60-1c disk refiner of the Andritz company was carried out according to the developed technique experimentally. This refiner grinds waste sorting of thermomechanical pulp at JSC Solikamskbumprom. The study of oscillations was performed using "Maple 15" mathematical software complex. The main research results are presented in figures 3 and 4.



a) b)

Figure 3. The amplitude of the longitudinal oscillations of the rotor at *i*=1: a) depending on B_1coefficient; b) depending on A_1.



a) b)

Figure 4. The amplitude of the longitudinal oscillations of the rotor at *i*=2:
a) depending on B_1coefficient; b) depending on A_1

The amplitude of oscillations of the rotor and stator of 54-60-1c refiner of the Andritz company when grinding waste sorting thermomechanical pulp in the longitudinal direction is presented in table 1.

Table 1. The amplitude of oscillation of the elements of 54-60-1c refiner of the Andritz company in the longitudinal direction.

Refiner element	Element friction	Amplitude of vibrations, microns		
		Theory	Experiment	Error, %
Rotor	Liquid	12	16	25
	Boundary	42	48	13
Stator	Liquid	8	10	20
	Boundary	16	21	24

Analyzing the data obtained, we can conclude that the amplitude of the stator oscillations is 1.6 - 2.3 times less than the amplitude of the rotor. This is due to the greater rigidity of the stator mounting to the refiner body. The rotor has less rigidity due to the presence of bearings. The amplitude of oscillations of the rotor and stator depends on the type of friction between the headsets. In case of boundary friction, the amplitude of oscillations of the stator and rotor increases by 2–3 times in comparison with liquid friction.

The error in determining the amplitude of oscillations of the rotor and stator does not exceed 25%. The error is caused by the error in determining the parameters of the elements of the refiner model and the error in the experiment.

The oscillation amplitude of the rotor and stator in the longitudinal direction is comparable with the gap between these elements. The gap between the rotor and the stator during operation of the refiner is regulated by the additive mechanism and can be hundredths of a millimeter. Therefore, it is recommended that when designing refiners, the development of methods and means of vibration protection, and during operation, to prevent boundary friction between headsets, i.e. when the amplitude of oscillations of the rotor and stator exceeds the gap between them.

4. Conclusion

A method for calculating the longitudinal vibrations of the mill has been developed and tested.

The stator oscillation amplitude is 1.6–2.3 times smaller than the rotor oscillation amplitude. This is due to the greater rigidity of the stator mounting to the refiner body.

The amplitude of oscillations of the rotor and stator depends on the type of friction between the headsets. In case of boundary friction, the amplitude of oscillations of the stator and rotor increases by 2–3 times in comparison with liquid friction.

The oscillation amplitude of the rotor and stator in the longitudinal direction is comparable with the gap between these elements. Therefore, it is recommended that when designing refiners, to develop methods and means of vibration protection, and during operation, to prevent boundary friction between headsets.

The developed calculation procedure can be used in other industries, for example, mining and metallurgy.

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