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AUTOMATION OF ISOGEOMETRIC FORMULATION AND EFFICIENCY CONSIDERATION

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Abstract. This paper deals with automation of the isogeometric finite element formulation. Isogeometric finite element is implemented in AceGen environment, which enables symbolic formulation of the element code and the expressions are automatically optimized. The automated code is tested for objectivity regarding numerical efficiency in a numeric test with the Cooke membrane. This test shows that automatic code generation optimizes the isogeometric quadrilateral element with linear Bezier splines to the degree of only twelve percent overhead against standard displacement quadrilateral element of four nodes. Additionaly, the automated isogeometric element code is tested on a set of standard benchmark test cases to further test the accurancy and efficiency of the presented isogeometric implementation. The isogeometric displacement brick element with quadratic Bezier splines is in all tests compared to a collection of standard displacement element formulations and a selection of EAS elements. The presented results show superior behaviour of the isogeometric displacement brick element with quadratic Bezier splines for coarse meshes and best convergence rate with mesh refinement in most test cases. Despite all optimization of the element code the biggest disadvantage of the isogeometric model remains the time cost of the isogeometric analysis. Thus, when considering the ratio between solution error and solution time, the use of stable EAS elements, like TSCG12, remains preferable.

1 INTRODUCTION

Isogeometric analysis bridges the gap between fields of Computer Aided Design (CAD) and Computer Aided Engineering (CAE) by unifying geometrical description (see [2]). It merges the models for design and analysis into one, providing unprecedented accuracy and robustness across a wide array of applications. The reason why isogeometric analysis is not widely used in the engineering circles is its numerical complexity. The main goal of this article is to present automated and optimized isogeometric analysis and to provide the valid comparison of the convergence ratios and the solution times between classical finite element analysis (FEA) and isogeometric analysis.

2 IMPLEMENTATION OF ISOGEOMETRIC MODEL

Isogeometric finite element is implemented in AceGen environment. AceGen generates automated and optimized code of the isogeometric element for the isogeometric analysis. AceGen environment enables many favourable options, including automatic derivation of element residual and tangent matrix, symbolic formulation of the element code and automatic optimization of the expressions.

The basis for the isogeometric analysis are NURBS functions which can exactly represent the actual geometry and the same basis is chosen for the approximation of the unknown solution fields. The control points define the control mesh, which is the convex hull of the actual geometry. Position of the control points and additional weights control the actual geometry. NURBS functions are built as the cartesian product of the Bezier splines that are local to parameter space and defined by knot vectors. Knot vectors span the parameter space and define element domains, where NURBS basis functions are smooth. Across knots, NURBS basis functions will be $\mathcal{C}^{p-m}$, where p is the Bezier spline order and m is the multiplicity of the knot in question. The smoothness of the NURBS ensures higher accuracy compared to piecewise polynomial basis functions commonly used in FEA. For implementation of the isogeometric analysis the typical FEA flowchart is chosen. In preprocessing the boundary value problem is defined, the isogeometric control mesh is generated and all geometrical data is prepared. The isogeometric mesh generator makes mesh refinement via knot insertion, defines the elements and assigns nodes, nodal weights and local knot vectors to each element. This is possible due to the regular shape of the parameter space. Elements are mapped from parameter space onto parent element domain which is bi-unit square in 2D or bi-unit cube in 3D. Like in FEA the loop goes through all of the elements and builds element stiffness matrices within each element. Integration is performed by gaussian quadrature on element level. In each Gauss points are the NURBS basis functions evaluated. For evaluation of the Jacobi matrix of the mapping from physical domain to the parent element domain, the automatic differentiation procedure in AceGen is used. As the loop through the elements is over, the global stiffness matrix and force vector are assembled.

3 OBJECTIVITY REGARDING NUMERICAL EFFICIENCY

The formulation of the standard displacement quadrilateral element with four nodes (Q1) is the same as the formulation of the isogeometric displacement quadrilateral element with linear Bezier splines (Q1B1). The numerical efficiency of both mentioned finite element models as also of the standard displacement quadrilateral element with nine nodes (Q2) and the isogeometric quadrilateral elements with quadratic and cubic Bezier splines labeled as Q1B2 and Q1B3 respectively are compared in a two-dimensional numeric test with the Cooke membrane. Geometry and material data for the test are given in Fig. 1. The average computation time pro iteration needed for building the tangent matrix of the problem is noted as evaluation time. From the comparison of the evaluation time ratios given in Table 1 can be concluded that the automated implementation of the isogeometric model is objective regarding numerical efficiency because the isogeometric model Q1B1 has only twelve percent overhead in evaluation time against the standard displacement model Q1.

