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ON THE STABILITY OF THE RECENTLY DEVELOPED NICE INTEGRATION SCHEME

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Abstract. The paper deals with the integration of elasto-plastic constitutive models using recently developed NICE integration scheme [1],[2]. The emphasis is put on the stability of the integration, since this issue was not sufficiently addressed in previous publications of the NICE.

Nonlinear boundary value problems are nowadays typically solved numerically using finite element method (FEM) with implicit “static” (e.g. ABAQUS/Standard) or explicit “dynamic” approach (e.g. ABAQUS/Explicit). The NICE scheme was primarily developed for the integration of elasto-plastic constitutive models within explicit integration of a given boundary value problem, as a replacement for traditionally used backward-Euler scheme. The simplicity of the implementation, more than satisfactory accuracy and low time consumption of calculation, certainly outperforms the properties of other available schemes, including properties of the backward-Euler scheme. The only open issue regarding the NICE scheme is its conditional stability, which originates from the integration of evolution equations in a “forward” manner, whereas the backward-Euler scheme exhibits unconditional stability.

The aim of this paper is to derive stable time increment for the NICE scheme and to show, that for practical quasi-static applications it is much larger than the stable time increment size given for the integration of dynamic boundary value problem equations.

1 INTRODUCTION

In the field of computational solid mechanics a nonlinear boundary value problem is nowadays solved efficiently with the finite element method (FEM), where the solution of an arbitrary constitutive behaviour is tackled numerically with an integration scheme. Due to rapid development and increasing complexity of advanced constitutive relations, researchers are obliged to implement their own material models into FEM framework. This action demands, however, considerable expertise, time and effort. The requirements of low computational cost together with sufficient accuracy and stability of the constitutive calculations, when the material model is not simple, are most significant [3].

Integration schemes can be classified within the categories of explicit (forward) and implicit (backward, trapezoidal, midpoint…) schemes. Implicit schemes are attractive because in case of elasto-plastic behaviour exhibited within an increment the consistency condition is, contrary to explicit schemes, automatically satisfied, while explicit methods are faster and easily programmed.

Recently we proposed a simple and efficient numerical approach [2], resulting in a new numerical scheme for the integration of nonlinear constitutive equations, in which a fulfilment of the consistency condition during the integration is radically improved with regard to the classical forward-Euler scheme. Explicit constitutive integration scheme, named NICE (Next Increment Corrects Error) successfully combines accuracy of implicit (backward-Euler) and simplicity of explicit (forward-Euler) schemes, and is a perfect choice for global explicit FE framework (e.g. ABAQUS/Explicit). In [1], a comprehensive theoretical background with the major properties of the NICE scheme is presented. Its advantages and superiority over other explicit schemes are: substantially improved accuracy, fast computing and, especially, very simple implementation of arbitrary constitutive models. The CPU time consumption has been proven to be, due to effectiveness of the used integration scheme, fully comparable to the performance experienced when the simulation is performed with ABAQUS/Explicit built-in constitutive models.

Our objective is to analyze numerical stability of the proposed numerical scheme since this issue was not sufficiently addressed in previous publications. Because explicit integration schemes are proven to be only conditional stable, the stability can present a serious limitation. In this paper it is shown, that the stability of the NICE integration scheme implemented in ABAQUS/Explicit is not a problem for the typical metal forming simulations, because it turns out that a stable increment size of the NICE scheme is several decades larger than a default increment size of ABAQUS/Explicit forming analyses. Consequently, in practice there is no need to be concerned about the instability of the analysis when using explicit NICE integration scheme in such cases.

2 CONSTITUTIVE MODELLING IN PLASTICITY

Firstly, let us briefly review the governing equations of incremental theory of plasticity, which can be optionally extended to embrace also other phenomena (path dependent hardening, damage, path dependency of fracture, etc.) observed in solids. The complete set of differential-algebraic equations that define an elastic-plastic constitutive model is given in the classical form of the system of differential-algebraic equations (DAE):

$$\begin{gathered}
\Phi = 0 \\
\mathrm{d}\sigma_{ij} = C_{ijkl}\left(\mathrm{d}\varepsilon_{kl} - \frac{\partial \psi}{\partial \sigma_{kl}}\mathrm{d}\lambda\right) \\
\mathrm{d}\sigma_{\mathrm{Y}} = \frac{\partial \sigma_{\mathrm{Y}}}{\partial \lambda}\mathrm{d}\lambda \\
\mathrm{d}\kappa_s = \frac{\partial \kappa_s}{\partial \lambda}\mathrm{d}\lambda, \quad s \in \{1,2,\ldots,p\}
\end{gathered} \tag{1}$$

where the variables $\Phi, \psi, \sigma_{ij}, \varepsilon_{ij}, \sigma_{\mathrm{Y}}$ represent respectively: yield function, plastic potential, stress tensor, strain tensor and yield stress. For consideration of other included phenomena, p additional state variables are introduced. Variable $\mathrm{d}\lambda$ denotes the plastic multiplier.

3 REDEFINITION OF STATE VARIABLES

For the sake of simplicity of further derivation and implementation let us redefine variables and generalize the constitutive model in a form, which is convenient to a wide range of plasticity models. For this purpose, we first introduce a generalized stacked vector/matrix notation of tensor variables (similar to Voigt notation).

Let a_{ij} and b_{ij} be two symmetric second order tensors $a_{ij} = a_{ji}$, $b_{ij} = b_{ji}$ and let C_{ijkl} be a fourth order symmetric tensor $C_{ijkl} = C_{jikl} = C_{ijlk}$, where $i, j, k, l \in \{1, 2, 3\}$. Let α_n and β_n be scalar values for every $n \in \{1, 2, \ldots, r\}$. Further, let us define the following stacked vectors and matrix:

$$\begin{aligned} \mathbf{a} &= \{a_{11}, a_{22}, a_{33}, a_{12}, a_{23}, a_{31}\}^{\mathrm{T}} \\ \mathbf{b} &= \{b_{11}, b_{22}, b_{33}, b_{12}, b_{23}, b_{31}\}^{\mathrm{T}} \\ \hat{\mathbf{a}} &= \{a_{11}, a_{22}, a_{33}, a_{12}, a_{23}, a_{31}, \alpha_1, \ldots, \alpha_r\}^{\mathrm{T}} \\ \hat{\mathbf{b}} &= \{b_{11}, b_{22}, b_{33}, b_{12}, b_{23}, b_{31}, \beta_1, \ldots, \beta_r\}^{\mathrm{T}} \end{aligned} \qquad \mathbf{C} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1123} & C_{1131} \\ C_{2211} & C_{2222} & C_{2222} & C_{2212} & C_{2223} & C_{2231} \\ C_{3311} & C_{3322} & C_{3322} & C_{3312} & C_{3323} & C_{3331} \\ C_{1211} & C_{1222} & C_{1222} & C_{1212} & C_{1223} & C_{1231} \\ C_{2311} & C_{2322} & C_{2322} & C_{2312} & C_{2323} & C_{2331} \\ C_{3111} & C_{3122} & C_{3122} & C_{3112} & C_{3123} & C_{3131} \end{bmatrix} \tag{2}$$

and introduce special matrix operator "$\cdot$", which should be clearly distinguished from the usual matrix multiplication ".".

With the newly introduced operator "$\cdot$" the tensor operations $a_{ij}b_{ij}$, $a_{ij}b_{ij} + \sum_{n=1}^{r} \alpha_n \beta_n$, $a_{ij}C_{ijkl}$ and $C_{ijkl}b_{kl}$ can be expressed as $a_{ij}b_{ij} = \mathbf{a} \cdot \mathbf{b}$, $a_{ij}b_{ij} + \sum_{n=1}^{r} \alpha_n \beta_n = \hat{\mathbf{a}} \cdot \hat{\mathbf{b}}$, $a_{ij}C_{ijkl} = \mathbf{a} \cdot \mathbf{C}$ and $C_{ijkl}b_{kl} = \mathbf{C} \cdot \mathbf{b}$.