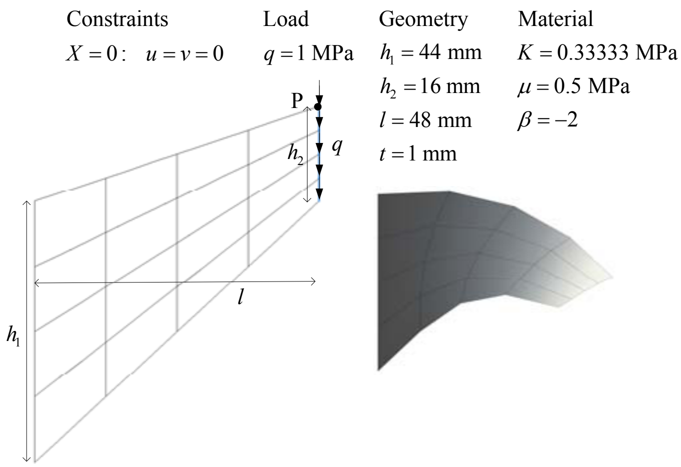

Figure 1: Cooke's membrane: system, load and material data and expected result.

FE	DOF	vert. displ.	eval. time	code size
Q1B1	40	36.6261 mm	1.12281	12238 bytes
Q1B2	60	45.011 mm	2.28856	24291 bytes
Q1B3	84	45.5034 mm	5.4093	42262 bytes
Q1	40	36.6261 mm	1	13236 bytes
Q2	144	45.4787 mm	2.656363	26480 bytes

Table 1: Results for Cooke membrane with 4×4 elements.

4 NUMERIC TESTS OF THE IMPLEMENTED ISOGEOMETRIC MODEL

The performance of the isogeometric model at large deformation problems is tested on a set of standard benchmark test cases and compared with other finite element codes. The benchmark test is designed to point out important properties of the proposed elements at large deformation problems. These include high accuracy, low mesh distortion sensitivity and locking free response for bending dominated situations, near incompressibility problems and in the limit very thin elements. The stability of different element formulations is also considered on the constant stress-strain tests cases. The three-dimensional isogeometric brick element with quadratic Bezier splines (H1B2) is in all tests compared to a standard displacement brick elements of eight and twenty nodes (H1 and H2 respectively), standard displacement tetrahedral elements of four and ten nodes (O1 and O2 respectively) as well as to a selected set of brick EAS elements (9-mode H1E9, 21-mode H1E21, 9-mode CG9 and 12-mode TSCG12), see [1], [3], [5] and [4].