Now, with the newly introduced notation, let us define expanded variables:

$$\begin{gathered} \mathrm{d}\boldsymbol{\varepsilon} = \{\mathrm{d}\varepsilon_{11} \quad \mathrm{d}\varepsilon_{22} \quad \mathrm{d}\varepsilon_{33} \quad \mathrm{d}\varepsilon_{12} \quad \mathrm{d}\varepsilon_{23} \quad \mathrm{d}\varepsilon_{31}\}^{\mathrm{T}} \\ \boldsymbol{\Sigma} = \{\boldsymbol{\sigma} \quad \sigma_{\mathrm{Y}} \quad \boldsymbol{\kappa}\}^{\mathrm{T}} = \{\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{12} \quad \sigma_{23} \quad \sigma_{31} \quad \sigma_{\mathrm{Y}} \quad \kappa_1 \quad \cdots \quad \kappa_p\}^{\mathrm{T}} \end{gathered} \tag{3}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1112} & C_{1123} & C_{1131} \\ C_{2211} & C_{2222} & C_{2233} & C_{2212} & C_{2223} & C_{2231} \\ C_{3311} & C_{3322} & C_{3333} & C_{3312} & C_{3323} & C_{3331} \\ C_{1211} & C_{1222} & C_{1233} & C_{1212} & C_{1223} & C_{1231} \\ C_{2311} & C_{2322} & C_{2333} & C_{2312} & C_{2323} & C_{2331} \\ C_{3111} & C_{3122} & C_{3133} & C_{3112} & C_{3123} & C_{3131} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \; ; \; \mathbf{R} = \begin{Bmatrix} \left[\mathbf{C} \cdot \dfrac{\partial \psi}{\partial \boldsymbol{\sigma}} \right]_{6x1} \\ \left[-\dfrac{\partial \sigma_{\mathrm{Y}}}{\partial \lambda} \right]_{1x1} \\ \left[-\dfrac{\partial \boldsymbol{\kappa}}{\partial \lambda} \right]_{px1} \end{Bmatrix}$$

and rewrite equations (1) into matrix form as:

$$\begin{gathered} \Phi = 0 \\ \mathrm{d}\boldsymbol{\Sigma} = \mathbf{C} \cdot \mathrm{d}\boldsymbol{\varepsilon} - \mathbf{R}\,\mathrm{d}\lambda \end{gathered} \tag{4}$$

4 NICE INTEGRATION SCHEME

The task of integration scheme is to calculate new stress state $\boldsymbol{\sigma}_{n+1}$ and other state variables at the end of each n-th increment for a given strain increment $\Delta\boldsymbol{\varepsilon}$, when a stress $\boldsymbol{\sigma}_n$ and other state variables at the beginning of the increment are known. Plasticity models are defined in a form of a system of differential-algebraic equations, where algebraic equation $\Phi = 0$ represents the consistency condition. In conventional numerical explicit schemes it is risky to transform the algebraic equation $\Phi = 0$ into a differential one as $\mathrm{d}\Phi = 0$, because this would lead finally to finite difference form $\Delta\Phi = 0$. Such approach is in accordance with forward-Euler approach for solving a set of DAEs, but in [1],[2] we widely discussed, why such approach is questionable. According to the NICE scheme, we suggest to perform a Taylor series expansion of $\Phi(\boldsymbol{\Sigma})$ on all unknown state variables instead, which leads to $\Phi + \Delta\Phi = 0$. Further, differential equations (4) must be transformed, according to forward-Euler approach, into a corresponding finite difference form. A set of equation is finally obtained, in which all state variables are known from the previous increment

$$\begin{gathered} \Phi_n + \left.\frac{\partial \Phi}{\partial \boldsymbol{\Sigma}}\right|_n \cdot \Delta\boldsymbol{\Sigma} = 0 \\ \Delta\boldsymbol{\Sigma} = \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon} - \mathbf{R}_n\,\Delta\lambda \end{gathered} \tag{5}$$

From the above set of equations the plastic multiplier can be extracted explicitly

$$\Delta\lambda = \frac{\Phi_n + \left.\dfrac{\partial \Phi}{\partial \boldsymbol{\Sigma}}\right|_n \cdot \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon}}{\left.\dfrac{\partial \Phi}{\partial \boldsymbol{\Sigma}}\right|_n \cdot \mathbf{R}_n} \tag{6}$$

Finally, the elasto-plastic update is made based on (6):

$$\mathbf{\Sigma}_{n+1} = \mathbf{\Sigma}_n + \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon} - \mathbf{R}_n \Delta\lambda \tag{7}$$

5 STABILITY OF NUMERICAL INTEGRATION

The purpose of the stability analysis is a determination of the circumstances under which a numerical scheme is able to attenuate perturbations of the initial state. The analysis is performed within the framework of linear analysis of numerical stability with assumption of small perturbations and follows the procedure described in [4]. Numerical integration scheme is said to be stable, if in the n-th increment initial perturbations $\delta\mathbf{\Sigma}_n$ are not amplified into larger perturbations $\delta\mathbf{\Sigma}_{n,m+1}$ at the end of the n-th increment. Further, due to conditional stability of the NICE scheme we introduce critical increment size $\Delta\boldsymbol{\varepsilon}_n^{\mathrm{cr}} = \alpha_n \Delta\boldsymbol{\varepsilon}$ as the α_n portion of the n-th load increment $\Delta\boldsymbol{\varepsilon}$. Evidently, if in the n-th increment $\alpha_n \geq 1$ the integration would proceed in a stable way, otherwise ($\alpha_n < 1$) instability occurs. Carrying out variational calculus of (4) and considering the fact that initial perturbations should not be amplified, the following criterion is obtained:

$$\alpha_n = \frac{\dfrac{2}{\max\left(-\boldsymbol{\omega}_n\right)} \left.\dfrac{\partial\Phi}{\partial\mathbf{\Sigma}}\right|_n \cdot \mathbf{R}_n - \Phi_n}{\left.\dfrac{\partial\Phi}{\partial\mathbf{\Sigma}}\right|_n \cdot \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon}} \quad ; \quad \boldsymbol{\omega}_n = \mathrm{eig}\left(-\left.\frac{\partial\mathbf{R}}{\partial\mathbf{\Sigma}}\right|_n \right) \tag{8}$$

6 NUMERICAL VALIDATION

6.1 Stability of numerical integration on mathematical example

In this subsection the stability issue will be elaborated on a problem, the form of which is similar to plasticity models. The problem is defined in terms of the following system of differential-algebraic equations (DAEs).

$$\begin{aligned} \Phi(\sigma_1, \sigma_2, \sigma_{\mathrm{Y}}) &= \frac{\sigma_1^2 + \sigma_2^2}{\sigma_{\mathrm{Y}}^2} - 1 = 0 \\ \mathrm{d}\sigma_1 &= E\,\mathrm{d}\varepsilon_1 - E\frac{\partial\Phi}{\partial\sigma_1}\mathrm{d}\lambda \\ \mathrm{d}\sigma_2 &= E\,\mathrm{d}\varepsilon_2 - E\frac{\partial\Phi}{\partial\sigma_2}\mathrm{d}\lambda \\ \mathrm{d}\sigma_{\mathrm{Y}} &= H\,\mathrm{d}\lambda \end{aligned} \tag{9}$$

Here, function Φ, which consists of the stress components σ_1, σ_2 and yield stress σ_{Y}, is analogous to the yield function. To introduce material hardening, σ_{Y} is defined as a function of plastic multiplier $\mathrm{d}\lambda$, multiplied with constant $H = 0.02$. In other evolution (differential) equations Hooke's law with $E = 200\mathrm{MPa}$ and associated flow rule are considered. In plasticity models the driven variable $\mathrm{d}\varepsilon_{ij}$ is obtained from the equilibrium calculation, which is not present in this case. Hence, we choose a fairly complex parametrically given loading path $t \in [0, 2\pi]$ as

$$\begin{Bmatrix} \mathrm{d}\varepsilon_1 \\ \mathrm{d}\varepsilon_2 \end{Bmatrix} = \begin{Bmatrix} -(R+r)\sin(t) + L\dfrac{R+r}{r}\sin\left(\dfrac{R+r}{r}t\right) \\ (R+r)\cos(t) - L\dfrac{R+r}{r}\cos\left(\dfrac{R+r}{r}t\right) \end{Bmatrix} \mathrm{d}t \tag{10}$$

with initial conditions $\varepsilon_1|_{t=0} = -0.1$, $\varepsilon_2|_{t=0} = 0$, where $R = 3/2$, $r = 1/2$ and $L = 2.1$.