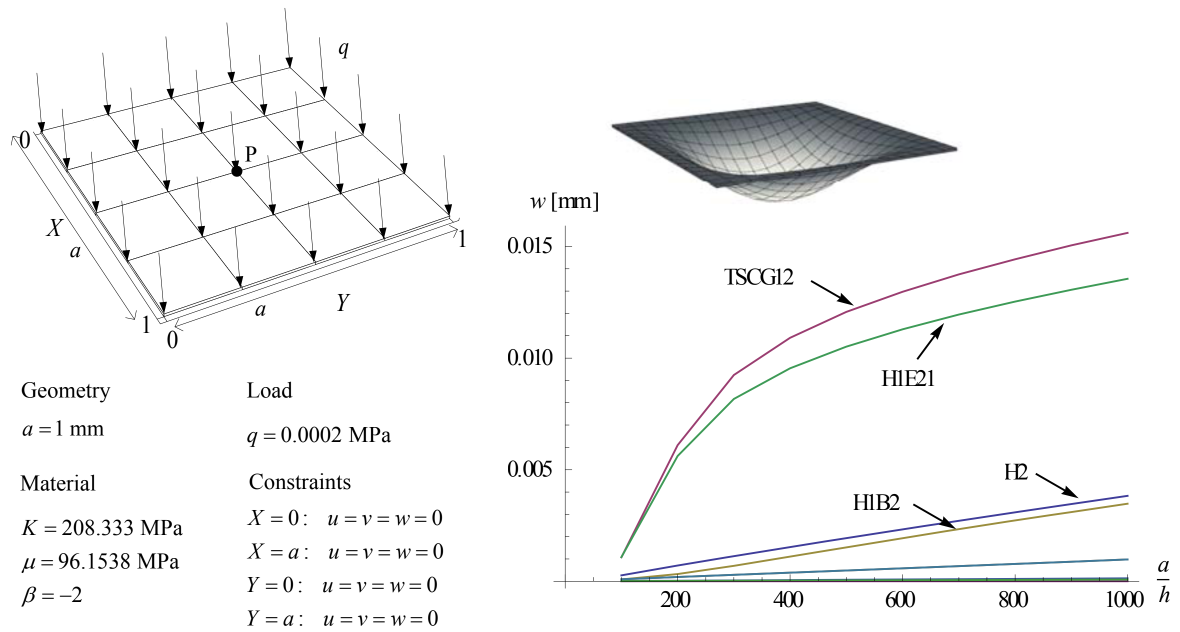

Figure 2: Thin plate: system, load and material data, typical deformed mesh and resulting deflection at the central point of the plate for all finite element formulations. TSCG12 element and the 21-mode H1E21 element are the only elements that do not exhibit severe shear locking behavior in this test

In Fig. 2 the geometry and material data are given for a test case with a thin clamped rectangular plate subjected to uniform loading. In Fig. 2 also the deflection at the center of the plate is presented for all elements as a function of the ratio between the length and the thickness of the plate. In this test the performance of finite element formulations is compared on the same element mesh of $4 \times 4 \times 1$ elements except for the isogeometric formulation the $5 \times 5 \times 1$ mesh is used to obtain an element node in the center of the plate. Best performance is achieved by the 12-mode element TSCG12 followed by the 21-mode element H1E21, all other elements exhibit severe shear locking behaviour.

5 FORMULATION EFFICIENCY CONSIDERATION

Isogeometric formulation efficiency is tested on numerical example of twisting a block around its axis. The upper surface of a block with fully constrained bottom surface is rotated around the center point for angle ϕ. Deplaning of the upper surface in is not restricted. Geometry and material data for the test are given in Fig. 3. The displacement of nodes on the upper surface is applied incrementally until the solution converges.

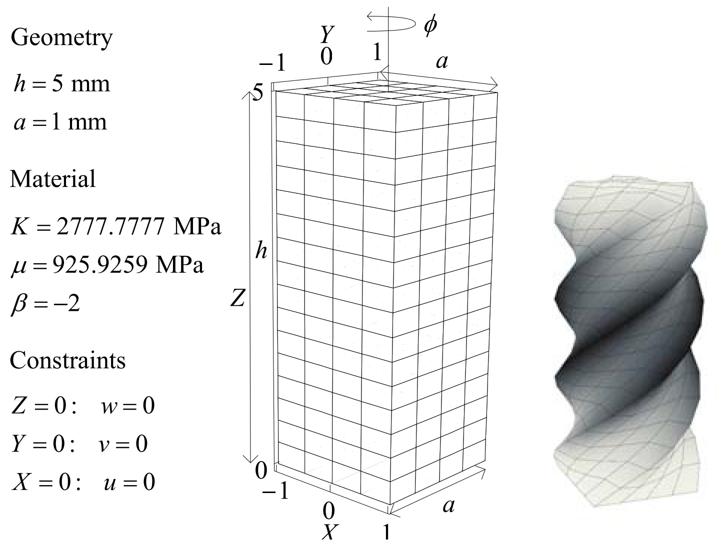

Figure 3: Twisting the block: geometry, load, material data and typical deformed mesh.

In Table 2 the maximal number of block turns ($\phi/2\pi$) with total solution time is given for various mesh densities and element formulations. Total solution time is a sum of total assembly time needed for building the tangent matrices and total linear solver time needed for solving the systems of equations. On Fig. 4 the total assembly time and total linear solver time are plotted with respect to the number of degrees of freedom for the isogeometric brick element with quadratic Bezier splines (H1B2), the standard brick displacement element with eight nodes (H1) and full quadratic standard brick displacement element with twenty-seven nodes (fH2). The results show exponential growth of linear solver time in case of the isogeometric formulation in comparison to linear growth of linear solver time for standard displacement element formulations. Assembly time for isogeometric formulation grows proportionately with the number of degrees of freedom but at much higher rate than for standard displacement elements. Here is to be noted that the