Using general notation, defined in (3), the problem can be rewritten in the matrix form (4) by yielding

$$\mathbf{C} = \begin{bmatrix} E & 0 \\ 0 & E \\ 0 & 0 \end{bmatrix} \quad ; \quad \mathbf{\Sigma} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_\mathrm{Y} \end{Bmatrix} \quad ; \quad \mathbf{R} = \begin{Bmatrix} 2E\sigma_1\sigma_\mathrm{Y}^{-2} \\ 2E\sigma_2\sigma_\mathrm{Y}^{-2} \\ -H \end{Bmatrix} \tag{11}$$

In this case the integration is driven with maximal increment size $\Delta t = \Delta t^{\mathrm{cr}}$ in such a way that integration is stable. For this purpose the size of each increment is calculated based on (8), stating $\alpha = 1$.

A conversion of given DAEs to difference equations according to NICE leads to explicit update of the state variables $\mathbf{\Sigma}$ based on equation (7):

$$\mathbf{\Sigma}_{n+1} = \mathbf{\Sigma}_n + \left(\mathbf{C} \cdot \left.\frac{\partial \boldsymbol{\varepsilon}}{\partial t}\right|_n \Delta t - \mathbf{R}_n \Delta\lambda \right) \quad ; \quad \Delta\lambda = \frac{\Phi_n + \left.\dfrac{\partial \Phi}{\partial \mathbf{\Sigma}}\right|_n \cdot \mathbf{C} \cdot \left.\dfrac{\partial \boldsymbol{\varepsilon}}{\partial t}\right|_n \Delta t}{\left.\dfrac{\partial \Phi}{\partial \mathbf{\Sigma}}\right|_n \cdot \mathbf{R}_n} \tag{13}$$

The integration was performed in Mathematica 8.0 with the NICE, forward-Euler and default built-in solution technique up to $t = 2\pi$, considering the starting point $\sigma_1|_1 = 0\mathrm{MPa}$, $\sigma_2|_1 = -5\mathrm{MPa}$ and $\sigma_\mathrm{Y}|_1 = 5\mathrm{MPa}$ at which the algebraic equation $\Phi_1 = 0$ is fulfilled.

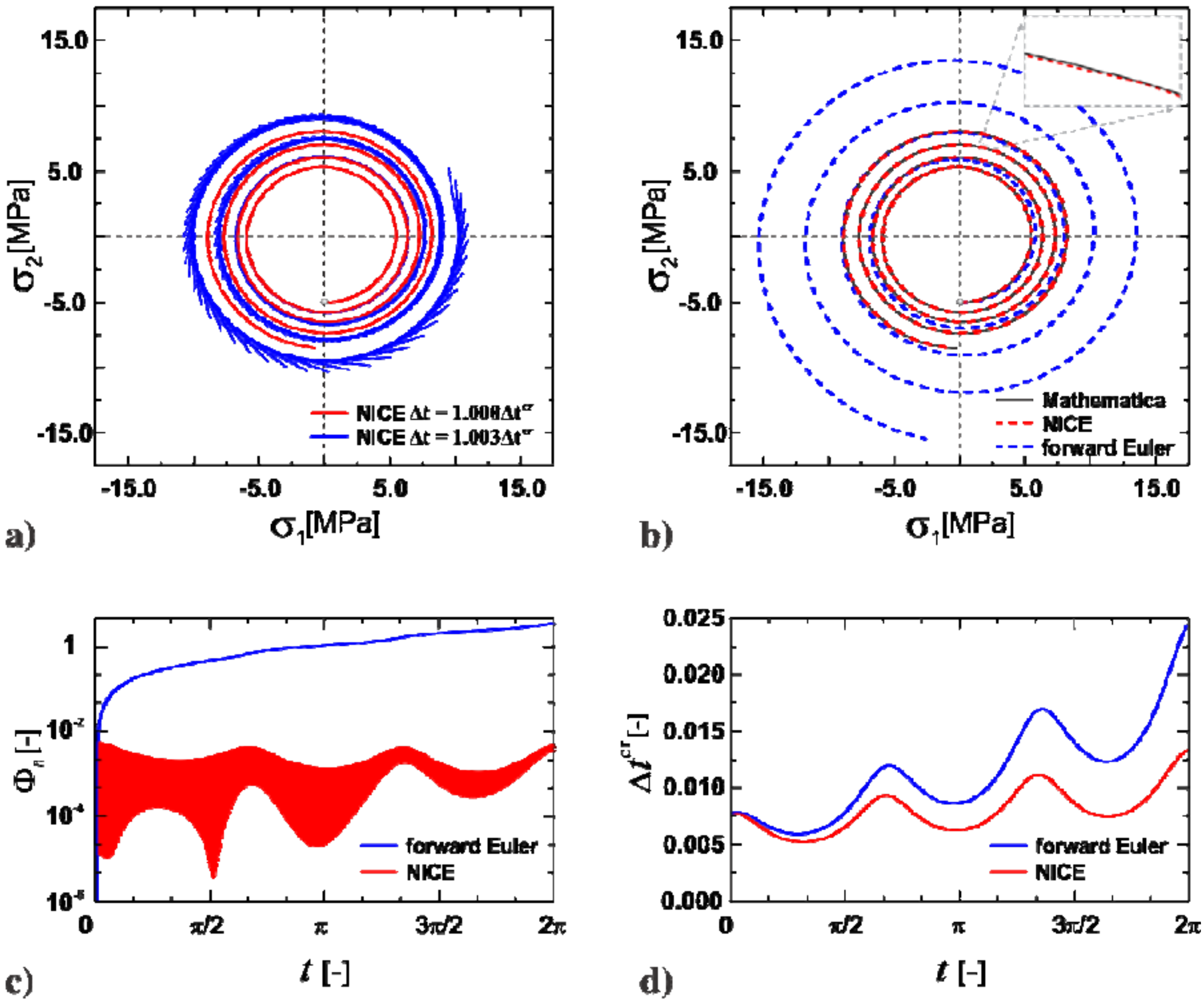

Figure 1: a) comparison of stable and unstable solution for NICE scheme, b) comparison of forward-Euler, NICE and the default Mathematica solution, c) drift from algebraic constraint, d) maximal stable increment size

Figure 1a shows that the maximal stable increment size Δt^{cr} is precisely determined in each increment and that the solution of the NICE scheme with $\Delta t = \Delta t^{\mathrm{cr}}$ is stable during the whole integration path. In Figure 1b, extremely superior accuracy of the NICE scheme in regard to forward-Euler scheme is proven. Also, one can see that the exhibited error of the NICE scheme regarding Mathematica solution is negligible. This is achieved with drift control from algebraic constraint, which is automatically included in the NICE scheme (see Figure 1c). Figure 1d depicts, that the maximal stable increment size for both schemes are the same at the beginning of integration. The difference in stable increment sizes during the integration is the consequence of drift from the algebraic constraint in case of forward-Euler scheme, which decrease curvature of the calculated stress path results in increase stable increment size.

6.2 Cup drawing simulation with NICE scheme and YLD2004-18p constitutive model

The above general formulation of the integration procedure is adopted to YLD2004-18p, a fairly complex advanced constitutive model which was developed to describe of plastic anisotropy [5],[6]. For implementation purpose the following vectors and matrices are defined in accordance with (3):

$$\mathsf{C} = \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix} \quad ; \quad \boldsymbol{\Sigma} = \begin{Bmatrix} \boldsymbol{\sigma} \\ \sigma_{\mathrm{Y}} \end{Bmatrix} \quad ; \quad \mathbf{R} = \begin{Bmatrix} \mathbf{C} \cdot \left. \dfrac{\partial \Phi}{\partial \boldsymbol{\sigma}} \right| \\ -\dfrac{H}{\sigma_{\mathrm{Y}}} \boldsymbol{\sigma} \cdot \dfrac{\partial \Phi}{\partial \boldsymbol{\sigma}} \end{Bmatrix} \tag{14}$$

Anisotropic yield function Φ is defined based on linear transformations of the stress deviator, where plastic anisotropy is described with 18 parameters. Their values have to be determined according to acquired experimental data. The entire model is defined in such a way, that it is able to take a general three dimensional stress state into account. A more detailed description of the model with the yield function formulation and corresponding derivatives is given in [5],[6].