Mesh	DOF	H1		Mesh	DOF	H2	
		nr. of turns	timing			nr. of turns	timing
$2 \times 2 \times 16$	414	3.6956	1.5810 s	$1 \times 1 \times 8$	414	1.9382	3.8130 s
$4 \times 4 \times 16$	1150	3.6075	8.8789 s	$2 \times 2 \times 8$	1150	2.1439	12.4620 s
$5 \times 5 \times 20$	2088	4.3424	26.7229 s	$2 \times 2 \times 14$	2050	2.0471	24.7900 s
$6 \times 6 \times 28$	4018	2.8728	49.1809 s	$3 \times 3 \times 14$	4018	1.7428	58.9359 s
$7 \times 7 \times 43$	8128	2.1872	97.8720 s	$4 \times 4 \times 17$	8100	1.4902	123.5119 s
Mesh	DOF	H1B2					
		nr. of turns	timing				
$2 \times 2 \times 8$	400	2.5947	12.5600 s				
$3 \times 3 \times 12$	925	2.6486	41.9469 s				
$4 \times 4 \times 18$	1980	2.1874	120.2559 s				
$5 \times 5 \times 27$	4018	1.7538	289.7069 s				
$7 \times 7 \times 33$	8100	1.5683	685.8650 s				

Table 2: Maximal number of block turns ($\phi/2\pi$) for converged solution of the twisting test with solution time.

standard brick displacement element with twenty-seven nodes (fH2) has the same number of integration points as the isogeometric brick element with quadratic Bezier splines.

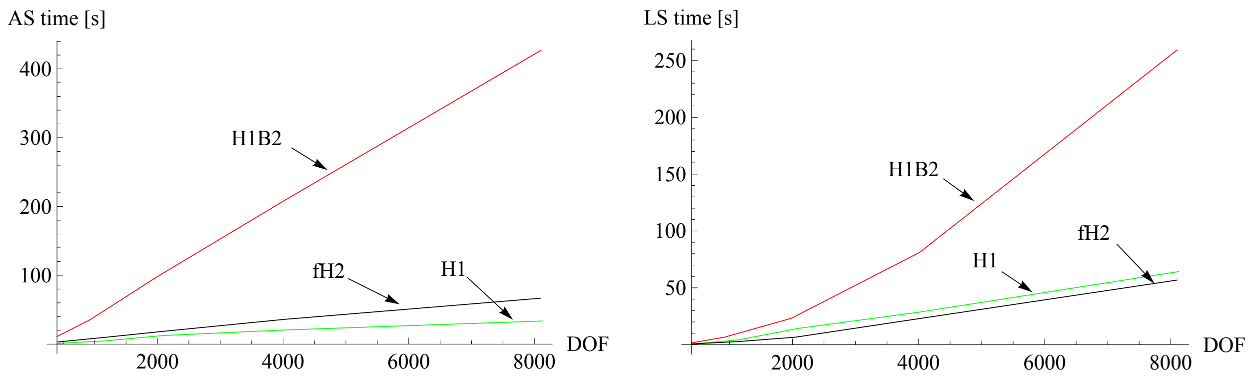

Figure 4: Assembly time (AS) and linear solver time (LS) for the chosen element formulations in the twisting block test.

6 CONCLUSIONS

The isogeometric finite element formulation is automated and optimized with automatic code generator AceGen. The evaluation time obtained in the numeric test with the Cooke membrane shows objectivity of the automated isogeometric element formulation regarding numerical efficiency. The automated isogeometric element code is additionally tested on a set of standard benchmark test cases. The three-dimensional isogeometric brick element with quadratic Bezier splines is in all tests compared to other finite element formulations implemented in AceGen. Finite element codes taken into consideration are isogeometric displacement brick element with quadratic Bezier splines, standard displacement brick elements with eight and twenty nodes, standard displacement tetrahedral elements with four and ten nodes, standard enhanced EAS brick elements with nine and twenty-one enhanced modes (see [1, 5]) and two different modified EAS brick elements with nine enhanced modes (see [3, 4]). The results obtained by the benchmark test show best convergence rate for the isogeometric formulation regarding number of degrees of freedom. The isogeometric element behaves exceptionally well in the test of formulation stability. On the other hand exhibits the isogeometric element severe shear locking in the case of in limit very thin plate. Despite all optimization, time efficiency tests show isogeometric analysis to be the most time consuming. This is shown in a numerical test of twisting a block around its axis. Assembly time for global tangent matrix and residual and linear solver time for isogeometric displacement brick element with quadratic Bezier splines grow at much higher rate then for other considered finite element formulations. From obtained results follows that the ratio between solution error and solution time is much better for considered EAS brick elements than for presented isogeometric formulation.

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