The considered YLD2004-18p material model was implemented into ABAQUS/Explicit, via User Material Subroutine (VUMAT) interface platform, with the derived stability criterion (8) used to monitor the stable increment size during the integration process. As a case study, deep drawing simulation of a cylindrical cup was performed, where all material model parameters (plastic anisotropy and hardening parameters) and cup forming process parameters (tolling geometry, friction, blank dimension and blank holder force) were completely adopted from [6].

Due to symmetry, only a quarter of the entire model was considered in the simulation. Figure 2a and 2b show the respective field distribution of equivalent plastic deformation and ratio α_n of critical stable increment size vs. ABAQUS/Explicit default increment size at approximately 80% of punch depth. One can observe, that at this increment the values of α_n are all in the range higher than 10^2, thus the default ABAQUS/Explicit increment size is at least two decades smaller than the NICE stable increment. In Figure 2c even more rigorous criterion is used to study the possible stability issues in this case. Namely, the envelope of minimal ratios α_n over the entire loading path is presented on the initial blank geometry. With such an approach the minimal value of ratio α_n, occurred at anytime and anywhere during the whole simulation can be extracted. Obviously, the most critical location regarding the stability is the location between two ears, where maximal equivalent plastic deformation is achieved. By observing Figure 2d, in which the evolution of ratio α_n in this critical location during simulation is shown, we can see that the smallest critical stable increment remains approximately 80 times larger than the default ABAQUS/Explicit load increment size. Finally, we can conclude that stability of the NICE scheme is not problematic, since the stable strain increment size is usually in practical quasi-static applications much larger than the stable increment size for the integration of dynamic boundary value problem equations.

In Figures 2e and 2f the consistency condition fulfilment during the integration is addressed. Figure 2e shows the distribution of Φ at approximately 80% of punch depth, whereas Figure 2f shows the evolution of Φ in the most critical location during the simulation. As expected (see [1],[2]) the consistency condition is very accurately fulfilled during simulation and is not an issue at all.

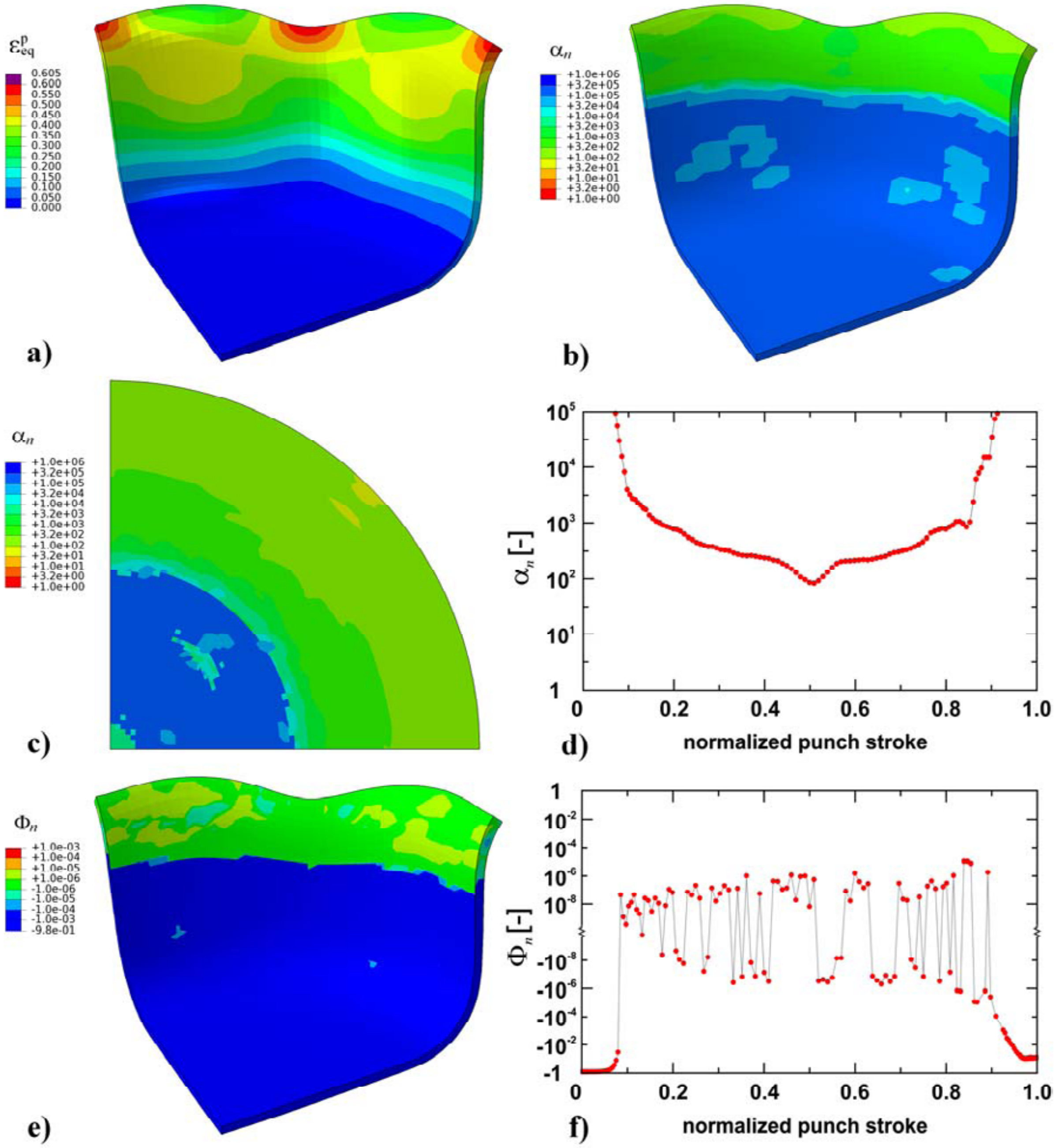

Figure 2: Deep drawing simulation of cylindrical cup using YLD2004-18p model: a) evolution of equivalent plastic strain, b) critical stable increment size vs. ABAQUS/Explicit default increment size, c) minimum envelope of all critical stable increment sizes over entire loading path, d) critical stable increment size vs. ABAQUS/Explicit default increment size evolution for the characteristic element over entire loading path e) drift from the algebraic constraint, f) drift from algebraic constraint for the characteristic element over entire loading path.

7 CONCLUSION

The main subject of the paper is the stability evaluation of the newly developed NICE integration scheme. It is shown on the deep drawing case that the stable increment size for the integration of constitutive equations with the NICE scheme is in quasi-static applications much larger than the stable increment size for the integration of dynamic boundary value problems.

Trough the examples exposed it was proven again that the proposed NICE integration scheme is a powerful integration tool for elasto-plastic constitutive models integration, in which the fulfilment of the consistency condition during the integration is radically improved in comparison with the classical forward-Euler scheme. In NICE scheme the accuracy of implicit (backward-Euler) and simplicity of explicit (forward-Euler) schemes has been successfully combined. In this regard the simplicity of the implementation, more than satisfactory accuracy and low time consumption of calculation, certainly outperforms the properties of other available schemes, including properties of the backward-Euler scheme.

The only open issue which remained unsolved until now was the stability limitation of NICE scheme. Analyses performed in the paper have shown that the stability of the NICE scheme is not problematic in quasi-static applications. The evaluated stable strain increment size is usually 10^{-4}–10^{-3} for the NICE scheme, thus, is more than sufficiently large for usual quasi-static simulations. Together with the stability evaluation and possibly control NICE integration scheme is a perfect choice for implementation of elasto-plastic constitutive models into dynamic explicit FEM programs.

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